Midterm I

Name:

For the section that you *attend* please indicate:
Instructor:(circle one) Clayton   Gangnon

TA: (circle one) Cheng   Li   Song
   Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.

2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating *clearly* that you have done so.

3. To get full credit, you must show your work. Partial credit will be awarded.

4. Do not dwell too long on any one question. Answer as many questions as you can.

5. Note that some questions have multiple parts. For some questions, these parts are independent, and so you can work on part (b) or (c) separately from part (a).

For graders' use:

Question	Possible Points	Score
1	20	
2	16	
3	22	
4	22	
5	20	
Total	100	

1. An experiment was conducted to determine the average bone density of old dogs (more than 8 years old). 18 old dogs were randomly sampled and the density of the femoral head (a portion of the leg bone) was determined. The values ranged from 18 to 62 and are summarized in the stem and leaf display below.

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    Old Dogs


    1|8
    2|23
    3|4789
    4|01669
    5|1479
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6|02
7|
8|
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(a) Comment briefly on the shape of this stem and leaf display and find the median bone density for this group of dogs.

(b) A second experiment was conducted that involved young (less than 3 year old) dogs. Their bone densities were:

77 54 78 76 82 65 72 76 60 54 60 62 69 70 64 71 77

Construct a stem and leaf display for the bone densities of the young dogs. (Construct your display beside the display given above for old dogs.)

(c) Determine the median bone density for the young dogs.

(d) *Without doing any calculations* indicate which group of dogs will have the larger sample mean. Explain your answer.

(e) *Without doing any calculations* indicate which group of dogs will have the larger sample variance. Explain your answer.

2. An experiment was conducted to study the food intake of bark beetles. It is known that the food intake of a bark beetle on a typical day is normally distributed, with a population mean of 16 and a population variance of 7.8. In an experiment, a researcher plans to measure the food intake of 18 independently sampled bark beetles and calculate S^2.

As best as you can using tables, provide bounds for $P(3.4 < S^2 < 9.4)$.

3. The height of a randomly sampled mature wheat plant follows a normal distribution with a mean of 90 cm and a variance of 8 cm^2. Suppose that a single plant is sampled at random; denote its height by X.

(a) Determine $P(89 < X < 93)$.

(b) If a plant has height X and is within the size range $89 < X < 93$ it is said to be acceptable for harvest. In an experiment, 100 plants were sampled at random. By making the appropriate assumptions, determine the probability of having 40 or more plants acceptable for harvest.

(c) What assumptions did you have to make to answer part (b)? Justify them as best you can.

4. (a) Suppose we have 240 independent observations $X_1, X_2, \ldots$ from $N(20, 180)$. What is the probability that $\bar{X} < 19$?

(b) Consider the random variable W whose distribution is given by: $P(W = 0) = 0.225$, $P(W = 20) = 0.550$, and $P(W = 40) = 0.225$.

  i. Calculate the expected value, $E(W)$.
  ii. Calculate the variance, $\text{Var}(W)$.
  iii. In an experiment, 240 observations were sampled at random from the distribution of W. Consider calculating the sample mean of these 240 observations; call that mean $\bar{W}$. What is the probability that $\bar{W} < 19$?

5. An experimental colony of ground squirrels has 7 squirrels in it. Ground squirrels usually hibernate, but there is a 0.2 chance that a given squirrel will stay awake (not hibernate). It is known that, in terms of hibernation, ground squirrels act independently of each other.

(a) What is the probability that exactly 2 ground squirrels will stay awake.

(b) Regardless of the actions of the squirrels, there is a base cost of $28 to maintain the colony of ground squirrels. Moreover, for each squirrel that stays awake, it costs additional money to maintain it. In particular, it costs $9 to maintain each squirrel that stays awake. If T represents the total cost of the experiment, what is the mean and variance of T? (Hint: $T = 28 + 9 \times X$.)