## Midterm I

Name: $\qquad$
For the section that you attend please indicate:
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## Instructions:

1. This exam is open book. You may use textbooks, notebooks, class notes, and a calculator.
2. Do all your work in the spaces provided. If you need additional space, use the back of the preceding page, indicating clearly that you have done so.
3. To get full credit, you must show your work. Partial credit will be awarded.
4. Do not dwell too long on any one question. Answer as many questions as you can.
5. Note that some questions have multiple parts. For some questions, these parts are independent, and so you can work on part (b) or (c) separately from part (a).

For graders' use:

| Question | Possible Points | Score |
| :---: | :---: | :---: |
| 1 | 16 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 24 |  |
| 5 | 20 |  |
| Total | 100 |  |

1. A study was conducted of the dry matter intake of cows from a Wisconsin herd which were fed a standard ration. The data ranged in value from 18.4 to 26.1 and are summarized in the stem and leaf display below:
$18 \mid 4$
19|1568
20|2237
21|36
22|9
23|
24|4
25|
26|1
(a) Comment briefly on the shape of this data set.
(b) Find the sample mean dry matter intake and the median dry matter intake for these cows.
(c) One of the boxplots below is the correct boxplot for these data. Choose the plot that you believe is the correct plot for these data and provide a brief explanation of your choice.

2. (a) A random sample of size 28 will be taken from a normal distribution with mean $\mu=18$ and variance $\sigma^{2}=49$ and the sample variance $S^{2}=\frac{1}{n-1} \sum_{i=1}^{28}\left(X_{i}-\bar{X}\right)^{2}$ will be calculated. Give bounds for the value of $P\left(S^{2}>90\right)$.
(b) The experiment in part (a) will be repeated, but the sample size will be increased to 280 . The following italicized statement is either True or False. Indicate whether it is True or False and explain your choice. For this new sample size of 280, the value of $P\left(S^{2}>90\right)$ will be smaller than it was in part (a).
(Hint: You do not need to do any calculations to answer part (b).)
3. The amount of protein excreted in the urine of a cat over a 24 hour period depends, in part, on its weight. This problem concerns cats that weigh 4 kg . If a cat is sampled at random from this population, the amount of protein that it will excrete is normally distributed, with a mean of 68 milligrams, and a variance of 900 milligrams ${ }^{2}$.
(a) What is the probability that, for a randomly sampled cat, its protein excretion will lie between 70 and 90 milligrams?
(b) Twelve cats are sampled at random from this population. What is the probability that, for these twelve cats, the sample mean amount of excreted protein lies between 60 and 70 milligrams?
4. A study was conducted of the number of "homes" occupied by a particular species of hermit crab over its lifespan. It was found that, if $X$ represents the number of homes occupied by an individual hermit crab, then $X$ has a distribution as follows:

| x | 1 | 2 | 10 |
| :---: | :---: | :---: | :---: |
| $\mathrm{p}(\mathrm{x})$ | .4 | .3 | .3 |

(a) For a given hermit crab, what is the expected number of homes that it occupies, and what is the variance of the number of homes that it occupies?
(b) Hermit crabs are called "unitary" if they occupy exactly one home during their lifespan. 75 hermit crabs were sampled at random, and it was observed, for each crab, whether it was unitary or not. For these 75 crabs, find, approximately, the probability that 28 or fewer crabs were unitary.
(c) State the assumptions that you made for part (b), and verify them, if possible.
5. (a) In a large plantation of blue spruce trees, the probability that a given tree will be damaged by spruce bud worm is 0.7 , and the damage on one tree is independent of damage on another tree. A total of 8 trees were randomly chosen to be studied for damage. What is the probability that exactly 6 of the 8 trees are damaged?
(b) In an extension of the above study, 4 plantations were examined. In each plantation, several trees were sampled and examined for disease. It is known that damage on trees is independent from tree to tree, although climate effects result in a different probability of disease for trees in the different plantations. The following information is available:

| Plantation | Probability of Damage for a Tree |
| :---: | :---: |
| A | .8 |
| B | .75 |
| C | .6 |
| D | .7 |

Suppose that $5,12,9$, and 8 trees were randomly sampled from plantations $\mathrm{A}, \mathrm{B}, \mathrm{C}$, and D respectively. Of the 34 trees that were observed, let $X$ represent the total number of trees that are damaged.
Find $E(X)$ and $\operatorname{Var}(X)$.

