Stat/For/Hort 571
Midterm I, Fall 2001
Brief Solutions

1. (a) The display appears roughly symmetric or bell-shaped, centered around 46. Median $x_{[0.50]} = 43.5$.

   (b) The stem and leaf displays for the two groups are

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Old Dogs          Young Dogs

1|8               1|
2|23              2|
3|4789            3|
4|01669           4|
5|1479            5|44
6|02              6|002459
7|                7|01266778
8|                8|2
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   (c) Median $y_{[0.50]} = 70$.

   (d) Young dogs. We can see this because the stem and leaf display for young dogs is generally shifted higher than for old dogs. (It is important to draw both displays using the same stems.)

   (e) Old dogs. This is apparent because the stem and leaf display for the old dogs is more spread out than for young dogs.

2. Since $V^2 = 17S^2/7.8 \sim \chi^2_{17}$,

$$\begin{aligned} P(3.4 < S^2 < 9.4) &= P(\frac{17(3.4)}{7.8} < V^2 < \frac{17(9.4)}{7.8}) \\ &= P(7.41 < V^2 < 20.49) \\ &= P(V^2 > 7.41) \\ &\quad -P(V^2 > 20.49). \end{aligned}$$

From Table B, $0.975 < P(V^2 > 7.41) < 0.990$ and $P(V^2 > 20.49) = 0.250$.
So, $0.725 < P(3.4 < S^2 < 9.4) < 0.740$.

3. (a) Since $X \sim N(90, 8)$,

$$\begin{aligned} P(89 < X < 93) &= P(\frac{89-90}{\sqrt{8}} < Z < \frac{93-90}{\sqrt{8}}) \\ &= P(-0.35 < Z < 1.06) \\ &= 1 - [P(Z < -0.35) \\ &\quad +P(Z > 1.06)] \\ &= 0.4922 \end{aligned}$$

   (b) Let Y = number of plants acceptable for harvest. Then $Y \sim B(100, 0.4922)$. Since $E(Y) = p = 49.22$ and $\text{Var}(Y) = np(1-p) = 24.99$, $Y \sim N(49.22, 24.99)$ approximately. So,

$$\begin{aligned} P(Y \geq 40) &\approx P(Z \geq \frac{40-49.22}{\sqrt{24.99}}) \\ &= P(Z \geq -1.84) \\ &= 1 - P(Z < -1.84) \\ &= 0.9671. \end{aligned}$$

   (c) The assumptions of a binomial distribution are basically provided (random sample, constant p, outcomes of either success or failure for each plant). In addition, to use the normal approximation to the binomial, we need to check that the sample size is sufficiently large. With $np = 49.22 \geq 5$ and $n(1-p) = 50.78 \geq 5$ it follows that the normal is a good approximation to the binomial.

4. (a) Since $\bar{X} \sim N(20, 180/240)$,

$$\begin{aligned} P(\bar{X} < 19) &= P(Z < \frac{19-20}{\sqrt{0.75}}) \\ &= P(Z < -1.15) \\ &= 0.1251. \end{aligned}$$

   (b) i. $E(W) = 0\times0.225+20\times0.550+40\times0.225 = 20$.

   ii. $\text{Var}(W) = (0-20)^2 \times 0.225 + (20-20)^2 \times 0.550 + (40-20)^2 \times 0.225 = 180$.

   iii. Assuming $n = 240$ is large enough for the CLT to provide a good approximation, $\bar{W} \sim N(20, 180/240)$. Hence, by the result in 4(a), $P(\bar{W} < 19) \approx 0.1251$. (To get full credit you had to note the use of the CLT.)

5. (a) Let X = the number of ground squirrels that stay awake. Then $X \sim B(7, 0.2)$, and

$$\begin{aligned} P(X = 2) &= \frac{7!}{2!5!}(0.2)^2(0.8)^5 \\ &= 0.275. \end{aligned}$$

   (b) From the properties of the binomial distribution, $E(X) = 7 \times 0.2 = 1.4$ and $\text{Var}(X) = 7 \times 0.2 \times 0.8 = 1.12$. From general properties of expectations and variances, $E(T) = 28 + 9 \times E(X) = 40.6$ and $\text{Var}(T) = 9^2 \times \text{Var}(X) = 90.72$.

Grade Distribution

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  100:9
90-99:53
80-89:40
70-79:27          mean = 83.2, median = 87
60-69:10
50-59:10
  <50:4
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