Stat/For/Hort 571 – Midterm I, Fall 99 – Brief Solutions

1. (a) The stem and leaf display is useful for a small data set such as this.

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 8.| 30
 9.| 91
10.| 9953
11.| 569
12.| 816
13.| 2
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The display appears to be roughly symmetrical, centered at about 11.

(b) The median is the middle value (8^{th}) of the 15 ordered observations.
Median = 10.9
Mean = $\Sigma y_i / n = 163.6/15 = 10.91$
Variance = $[1/(n-1)][\Sigma y_i^2 - (\Sigma y_i)^2/n] =$
$(1/14)[1819.34 - (163.6)^2/15] = 2.501$

(c) We want $P(S^2 < 2.501)$. We must convert this to a probability statement about $V^2 = (n-1)S^2/\sigma^2$ because our tables are in terms of V^2. Thus,

$$P\left(V^2 < \frac{(n-1)2.5}{\sigma^2}\right) = P\left(V^2 < \frac{14\times 2.5}{4}\right) = P\left(V^2 < 8.75\right)$$

We compute $1 - P(V^2 > 8.75)$ where V^2 has 14 df. Then $.10 < P(S^2 < 2.5) < .25$. [Draw a picture!]

2. Let μ be the mean height of the pin oak seedlings. Then

H_0: $\mu = 1.3$ m and H_A: $\mu \neq 1.3$ m

We find the mean height is $8.7/8 = 1.0875$ and we are given that $\sigma^2 = 0.10$. Thus

$$z = \frac{y - \mu}{\sigma/\sqrt{n}} = \frac{1.0875 - 1.3}{.10/\sqrt{8}} = -1.901$$

The p-value is $2P(Z \le -1.901) = 2P(Z \ge 1.901) = 2 \times .0287 = .0574$. Thus, the results are significant at 10% but not at 5% or 1%.

3. (a) We want to find y_* such that $P(\overline{Y} < y_*) = 0.8$ when $\overline{Y} \sim N(3.62, .025/18)$. Note that $P(\overline{Y} < y_*) = .8$ implies $P(\overline{Y} > y_*) = .2$. From Table A we see that $P(Z > .84) = .2$.

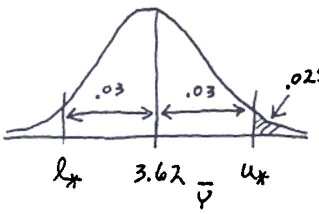
Thus $\frac{y_* - \mu}{\sigma/\sqrt{n}} = .84$. Solving for y_*, we find

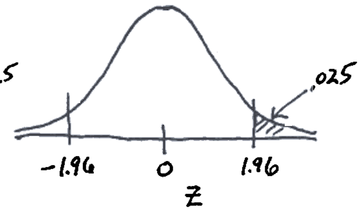
$y_* = .84 \times (\sqrt{.025}/\sqrt{18}) + 3.62 = 3.651$. [Draw a picture!]

(b) We want $P(l_* \le \overline{Y} \le u_*) = .95$. Converting to z-scores

$P\left(\frac{l_* - \mu}{\sigma/\sqrt{n}} \le Z \le \frac{u_* - \mu}{\sigma/\sqrt{n}}\right) = .95$. At the upper end of the interval

we have $P\left(Z \ge \frac{u_* - \mu}{\sigma/\sqrt{n}}\right) = .025$ and we know $u_* - \mu = .03$ (half of the interval).





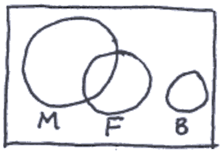
Using Table A we find $P(Z \ge 1.96) = .025$ so we set

$$\frac{u_* - \mu}{\sigma/\sqrt{n}} = \frac{.03}{\sqrt{.025}/\sqrt{n}} = 1.96 \text{ or } \sqrt{n} = (1.96\times\sqrt{.025})/.03 = 10.33.$$

Then n = 107.

4. First consider the entire sample space (I = in, O = out). Remember that Butch will never be in the room with a cat. Here Y is the number of animals in the living room.

M	F	B	Y	probability
I	I	I	3	0
I	I	O	2	.6 × .4 = .24
I	O	I	2	0
O	I	I	2	0
I	O	O	1	.6 × (1-.4) = .36
O	I	O	1	(1-.6) × .4 = .16
O	O	I	1	.2
O	O	O	0	1 – (.24 + .36 + .16 + .2) = .04



(a) Because the cats behave independently, the probability that Frisky is in the room is not affected by Mittens' behavior, and the probability is 0.4.

(b) This probability is 0 because Butch and Frisky are never in the living room together.

(c) From the table above, the probability that only Frisky is in is 0.16.

(d) Here p(0) = 0.04, p(1) = 0.72, p(2) = 0.24, p(3) = 0. Then $\Sigma x p(x) = 0\times0.04 + 1\times.72 + 2\times.24 + 3\times0 = 1.2$.

5. (a) $\mu = np = 12\times.2 = 2.4$ and $\sigma^2 = np(1-p) = 12\times.2\times.8 = 1.92$

(b) From (a), $\mu - \sigma = 2.4 - \sqrt{1.92} = 1.014$. We want $P(Y < 1.014)$, which for a discrete random variable is $P(Y = 0 \text{ or } Y = 1) = P(Y = 0) + P(Y = 1)$. Then

$P(Y = 0) = \frac{12!}{12!0!}\times.2^0\times.8^{12} = .069$ and

$P(Y = 1) = \frac{12!}{1!11!}.2^1.8^{11} = .206$ and

$P(Y = 0) + P(Y = 1) = .275$.

(c) Because np = 20 > 5 and n(1-p) = 80 > 5, we can apply the normal approximation to the binomial distribution. Then $P(Y_{NA} < \mu - \sigma)$ is just the probability that a normal RV is below a point 1 standard deviation below the mean, so $P(Y_{NA} < \mu - \sigma) = P(Z < -1) = .1587$.

Grade Distribution

100:2
90-99:27
80-89:35
70-79:36
60-69:33
50-59:9
<50:13

median = 76