


Q1-1: Consider Instance space  $X$ , Hypothesis space  $H$ , training examples  $D$ . Suppose that every  $h$  in our version space  $VS_{H,D}$  is consistent with  $m$  training examples. Which of the following statement(s) is/are TRUE?

- A. *The version space  $VS_{H,D}$  is  $\epsilon$ -exhausted with respect to  $c$  and  $D$  if every hypothesis  $h$  in  $VS_{H,D}$  has training error  $< \epsilon$ .*
- B. *“The probability that  $VS_{H,D}$  is not  $\epsilon$ -exhausted  $\leq |H|e^{-\epsilon m}$ ” provides a bound for the probability that ANY learner will output a hypothesis  $h$  with error  $> \epsilon$ .*

1. Both the statements are TRUE.
2. Statement A is TRUE, but statement B is FALSE.
3. Statement A is FALSE, but statement B is TRUE.
4. Both the statements are FALSE.

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The version space  $VS_{H,D}$  is  $\epsilon$ -exhausted with respect to  $c$  and  $D$  if every hypothesis  $h$  in  $VS_{H,D}$  has **true** error  $< \epsilon$


“The probability that  $VS_{H,D}$  is not  $\epsilon$ -exhausted  $\leq |H|e^{-\epsilon m}$ ” provides a bound for the probability that ANY **consistent** learner will output a hypothesis  $h$  with error  $> \epsilon$ .

Q1-2: We know that Probability that the version space is not  $\varepsilon$ -exhausted after  $m$  training examples is at most  $|H|e^{-\varepsilon m}$  where symbols have their usual meanings. From this we can derive that if  $error_{train}(h) = 0$ , then with probability at least  $(1 - \delta)$ ,  $error_{true}(h) \leq 1/m * A$  where  $A$  is

1.  $\ln (|H|) + \ln (\delta)$
2.  $\ln (|H|) + \ln (1/\delta)$
3.  $\ln (1/|H|) + \ln (\delta)$
4.  $\ln (1/|H|) + \ln (1/\delta)$

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We can write the bound as follows:

$$\Pr[(\exists h \text{ in } H) \text{ s.t. } (error_{train}(h) = 0) \text{ AND } (error_{true}(h) > \varepsilon)] \leq |H| e^{-\varepsilon m} = \delta$$

Hence, with at least probability  $(1 - \delta)$ ,  $error_{true}(h) \leq \varepsilon$ , where  $\varepsilon$  can be obtained by solving  $|H| e^{-\varepsilon m} = \delta$ .

Q2-1: Which of the following statement(s) is/are TRUE?

- A. PAC analysis formalizes the learning task and allows for non-perfect learning.*
- B. Finding a consistent hypothesis is easier for smaller concept classes.*
- C. In PAC analysis, we are trying to bound training error of a hypothesis.*

- 1. True, True, True
- 2. True, False, False
- 3. True, False, True
- 4. False, False, False

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- C. In PAC analysis, we are trying to bound training error of a hypothesis.*

1. True, True, True

2. True, False, False



3. True, False, True

4. False, False, False

Q2-2: Similar to “PAC analysis example: learning decision trees”, consider the case when each instance has  $n$  features and each feature has 3 possible values. Let learned hypotheses are DTs of depth 2 using only 2 variables.

What is  $|H|$ ? Note:  $nC2 = \binom{n}{2}$

1.  $nC3 \times 2^6$
2.  $nC3 \times 2^9$
3.  $nC2 \times 2^9$
4.  $nC2 \times 2^6$

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#possible split choices (choose 2 features out of  $n$ ) =  $nC2$

#leaves = 9. Each leaf has 2 possible labellings. So, #possible leaf labellings =  $2^9$



Q3-1: Which of the following statement(s) is/are TRUE?

A. *Agnostic PAC learnability requires learning a hypothesis with true error at most  $\epsilon$*

B. *In the case of agnostic PAC learning, if  $error_{train}(h_{best}) = 0$ , then with probability at least  $(1 - \delta)$ ,  $error_{true}(h_{best}) \leq (1/2m) \sqrt{[\ln |H| + \ln(1/\delta)]}$*

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1. True, True
2. True, False
3. False, True
4. False, False



Being agnostic PAC learnable is a stronger condition, since for all distributions, we can get close to the optimal error.

We want to get the expression for  $\epsilon$  in agnostic PAC learning case.  
Solve for  $\epsilon$  using:

$$m \geq \frac{1}{2\epsilon^2} \left( \ln |H| + \ln \left( \frac{1}{\delta} \right) \right)$$

Q3-2: What's the VC dimension of a decision stump in  $\mathbf{R}^2$ ? A decision stump is of the form  $f(x) = \text{sign}(x_i > b)$  or  $f(x) = \text{sign}(x_i < b)$  for one feature  $x_i$  and some bias parameter  $b$ .

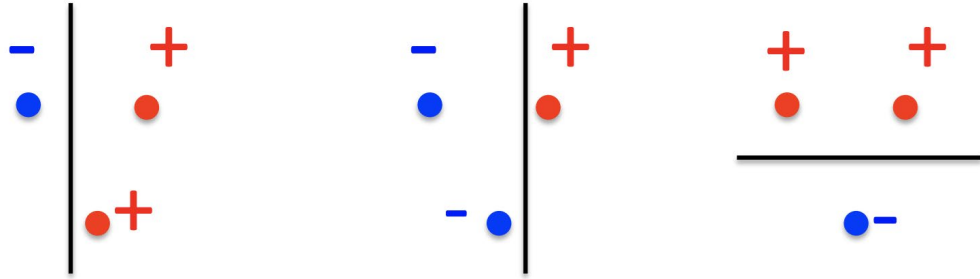
1. VC = 1
2. VC = 2
3. VC = 3
4. VC = 4

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- 1. VC = 1
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VC  $\geq 3$ : All possible labelings of 3 points can be shattered.



VC = 3: For all placements of 4 points, there exist a labeling that can't be shattered.

