CS 540 Introduction to Artificial Intelligence
Logic

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Sep 23, 2021

Announcements

- **Homeworks:**
  - HW2 due Tuesday.
- Class roadmap:

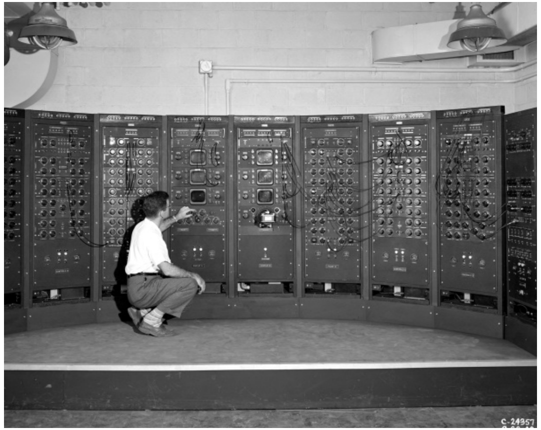
Tuesday, Sep 14	Probability
Thursday, Sep 16	Linear Algebra and PCA
Tuesday, Sep 21	Statistics and Math Review
Thursday, Sep 23	**Introduction to Logic**
Tuesday, Sep 28	Natural Language Processing

Fundamentals

Logic & AI

Why are we studying logic?

- **Traditional** approach to AI ('50s-'80s)
  - "Symbolic AI"
  - The Logic Theorist (1956)
    - Proved a bunch of theorems!
- Logic also the language of:
  - Knowledge rep., databases, theory of computing, etc.



Symbolic Techniques in AI

Lots of systems based on symbolic approach:

- Ex: expert systems, planning, more
- Playing great chess

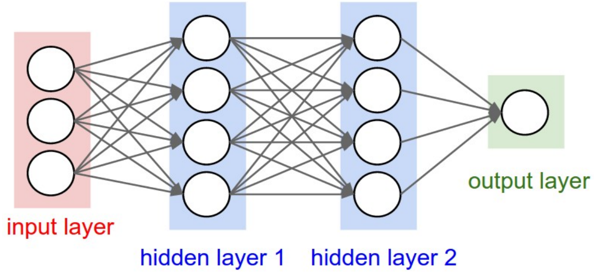
- Less popular recently

J. Gardner

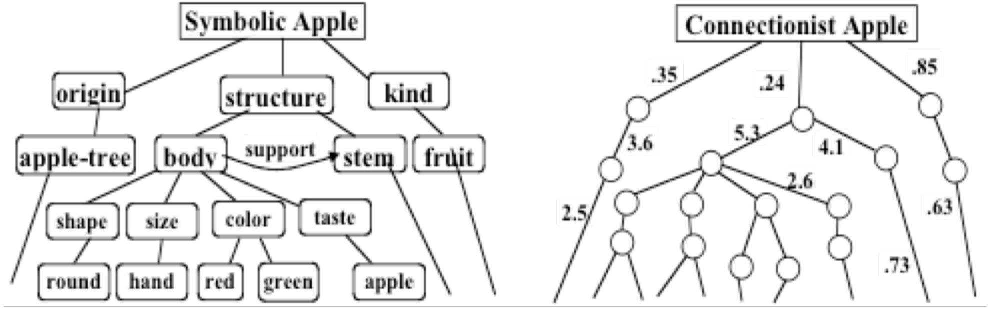
Symbolic vs Connectionist

Rival approach: **connectionist**

- Probabilistic models
- Neural networks
- **Extremely popular** last 20 years



Stanford CS231n

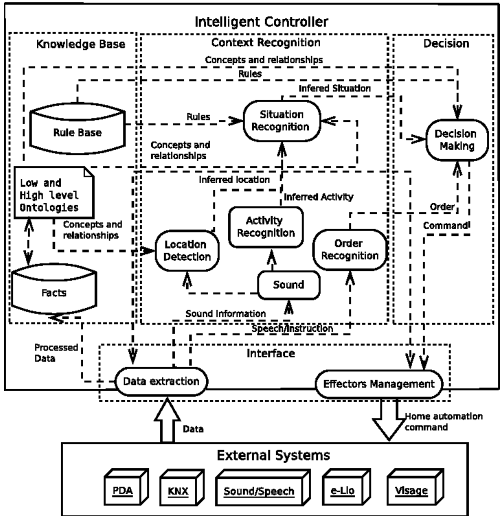


M. Minsky

Symbolic vs Connectionist

Analogy: Logic versus probability

- Which is better?
- Future: combination; best-of-both-worlds
  - Actually been worked on:
  - **Example**: Markov Logic Networks

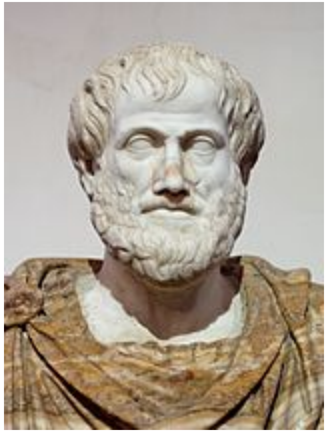


Outline

- Introduction to logic
  - Arguments, validity, soundness
- Propositional logic
  - Sentences, semantics, inference
- First order logic (FOL)
  - Predicates, objects, formulas, quantifiers

Basic Logic

- Arguments, premises, conclusions
  - Argument: a set of sentences (premises) + a sentence (a conclusion)
  - **Validity:** argument is valid iff it's necessary that if all premises are true, the conclusion is true
  - **Soundness:** argument is sound iff valid & premises true
  - **Entailment:** when valid arg., premises entail conclusion



Propositional Logic Basics

Logic Vocabulary:

- Sentences, symbols, connectives, parentheses
  - Symbols: P, Q, R, … (**atomic** sentences)
  - Connectives:

$\wedge$	and	[conjunction]
$\vee$	or	[disjunction]
$\Rightarrow$	implies	[implication]
$\Leftrightarrow$	is equivalent	[biconditional]
$\neg$	not	[negation]

  - Literal: P or negation $\neg$P

Propositional Logic Basics

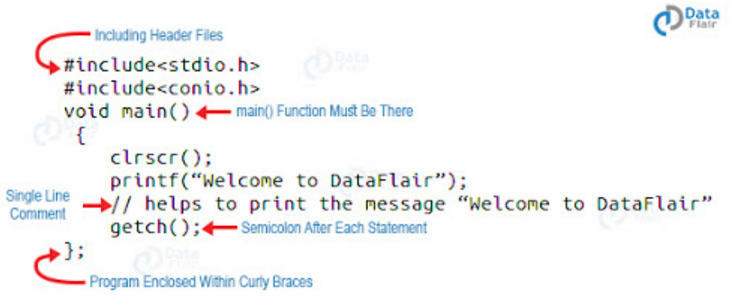
Examples:

- $(P \vee Q) \Rightarrow S$
  - "If it is cold or it is raining, then I need a jacket"
- $Q \Rightarrow P$
  - "If it is raining, then it is cold"
- $\neg R$
  - "It is not hot"

Propositional Logic Basics

Several rules in place

- Precedence: $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- Use parentheses when needed
- Sentences: **well-formed** or not well-formed:
  - $P \Rightarrow Q \Rightarrow S$ **X (not associative!)**



Sentences & Semantics

- Think of symbols as defined by user
- Sentences: built up from symbols with connectives

  - **Interpretation**: assigning True / False to symbols
  - **Semantics**: interpretations for which sentence evaluates to True
  - **Model**: (of a set of sentences) interpretation for which all sentences are True

Evaluating a Sentence

- Example:

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

- Note:
  - If P is false, P⇒Q is true regardless of Q (“5 is even implies 6 is odd” is True!)
  - Causality unneeded: “5 is odd implies the Sun is a star” is True!)

Evaluating a Sentence: Truth Table

- **Ex**:

P	Q	R	¬P	Q∧R	¬P∨Q∧R	¬P∨Q∧R⇒Q
0	0	0	1	0	1	0
0	0	1	1	0	1	0
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	1
1	0	1	0	0	0	1
1	1	0	0	0	0	1
1	1	1	0	1	1	1

- **Satisfiable**
  - There exists some interpretation where sentence true

Break & Quiz

Q 1.1: Suppose P is false, Q is true, and R is true. Does this assignment satisfy

(i) $\neg(\neg p \rightarrow \neg q) \wedge r$

(ii) $(\neg p \vee \neg q) \rightarrow (p \vee \neg r)$

- A. Both
- B. Neither
- C. Just (i)
- D. Just (ii)

Break & Quiz

Q 1.2: Let A =“Aldo is Italian” and B =“Bob is English”. Formalize “Aldo is Italian or if Aldo isn’t Italian then Bob is English”.

- a. $A \lor (\neg A \rightarrow B)$
- b. $A \lor B$
- c. $A \lor (A \rightarrow B)$
- d. $A \rightarrow B$

Break & Quiz

Q 1.3: How many different assignments can there be to

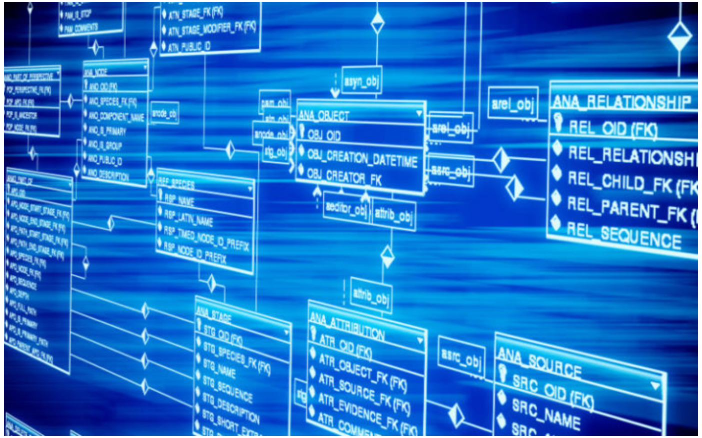
$(x_1 \wedge y_1) \vee (x_2 \wedge y_2) \vee \ldots \vee (x_n \wedge y_n)$

- A. 2
- B. 2^n
- C. 2^{2n}
- D. 2n

Knowledge Bases

- **Knowledge Base (KB)**: A set of sentences
  - Like a long sentence, connect with conjunction

Model of a KB: interpretations where all sentences are True

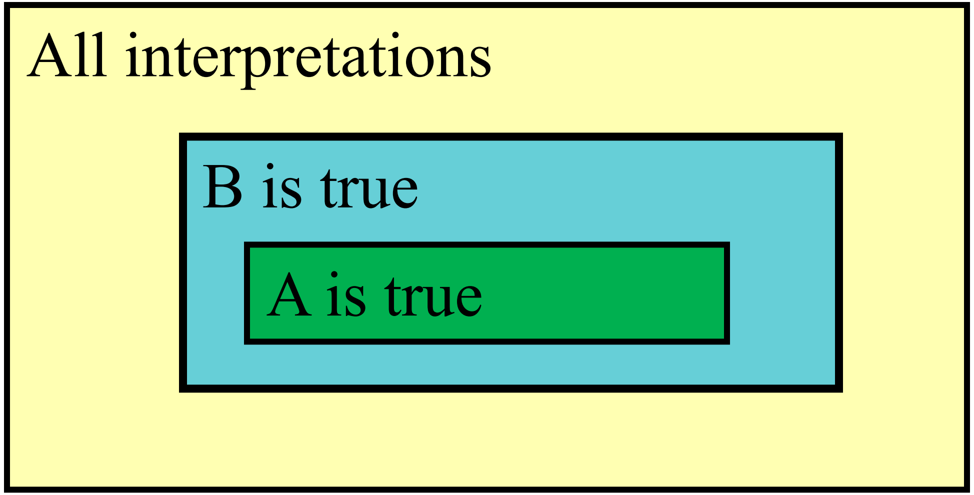


Goal: inference to discover new sentences

Entailment

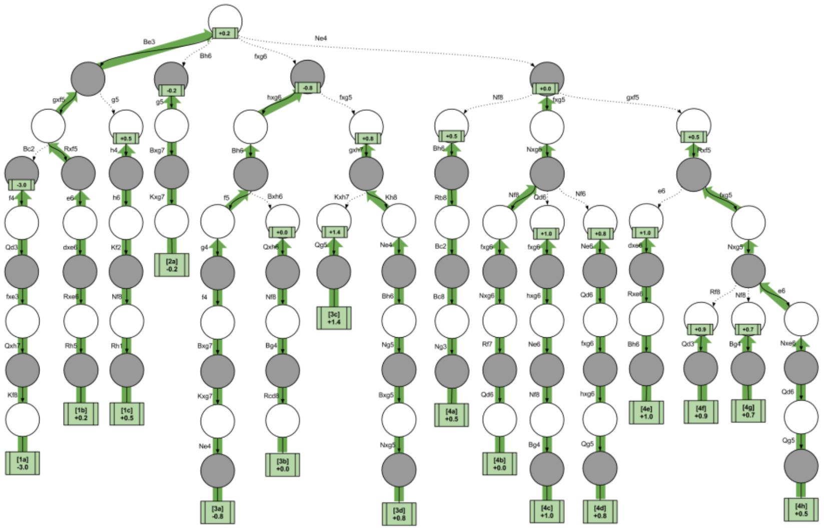
Entailment: a sentence logically follows from others

- Like from a KB. Write A ⊨ B
- A ⊨ B iff in every interpretation where A is true, B is also true



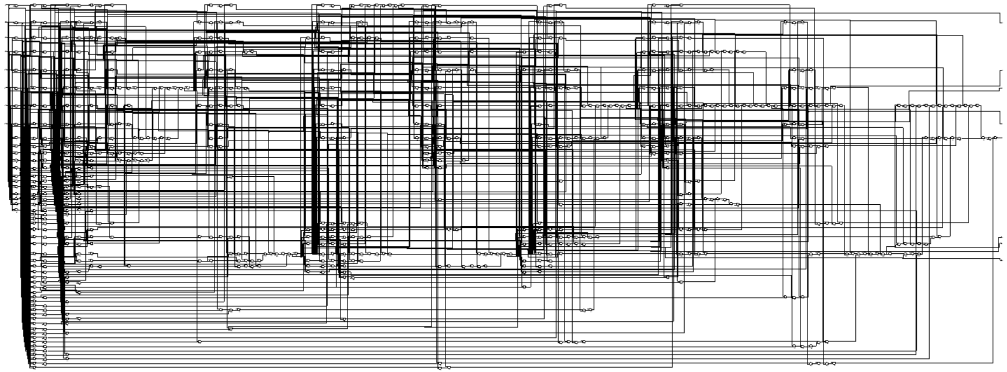
Inference

- Given a set of sentences (a KB), **logical inference** creates new sentences
  - Compare to prob. inference!
- **Challenges**:
  - Soundness
  - Completeness
  - Efficiency



Methods of Inference: **1. Enumeration**

- Enumerate all interpretations; look at the truth table
  - "Model checking"
- Downside: 2^n interpretations for n symbols



S. Leadley

Methods of Inference: **2. Using Rules**

- *Modus Ponens*: $(A \Rightarrow B, A) \models B$
- And-elimination
- Many other rules
  - Commutativity, associativity, de Morgan's laws, distribution for conjunction/disjunction

Methods of Inference: **3. Resolution**

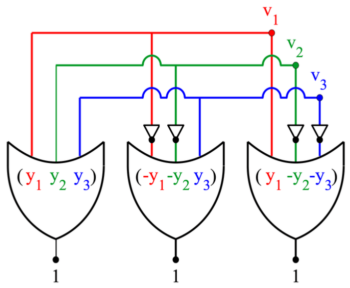
- Convert to special form and use a single rule
- **Conjunctive Normal Form** (CNF)

$$\underbrace{(\neg A \vee B \vee C)}_{\text{a clause}} \wedge (\neg B \vee A) \wedge (\neg C \vee A)$$

a clause

Conjunction of clauses; each clause disjunction of literals

- Simple rules for converting to CNF



Methods of Inference: **3. Resolution**

Start with our KB and **query** B

- Add $\neg B$
- Show that this leads to a contradiction
- Take clauses with a symbol and its complement
  - Merge, throw away symbol: $P \vee Q \vee R$, $\neg Q \vee S \vee T$: $P \vee R \vee S \vee T$
  - If no symbol left, KB entails B
  - No new clauses, KB does not entail B

Break & Quiz

Q 2.1: What is the CNF for $(\neg p \wedge \neg(p \Rightarrow q))$

- A. $(\neg p \wedge \neg(p \Rightarrow q))$
- B. $(\neg p) \wedge (\neg p \vee \neg q)$
- C. $(\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$
- D. $(\neg p \vee \neg q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (p \vee q)$

Break & Quiz

Q 2.2: Which has more rows: a truth table on *n* symbols, or a joint distribution table on *n* binary random variables?

- A. Truth table
- B. Distribution
- C. Same size
- D. It depends

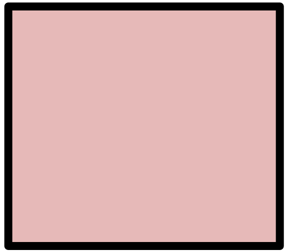
First Order Logic (FOL)

Propositional logic has some limitations

- Ex: how to say “all squares have four sides”
- No context, hard to generalize; express facts

FOL is a more expressive logic; works over

- Facts, Objects, Relations, Functions



First Order Logic (FOL)

Basics:

- Constants: “16”, “Green”, “Bob”
- Functions: map objects to objects
- Predicates: map objects to T/F:
  - Greater(5,3)
  - Color(grass, green)

First Order Logic (FOL)

Basics:

- Variables: x, y, z
- Connectives: Same as propositional logic
- Quantifiers:
  - $\forall$ universal quantifier: $\forall$**`x human(x)`** $\Rightarrow$ **`mammal(x)`**
  - $\exists$ existential quantifier