Stat/For/Hort 571 — Midterm II, Fall 2000 — Brief Solutions

1. (a) This is a paired design, with the pairing based on the age and initial weight. Let $D = Y_B - Y_A$, where Y_A and Y_B are the weight losses within a group for Program A and B respectively. We get $d_i = 3, 5, 0, -1, 3, 1$. Thus, $\bar{d} = 1.833$ and $s_d^2 = 4.966$. To test $H_0 : \mu_D = 0$ vs $H_A : \mu_D \neq 0$, we use

$$t = \frac{1.833 - 0}{\sqrt{4.966/6}} = 2.015 \text{ on 5 df.}$$

From tables, p-value $= 2 \times P(T \geq 2.015) \approx 0.10$. There is very marginal evidence of difference between the two programs.

   (b) The D's are independent and have a normal distribution.

2. (a) SSError $= 4 \times (4.01^2 + 4.48^2 + 3.99^2 + 3.02^2) = 244.76$.

Source	df	SS	MS
Diet	3	174.47	58.16
Error	16	244.76	15.30
Total	19	419.23	

So, $F = 58.16/15.30 = 3.80$ on $(3, 16)$ df. From tables, $F_{3,16,0.05} = 3.24$ and $F_{3,16,0.01} = 5.29$, so $0.01 <$ p-value < 0.05. There is moderately strong evidence against the null hypothesis that all diets have the same mean coefficients of digestibility.

   (b) SSError $= 7 \times (4.01^2 + 4.48^2) + 1 \times (3.99^2 + 3.02^2) = 278.09$.

Source	df	SS	MS
Diet	3	111.11	37.04
Error	16	278.09	17.38
Total	19	389.20	

So $F = 37.04/17.38 = 2.13$ on $(3, 16)$ df. From tables, $F_{3,16,0.25} = 1.51$ and $F_{3,16,0.10} = 2.46$, so $0.10 <$ p-value < 0.25. There is no evidence against the null hypothesis that all diets have the same mean coefficients of digestibility.

   (c) They are different, because the data are different. In particular, SSError is bigger in (b), since there are more observations on Treatment 1 and 2 which have larger variances. Also, note that SSTrt is smaller in (b).

3. (a) With $n = 9$, $s^2 = 15$, $\chi^2_{8,0.025} = 17.53$, and $\chi^2_{8,0.975} = 2.18$, we have,

$$\frac{(n-1)s^2}{\chi^2_{n-1,0.025}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{n-1,0.975}},$$

which gives a CI of $6.85 \leq \sigma^2 \leq 55.05$.

   (b) $v^2 = (n-1)s^2/6.8 = 120/6.8 = 17.64$. From χ^2_8 in tables, p-value < 0.05. Hence, reject H_0 at the 5% level, but barely.

   (c) $v^2 = (n-1)s^2/56 = 120/56 = 2.14$. From χ^2_8 in tables, p-value < 0.05. Hence, reject H_0 at the 5% level, but barely.

   (d) Yes. From (b) and (c), we can see that 6.8 and 56 are barely rejected; from (a), each of these is barely not in the CI. Hence, there is an exact correspondence.

4. (a) True. The 95% CI for μ_1 is $\bar{y}_1 \pm t_{n_1-1,0.025}\sqrt{s_1^2/n_1} = 28.6 \pm 2.145\sqrt{10.2/15} = 28.6 \pm 1.77$. The 95% CI for μ_2 is $\bar{y}_2 \pm t_{n_2-1,0.025}\sqrt{s_2^2/n_2} = 80.4 \pm 2.571\sqrt{3.2/6} = 80.4 \pm 1.88$. The half-width, and hence the width of the CI is larger for μ_2.

   (b) False. Let $\gamma =$ the probability of $\bar{Y}$ exceeding the threshold under H_0 on a single trial. Then,

$$\gamma = P(\bar{Y} > 25.0 | \mu = 25) = P(Z > 0) = 0.5.$$

For the test that is based on all 4 trials,

$$\alpha = P(\text{reject } H_0 \mid H_0) = \gamma \times \gamma \times \gamma \times \gamma = (0.5)^4 = 0.0625.$$

5. Under $H_A : p = 0.85$, $\hat{p} \sim N\Big(0.85, (0.85 \times 0.15)/n\Big)$. We want

$$0.90 = P(\hat{p} \geq 0.81 | p = 0.85) \approx P(Z \geq \frac{0.81 - 0.85}{\sqrt{0.85 \times 0.15/n}}).$$

From tables, $0.90 = P(Z \geq -1.282)$. Thus $\frac{0.81-0.85}{\sqrt{0.85 \times 0.15/n}} = -1.282$. It follows that $n = 130.97$ and is rounded up to 131. Normal approximation is justified, because $np > 5$ and $n(1-p) > 5$ with $n = 131$ and $p = 0.85$.

Grade Distribution

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  100:3
90-99:30
80-89:39
70-79:44          mean = 77, median = 79
60-69:22
50-59:6
  <50:6
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