Stat/For/Hort 571 **Nordheim and Zhu** **October 10, 2000**

Midterm I

Name:____

For the section that you *attend* please indicate:
Instructor:(circle one) Nordheim Zhu

TA: (circle one) Song P. Yan M. Yuan

Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.
2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.
3. To get full credit, you must show your work. Partial credit will be awarded.
4. Do not dwell too long on any one question. Answer as many questions as you can.
5. Note that some questions have multiple parts. For some questions, these parts are independent; in such cases you can work, for example, on part (b) or (c) separately from part (a).

For graders' use:

Question	Possible Points	Score
1	18	
2	24	
3	20	
4	22	
5	16	
Total	100	

1. (a) A random sample of 10 small ponds in southeastern Minnesota was selected and the pH was determined for each. The recorded values are:

   7.4 6.7 7.2 6.1 6.7 7.9 6.8 6.4 6.6 7.0

   Make a useful display of these data. Comment on the display. Find the sample median.

   (b) It is known that the pH values for ponds in southwestern Maine are distributed approximately normally with mean 6.45 and variance 0.30. Suppose that a random sample of 15 ponds is selected. If $\bar{Y}$ is the random variable for the sample mean for this random sample, find $P(\bar{Y} < 6.60)$.

2. (a) Let $Y_1 \sim B(8, 0.7)$. Find $\text{Prob}(Y_1 \geq 7)$.

   (b) Let $Y_2 \sim N(5.6, 1.68)$. Find $\text{Prob}(Y_2 \geq 7)$.

   (c) Verify that the means of Y_1 and Y_2 are the same and that the variances are also the same.

   (d) Taking into account the equalities verified in (c), if your answers to (a) and (b) are the same, explain why. If your answers are different, explain why.

3. (a) For a random sample of size 9 from $N(30, 25)$, let $S_1^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2$. Find c so that $P(S_1^2 > c) = 0.01$.

   (b) Take a second random sample of size 9, independent of the first, from the same distribution as in (a). Using the value of c you computed in (a), find the probability that S_1^2 (from the first random sample) and S_2^2 (from the second random sample) are both greater than the value c.

4. Consider a lottery game in which you have a chance to receive \$0, \$1, or \$100. The probabilities for each possible result are as follows: $p(0) = 0.80$; $p(1) = 0.18$; $p(100) = 0.02$.

   (a) Suppose that you need to pay \$2 each time you play this game. In the long run would the company (or state) running this lottery game make money or lose money? (Hint: Think in terms of expectation of a random variable.)

   (b) Suppose you play this game 100 times (independently). Let $\bar{Y}$ be the mean amount of money you receive. Find $P(\bar{Y} \geq 2.00)$.

   (c) What assumption did you have to make to perform the calculation in part (b)? Is the assumption reasonable?

5. Consider a scientific study to assess the strength of a new kind of artificial ligament. Suppose that n independent artificial ligaments are tested for rupture where the probability of rupture for a single ligament is 0.4. Assume that the binomial assumptions hold. Let Y = number of ligaments that rupture. Let $\hat{p} = \frac{Y}{n}$, the proportion of ligaments that rupture. Find n so that $P(\hat{p} > 0.45) = 0.05$. (Hint: Assume n is large enough so that the normal approximation holds. After you calculate n, justify the normal approximation.)