# Reinforcement Learning Part 1

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Computer Sciences 760
Fall 2017

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Some of the slides in these lectures have been adapted/borrowed from materials developed by Mark Craven, David Page, Jude Shavlik, Tom Mitchell, Nina Balcan, Matt Gormley, Elad Hazan, Tom Dietterich, and Pedro Domingos.

## Goals for the lecture

you should understand the following concepts

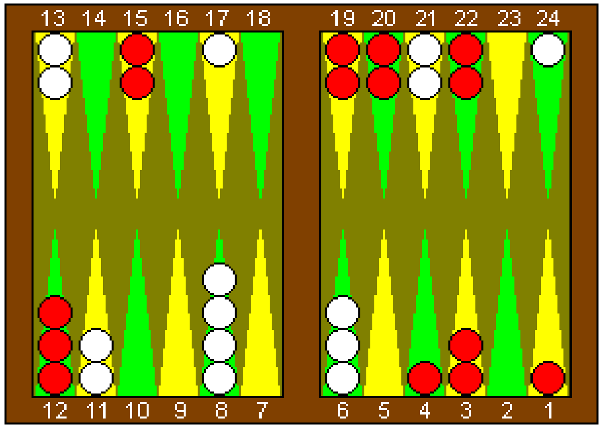
- the reinforcement learning task
- Markov decision process
- value functions
- value iteration
- Q functions
- Q learning

# Reinforcement learning (RL)

Task of an agent embedded in an environment

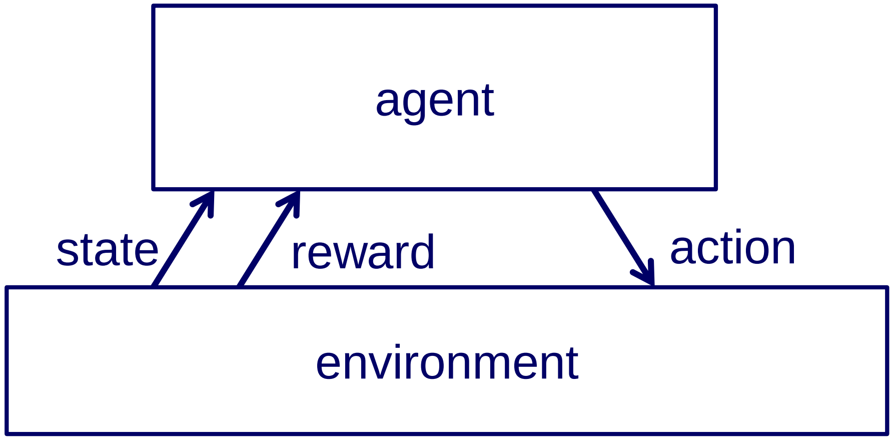
```
repeat forever
    1) sense world
    2) reason
    3) choose an action to perform
    4) get feedback (usually reward = 0)
    5) learn
```

the environment may be the physical world or an artificial one



# Reinforcement learning

- set of states $S$
- set of actions $A$
- at each time $t$, agent observes state $s_t \in S$ then chooses action $a_t \in A$
- then receives reward $r_t$ and changes to state $s_{t+1}$



$$\begin{tikzcd} s_0 \arrow[r, "a_0", "r_0"'] & s_1 \arrow[r, "a_1", "r_1"'] & s_2 \arrow[r, "a_2", "r_2"'] & {} \end{tikzcd}$$

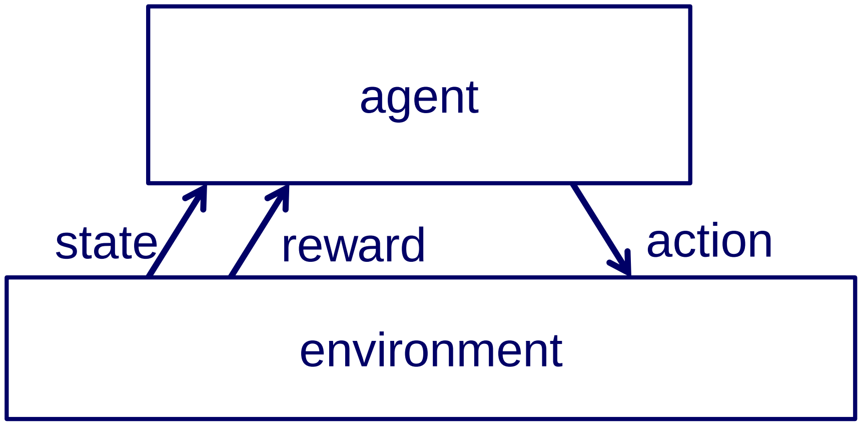
# Reinforcement learning as a Markov decision process (MDP)

- Markov assumption

$$P(s_{t+1} \mid s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(s_{t+1} \mid s_t, a_t)$$

- also assume reward is Markovian

$$P(r_{t+1} \mid s_t, a_t, s_{t-1}, a_{t-1}, \ldots) = P(r_{t+1} \mid s_t, a_t)$$



$$\begin{tikzcd} s_0 \arrow[r, "a_0", "r_0"'] & s_1 \arrow[r, "a_1", "r_1"'] & s_2 \arrow[r, "a_2", "r_2"'] & {} \end{tikzcd}$$

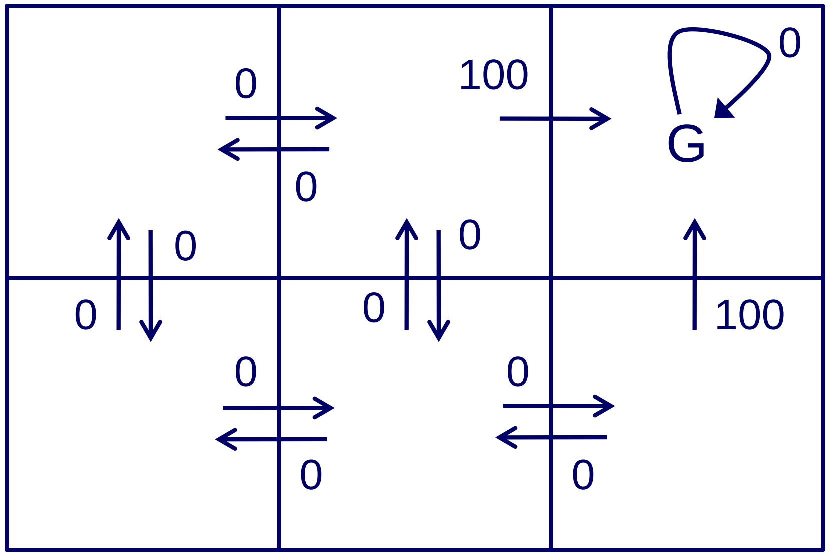
Goal: learn a policy $\pi : S \rightarrow A$ for choosing actions that maximizes

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \quad \text{where } 0 \leq \gamma < 1$$

for every possible starting state $s_0$

# Reinforcement learning task

- Suppose we want to learn a control policy $\pi : S \rightarrow A$ that maximizes $\sum_{t=0}^{\infty} \gamma^t E[r_t]$ from every state $s \in S$



each arrow represents an action $a$ and the associated number represents deterministic reward $r(s, a)$

# Value function for a policy

- given a policy $\pi : S \rightarrow A$ define

$$V^{\pi}(s) = \sum_{t=0}^{\infty} \gamma^{t} E[r_{t}]$$

assuming action sequence chosen according to $\pi$ starting at state $s$

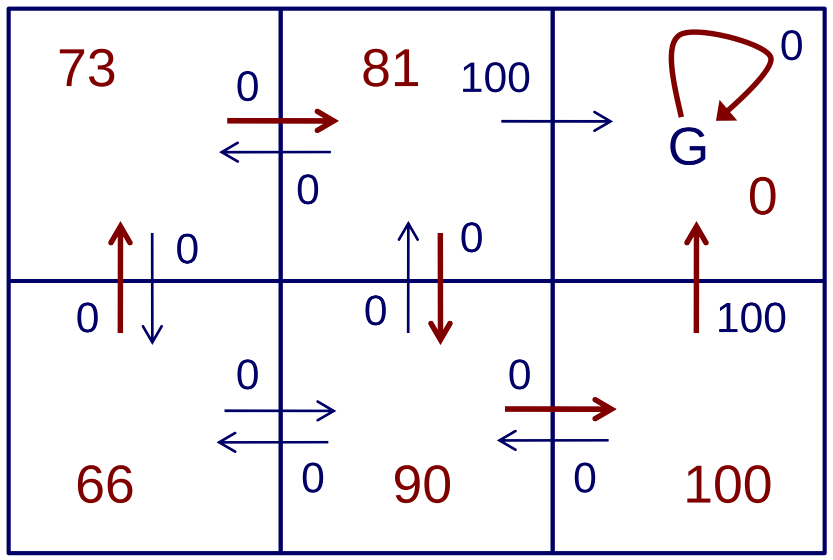
- we want the optimal policy $\pi^{*}$ where

$$p^{*} = \arg\max_{p} V^{p}(s) \quad \text{for all } s$$

we'll denote the value function for this optimal policy as $V^{*}(s)$

# Value function for a policy π

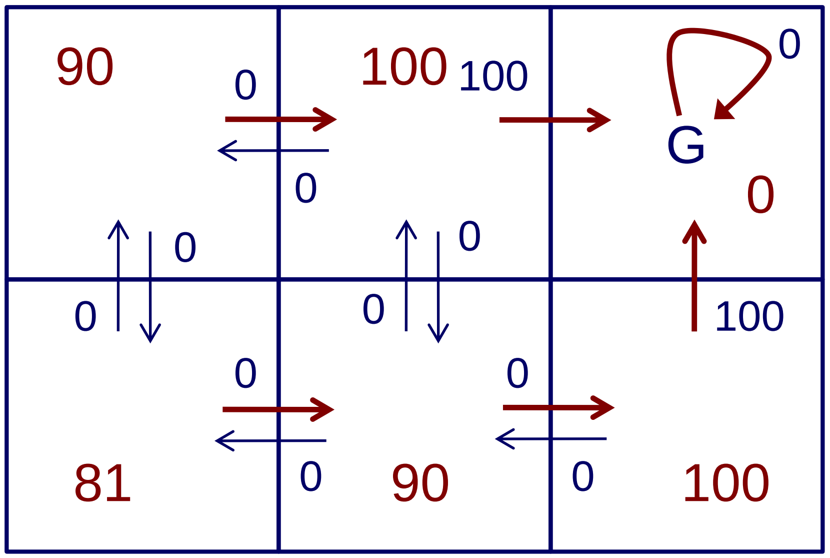
- Suppose π is shown by red arrows, $\gamma = 0.9$



$V^{\pi}(s)$ values are shown in red

# Value function for an optimal policy $\pi^*$

- Suppose $\pi^*$ is shown by red arrows, $\gamma = 0.9$



$V^*(s)$ values are shown in red

# Using a value function

If we know $V^*(s)$, $r(s_t, a)$, and $P(s_t \mid s_{t-1}, a_{t-1})$ we can compute $\pi^*(s)$

$$\pi^*(s_t) = \arg\max_{a \in A} \left[ r(s_t, a) + \gamma \sum_{s \in S} P(s_{t+1} = s \mid s_t, a) V^*(s) \right]$$

## Value iteration for learning $V^*(s)$

```
initialize $V(s)$ arbitrarily
loop until policy good enough
{
    loop for $s \in S$
    {
        loop for $a \in A$
        {
            $Q(s,a) \leftarrow r(s,a) + \gamma \sum_{s' \in S} P(s'|s,a) V(s')$
        }
        $V(s) \leftarrow \max_a Q(s,a)$
    }
}
```

## Value iteration for learning $V^*(s)$

- $V(s)$ converges to $V^*(s)$
- works even if we randomly traverse environment instead of looping through each state and action methodically
  - but we must visit each state infinitely often
- implication: we can do online learning as an agent roams around its environment
- assumes we have a model of the world: i.e. know $P(s_t \mid s_{t-1}, a_{t-1})$
- What if we don't?

# $Q$ learning

define a new function, closely related to $V^*$

$$V^*(s) \leftarrow E\left[r(s, \pi^*(s))\right] + \gamma E_{s'|s,\pi^*(s)}\left[V^*(s')\right]$$

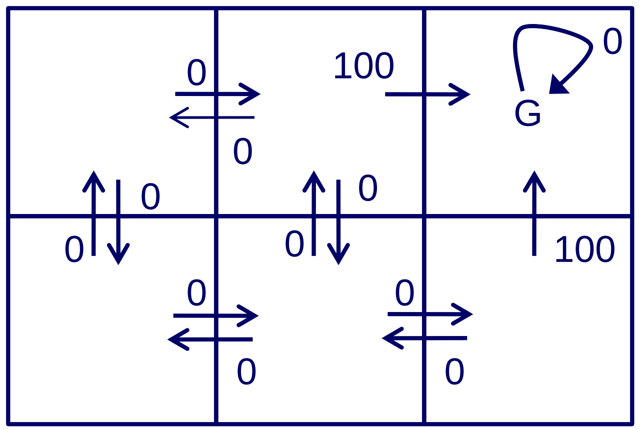
$$Q(s,a) \leftarrow E\left[r(s,a)\right] + \gamma E_{s'|s,a}\left[V^*(s')\right]$$

if agent knows $Q(s, a)$, it can choose optimal action without knowing $P(s' \mid s, a)$

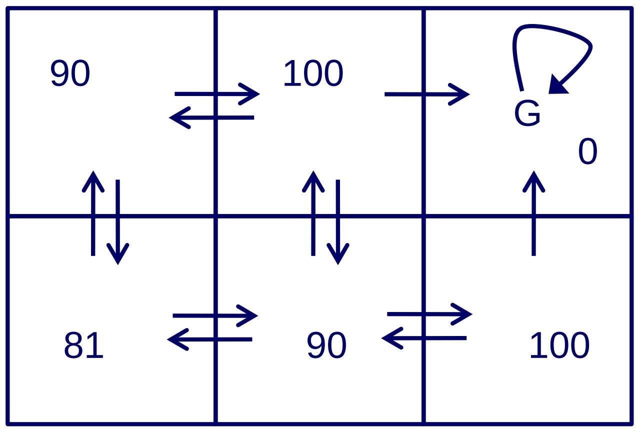
$$\pi^*(s) \leftarrow \arg\max_a Q(s,a) \qquad V^*(s) \leftarrow \max_a Q(s,a)$$

and it can learn $Q(s, a)$ without knowing $P(s' \mid s, a)$

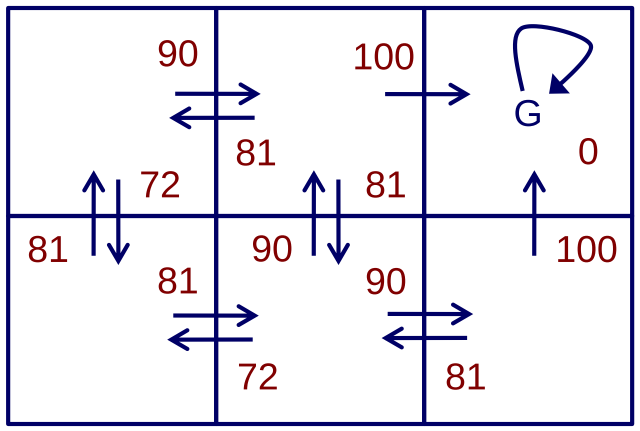
# $Q$ values



$r(s, a)$ (immediate reward) values



$V^*(s)$ values

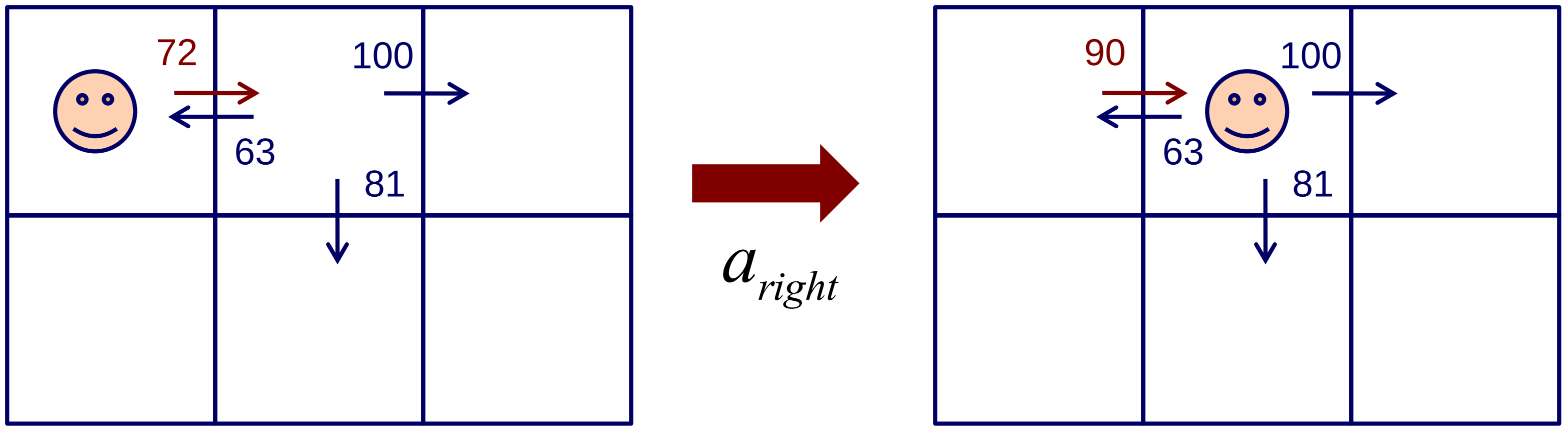


$Q(s, a)$ values

# $Q$ learning for deterministic worlds

```
for each $s$, $a$ initialize table entry $\hat{Q}(s,a) \leftarrow 0$
observe current state $s$
do forever
        select an action $a$ and execute it
        receive immediate reward $r$
        observe the new state $s'$
        update table entry
            $\hat{Q}(s,a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s',a')$
        $s \leftarrow s'$
```

# Updating $Q$



$$\hat{Q}(s_1, a_{right}) \leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a')$$
$$\leftarrow 0 + 0.9 \max\{63, 81, 100\}$$
$$\leftarrow 90$$

## $Q$ learning for *nondeterministic* worlds

```
for each $s$, $a$ initialize table entry $\hat{Q}(s,a) \leftarrow 0$
observe current state $s$
do forever
        select an action $a$ and execute it
        receive immediate reward $r$
        observe the new state $s'$
        update table entry
            $\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n\left[r + \gamma \max_{a'} \hat{Q}_{n-1}(s',a')\right]$
        $s \leftarrow s'$
```

where $\alpha_n$ is a parameter dependent on the number of visits to the given $(s, a)$ pair

$$a_n = \frac{1}{1 + \text{visits}_n(s,a)}$$

# Convergence of $Q$ learning

- $Q$ learning will converge to the correct $Q$ function
  - in the deterministic case
  - in the nondeterministic case (using the update rule just presented)

- in practice it is likely to take many, many iterations