Stat/For/Hort 571
Midterm I, Fall 2002
Brief Solutions

1. (a) The display appears skewed.

   (b) Mean $\bar{x} = 21.007$.
   Median $x_{[0.50]} = \frac{20.2+20.3}{2} = 20.25$.

   (c) B. We can see this because 50% of data is between 19.5 and 21.8 (inside the box).

2. (a) Since $V^2 = 27S^2/49 \sim \chi^2_{27}$,

$$\begin{aligned} P(S^2 > 90) &= P(V^2 > \frac{27(90)}{49}) \\ &= P(V^2 > 49.59) \end{aligned}$$

   From Table B, $0.005 < P(V^2 > 49.59) < 0.01$.

   (b) The statement is true. S^2 is centered at σ^2. As the sample size increases, S^2 becomes a better estimate of σ^2. More specifically, the distribution of S^2 becomes more tightly centered on $\sigma^2 = 49$, and so $P(S > 90)$ becomes smaller.

3. (a) Since $X \sim N(68, 900)$,

$$\begin{aligned} & P(70 < X < 90) \\ =\ & P(\frac{70-68}{\sqrt{900}} < Z < \frac{90-68}{\sqrt{900}}) \\ =\ & P(0.07 < Z < 0.73) \\ =\ & P(Z > 0.07) - P(Z > 0.73) \\ =\ & 0.4721 - 0.2327 \\ =\ & 0.2394 \end{aligned}$$

   (b) Since $\bar{X} \sim N(68, 900/12) = N(68, 75)$,

$$\begin{aligned} & P(60 < X < 70) \\ =\ & P(\frac{60-68}{\sqrt{75}} < Z < \frac{70-68}{\sqrt{75}}) \\ =\ & P(-0.92 < Z < 0.23) \\ =\ & 1 - 0.4090 - 0.1788 \\ =\ & 0.4122 \end{aligned}$$

4. (a) $E(X) = 1 \times 0.4 + 2 \times 0.3 + 10 \times 0.3 = 4$, and

$$\begin{aligned} & Var(X) \\ =\ & (1-4)^2 \times 0.4 + (2-4)^2 \times 0.3 + (10-4)^2 \times 0.3 \\ =\ & 15.6. \end{aligned}$$

   (b) Since $X \sim B(75, 0.4)$,

$$\begin{aligned} P(X \le 28) &\approx P(Z \le \frac{28 - 75 \times 0.4}{\sqrt{75 \times 0.4 \times 0.6}}) \\ &= P(Z \le -0.47) \\ &= P(Z \ge 0.47) \\ &= 0.3192 \end{aligned}$$

   (c) $np = 75 \times 0.4 > 5$ and $n(1-p) = 75 \times 0.6 > 5$.

5. (a) Let Y = the number of trees that are damaged. Then $Y \sim B(8, 0.7)$, and

$$\begin{aligned} P(Y = 6) &= \frac{8!}{6!2!}(0.7)^6(0.3)^2 \\ &= 0.296. \end{aligned}$$

   (b) Let X_A be the number of trees sampled from plantation A that are damaged. Define X_B, X_C, X_D similarly. Then

$$X = X_A + X_B + X_C + X_D,$$

   and

$$\begin{aligned} X_A &\sim B(5, 0.8) \\ X_B &\sim B(12, 0.75) \\ X_C &\sim B(9, 0.6) \\ X_D &\sim B(8, 0.7). \end{aligned}$$

   Therefore,

$$\begin{aligned} & E(X) \\ =\ & E(X_A) + E(X_B) + E(X_C) + E(X_D) \\ =\ & 5 \times 0.8 + 12 \times 0.75 + 9 \times 0.6 + 8 \times 0.7 \\ =\ & 24. \end{aligned}$$

   Note: Although you can get the same conclusion using $E(X) = \sum xp(x)$, this formula is not applicable here. Specifically, the probabilities listed in the problem (.8, .75, .6, .7) do not correspond to a single random variable X (the probabilities do not add up to 1).
   Since X_A, X_B, X_C, and X_D are independent, we have

$$\begin{aligned} & V(X) \\ =\ & V(X_A) + V(X_B) + V(X_C) + V(X_D) \\ =\ & 5 \times 0.8 \times 0.2 + 12 \times 0.75 \times 0.25 \\ & +9 \times 0.6 \times 0.4 + 8 \times 0.7 \times 0.3 \\ =\ & 6.89. \end{aligned}$$

Grade Distribution

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  100:7
 90-99:59
 80-89:32
 70-79:21         mean = 85.95, median = 90
 60-69:5
 50-59:4
   <50:3
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