Stat/For/Hort 571
Midterm I, Fall 2003
Brief Solutions

1. (a) A stem-and-leaf display:

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0 | 4
0 | 78
1 | 1344
1 | 67
2 | 1
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   (b) Since $y_{0.25} = 8$ and $y_{0.75} = 16$, IQR = 8.

2. Let A denote the event that Habibi barks and let B denote the event that Spotty barks. We have $P(A) = 0.5$ and $P(B) = 0.7$.

   (a) The event that both dogs bark corresponds to the event of "A and B". By independence, $P(A \text{ and } B) = P(A) \times P(B) = 0.35$.

   (b) The event that at least one dog barks corresponds to the event of "A or B". Thus $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.5 + 0.7 - 0.35 = 0.85$.

   (c) The probability distribution of Y is:

y	0	1	2
$p(y)$	0.15	0.5	0.35

   Note that from (a), $P(Y = 2) = 0.35$, $P(Y = 1) = P(Y \geq 1) - P(Y = 2) = 0.85 - 0.35 = 0.5$, and $P(Y = 0) = 1 - P(Y \geq 1) = 1 - 0.85 = 0.15$. Thus $E(Y) = 0 \times 0.15 + 1 \times 0.5 + 2 \times 0.35 = 1.2$ and $Var(Y) = (0 - 1.2)^2 \times 0.15 + (1 - 1.2)^2 \times 0.5 + (2 - 1.2)^2 \times 0.35 = 0.46$.

3. (a) Since $V^2 = \dfrac{(10-1)S^2}{9} \sim \chi^2_9$,

$$\begin{aligned} & P(8.34 \leq S^2 \leq 14.68) \\ = \ & P\left(\frac{(10-1)8.34}{9} \leq V^2 \leq \frac{(10-1)14.68}{9}\right) \\ = \ & P(8.34 \leq V^2 \leq 14.68) \\ = \ & P(V^2 \geq 8.34) - P(V^2 \geq 14.68) \\ = \ & 0.5 - 0.1 = 0.4. \end{aligned}$$

   (b) Since $\bar{Y} \sim N(20, \frac{9}{10})$, $Z = \dfrac{\bar{Y} - 20}{\sqrt{\frac{9}{10}}} \sim N(0,1)$. We know that $P(Z \leq 1.645) = 0.95$, thus

$$\begin{aligned} 0.95 &= P\left(\frac{\bar{Y} - 20}{\sqrt{\frac{9}{10}}} \leq 1.645\right) \\ &= P(\bar{Y} \leq 1.645 \times \sqrt{\frac{9}{10}} + 20) \end{aligned}$$

   and $y^* = 1.645 \times \sqrt{\frac{9}{10}} + 20 = 21.56$.

   (c) False. Since $Var(\bar{Y}) = \frac{9}{n}$, by the same argument as in (b), $y = 1.645 \times \sqrt{\frac{9}{n}} + 20$. The value y is smaller than y^*, because the variance $Var(\bar{Y})$ is smaller, when the sample size n is larger than 10.

4. (a) Let Y denote the number of infected plants among the 15 plants. Then Y is binomial with $n = 15$ and $p = 0.1$. The probability that at least 1 plant is infected is

$$\begin{aligned} P(Y \geq 1) &= 1 - P(Y < 1) \\ &= 1 - P(Y = 0) \\ &= 1 - (0.9)^{15} = 0.794. \end{aligned}$$

   Note that by using the complement of the event of interest, we could avoid computing $P(Y \geq 1) = P(1) + P(2) + \cdots + P(15)$.

   (b) Here W is binomial with $n = 150$ and $p = 0.1$. The mean and variance of W are $np = 15$ and $np(1-p) = 13.5$. Since $np = 15 \geq 5$ and $n(1-p) = 135 \geq 5$, we can use $W_{\text{NA}} \sim N(15, 13.5)$. Since

$$\begin{aligned} 0.90 &= P(W \geq a) \\ &\approx P(W_{\text{NA}} \geq a) \\ &= P\left(\frac{W_{\text{NA}} - 15}{\sqrt{13.5}} \geq \frac{a - 15}{\sqrt{13.5}}\right) \\ &= P(Z \geq \frac{a-15}{\sqrt{13.5}}). \end{aligned}$$

   and $P(Z \geq -1.282) = 0.90$, we set $\dfrac{a-15}{\sqrt{13.5}} = -1.282$. Thus $a = 15 - 1.282 \times \sqrt{13.5} = 10.29$, or $a = 10$.

Grade Distribution

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  100:21
90-99:59
80-89:22
70-79:19          mean = 88, median = 92
60-69:7
50-59:4
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