CS540 Introduction to Artificial Intelligence

Convolutional Neural Networks (II)

Yudong Chen
University of Wisconsin-Madison

November 4, 2021

Slides created by Sharon Li [modified by Yudong Chen]

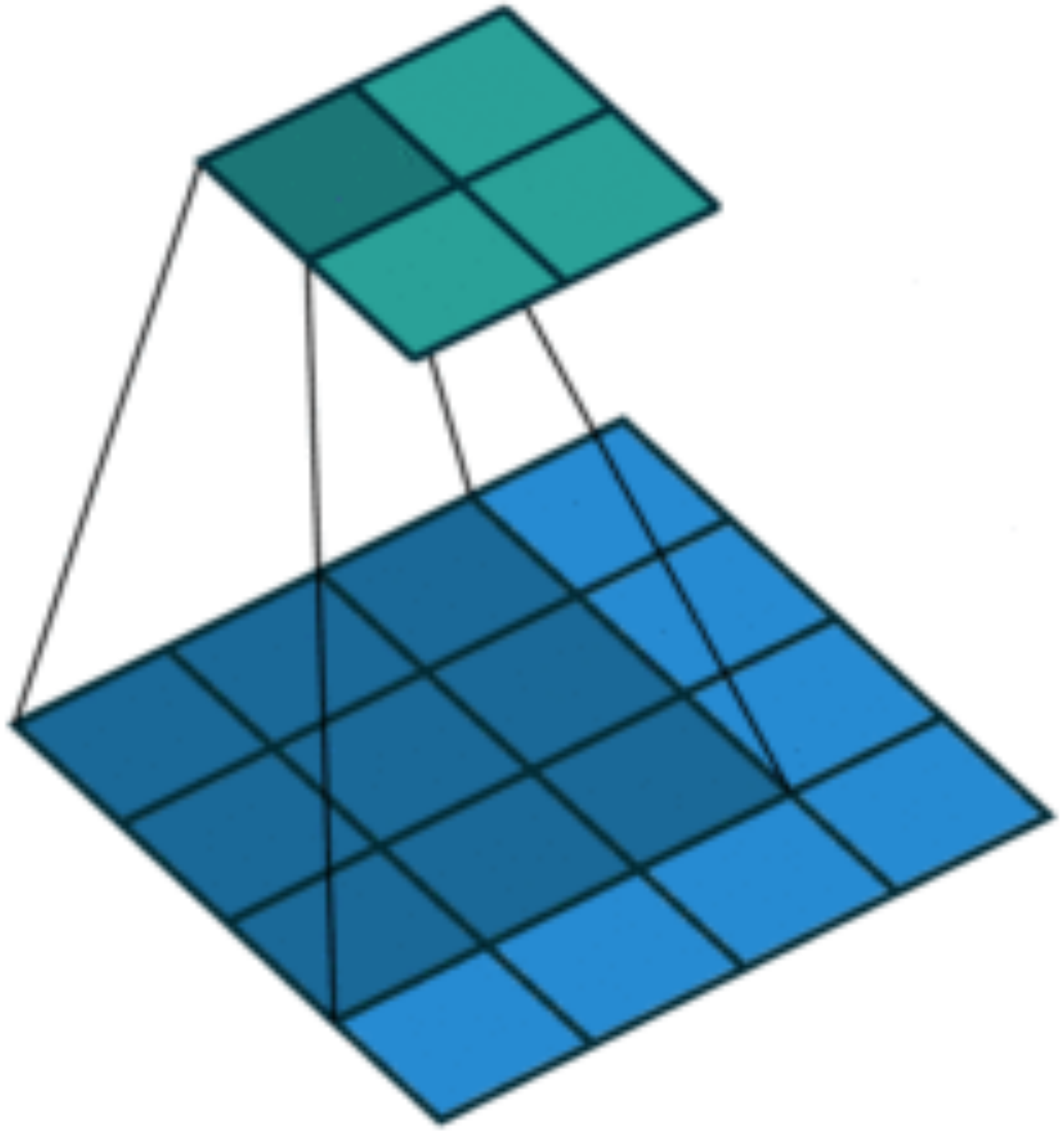
Outline

- Brief review of convolutional computations
- Convolutional Neural Networks
    - LeNet (first conv nets)
    - AlexNet
    - VGG

Review: 2-D Convolution

$$\underset{\text{Input}}{\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}} * \underset{\text{Kernel}}{\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}} = \underset{\text{Output}}{\begin{bmatrix} 19 & 25 \\ 37 & 43 \end{bmatrix}}$$

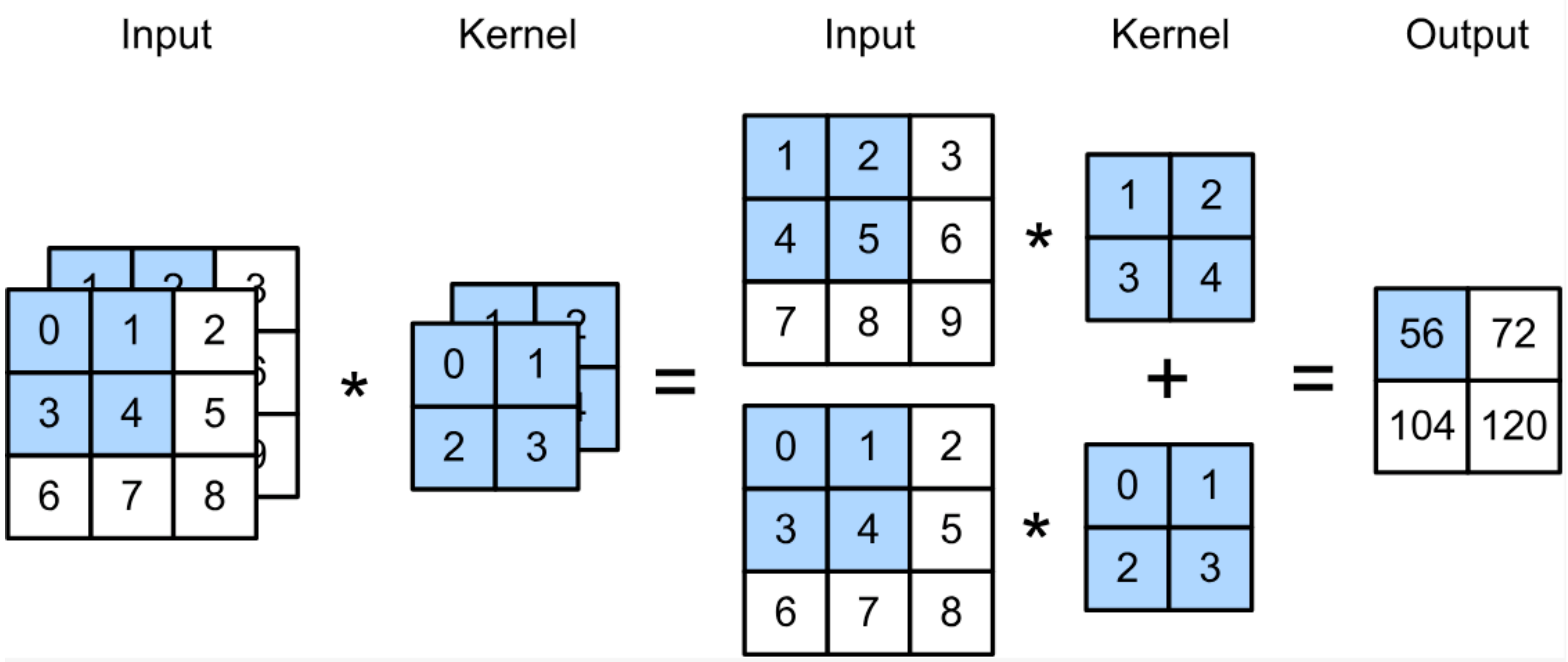
$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19,$$
$$1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25,$$
$$3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37,$$
$$4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43.$$



(vdumoulin@ Github)

Review: Multiple Input Channels

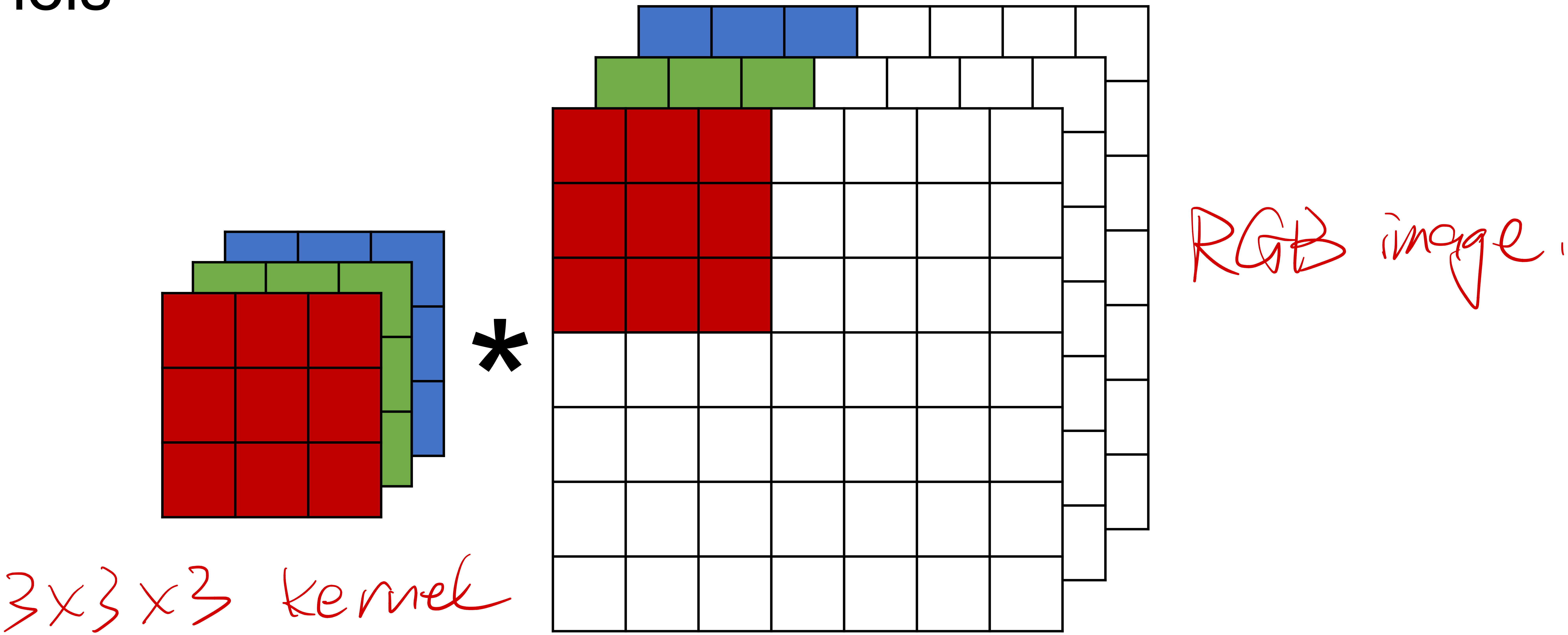
- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels



$$
\begin{aligned}
&(1\times 1+2\times 2+4\times 3+5\times 4)\\
&+(0\times 0+1\times 1+3\times 2+4\times 3)\\
&=56
\end{aligned}
$$

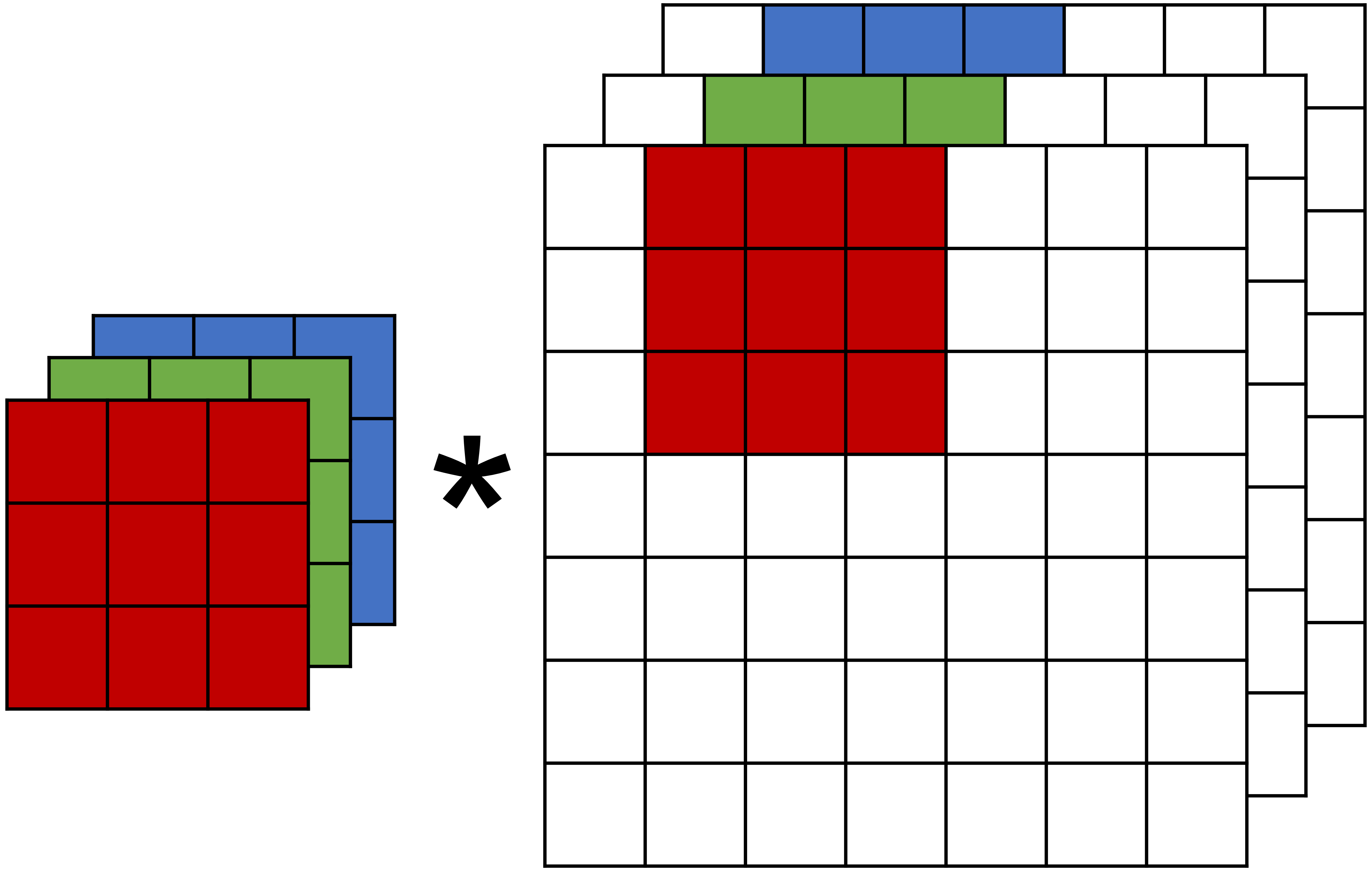
Review: Multiple Input Channels

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels



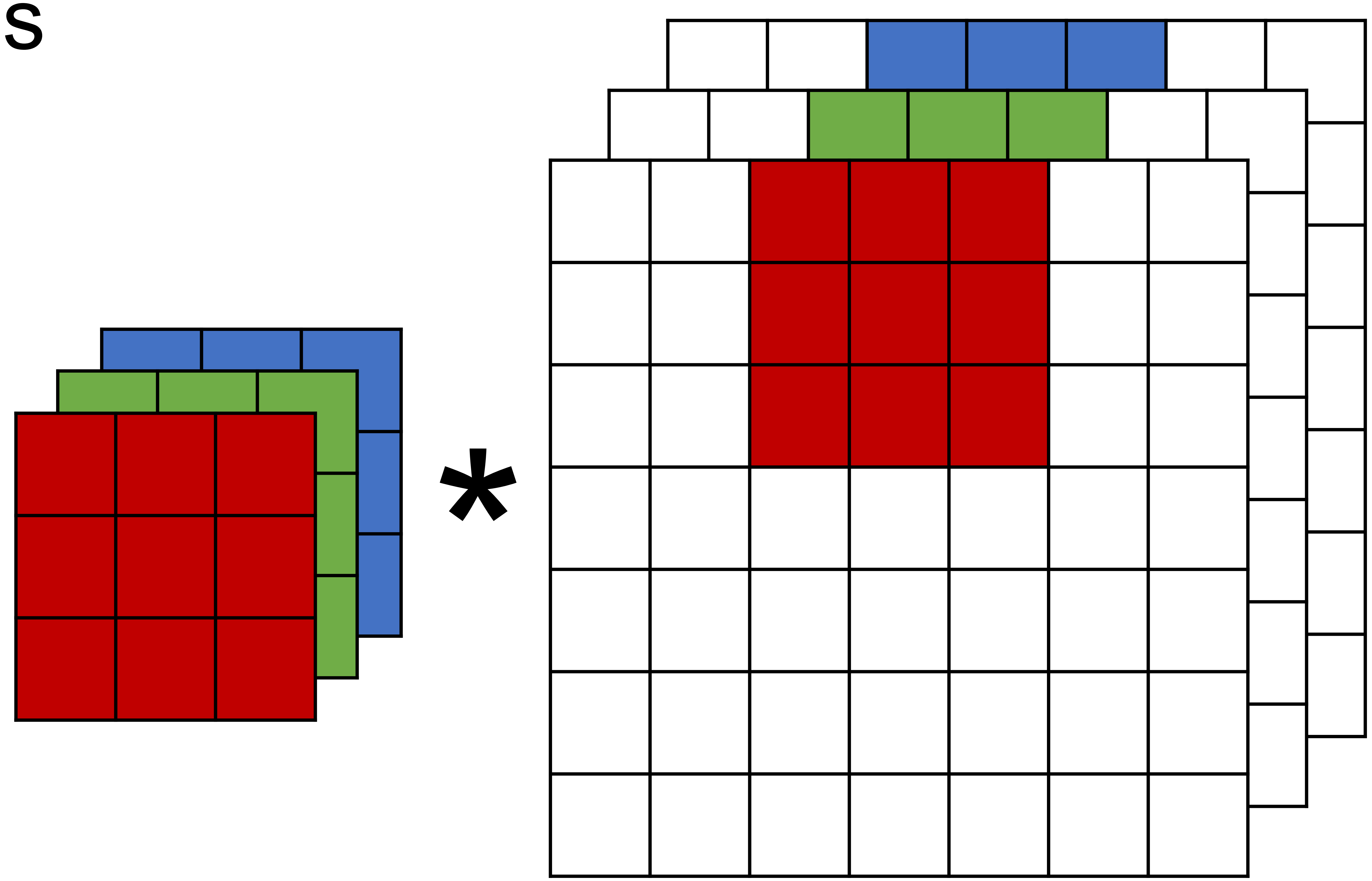
Review: Multiple Input Channels

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels



Review: Multiple Input Channels

- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels



Output shape

Kernel/filter size

$$\lfloor (n_h - k_h + p_h + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w)/s_w \rfloor$$

Input size

Pad

Stride

Consider a convolution layer with 16 filters. Each filter has a size of 11x11x3, a stride of 2x2. Given an input image of size 22x22x3, if we don't allow a filter to fall outside of the input, what is the output size?

- 11x11x16
- 6x6x16
- 7x7x16
- 5x5x16

$k_h \times k_w \times c_i$ c_o Kernels

Consider a convolution layer with 16 filters. Each filter has a size of 11x11x3, a stride of 2x2. Given an input image of size 22x22x3, if we don't allow a filter to fall outside of the input, what is the output size?

$n_h \times n_w$ $s_h \times s_w$

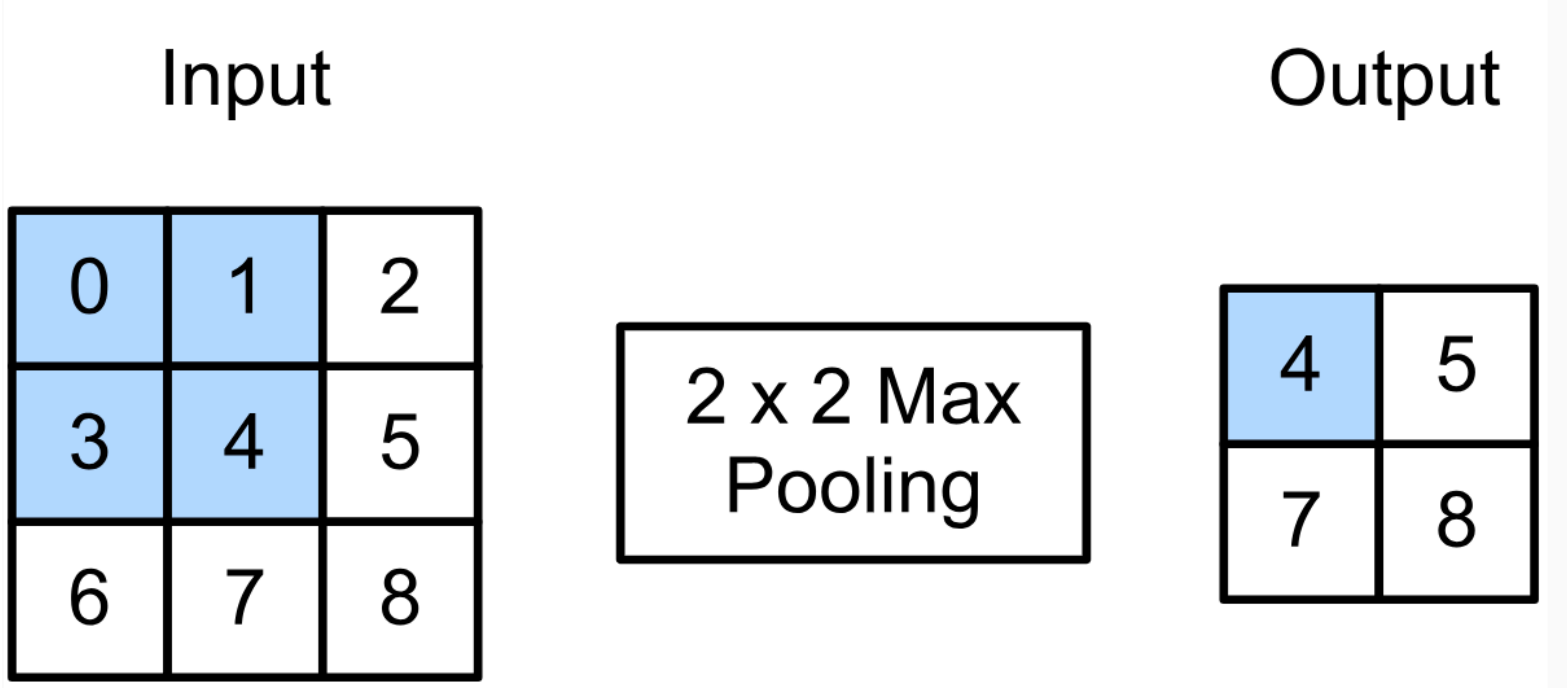
- 11x11x16
- 6x6x16
- 7x7x16
- 5x5x16

$$\lfloor (n_h - k_h + \underbrace{p_h}_{=0} + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + \underbrace{p_w}_{=0} + s_w)/s_w \rfloor$$

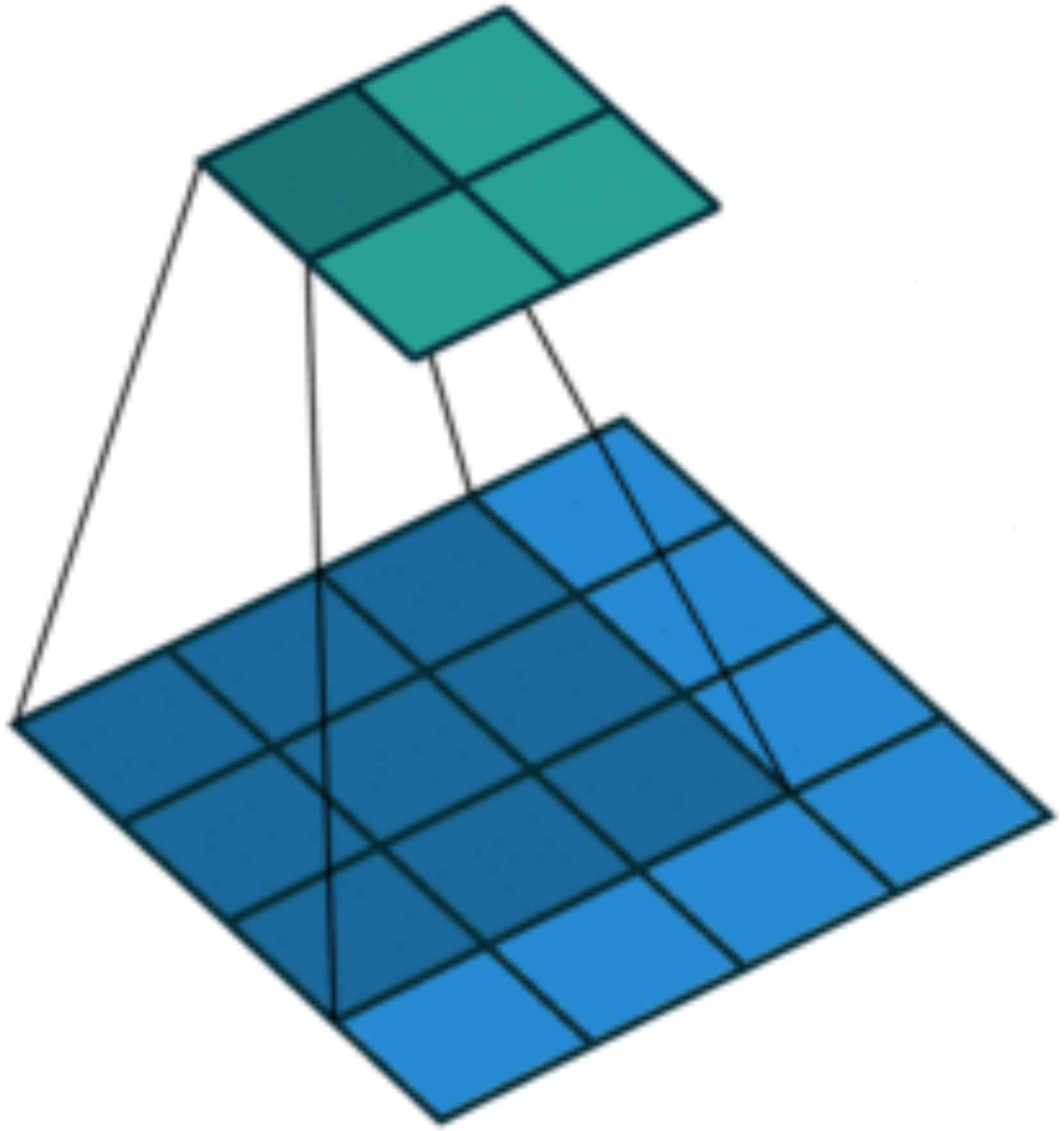
Pooling Layer

2-D Max Pooling

- Returns the maximal value in the sliding window



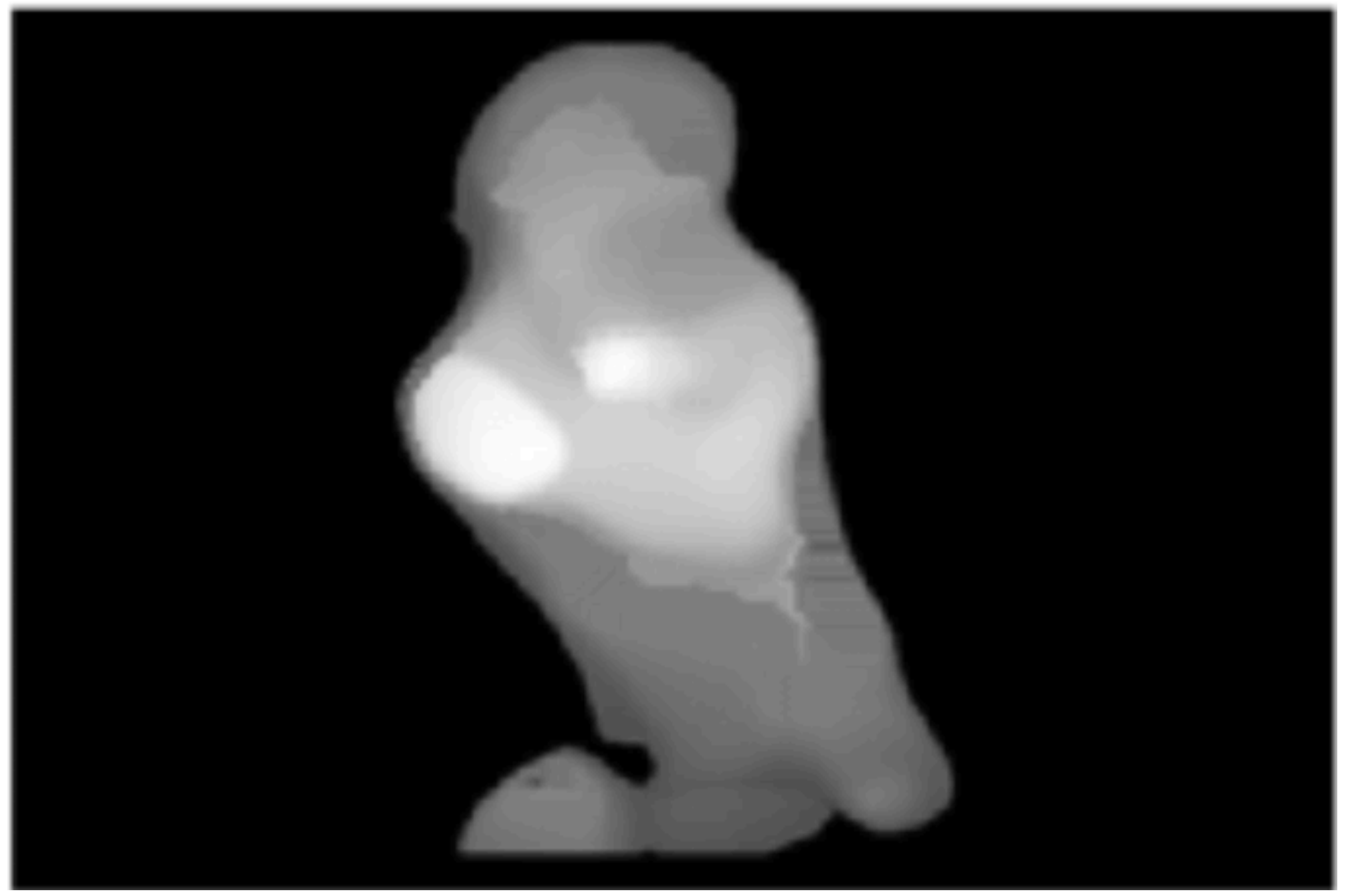
$$\max(0,1,3,4) = 4$$



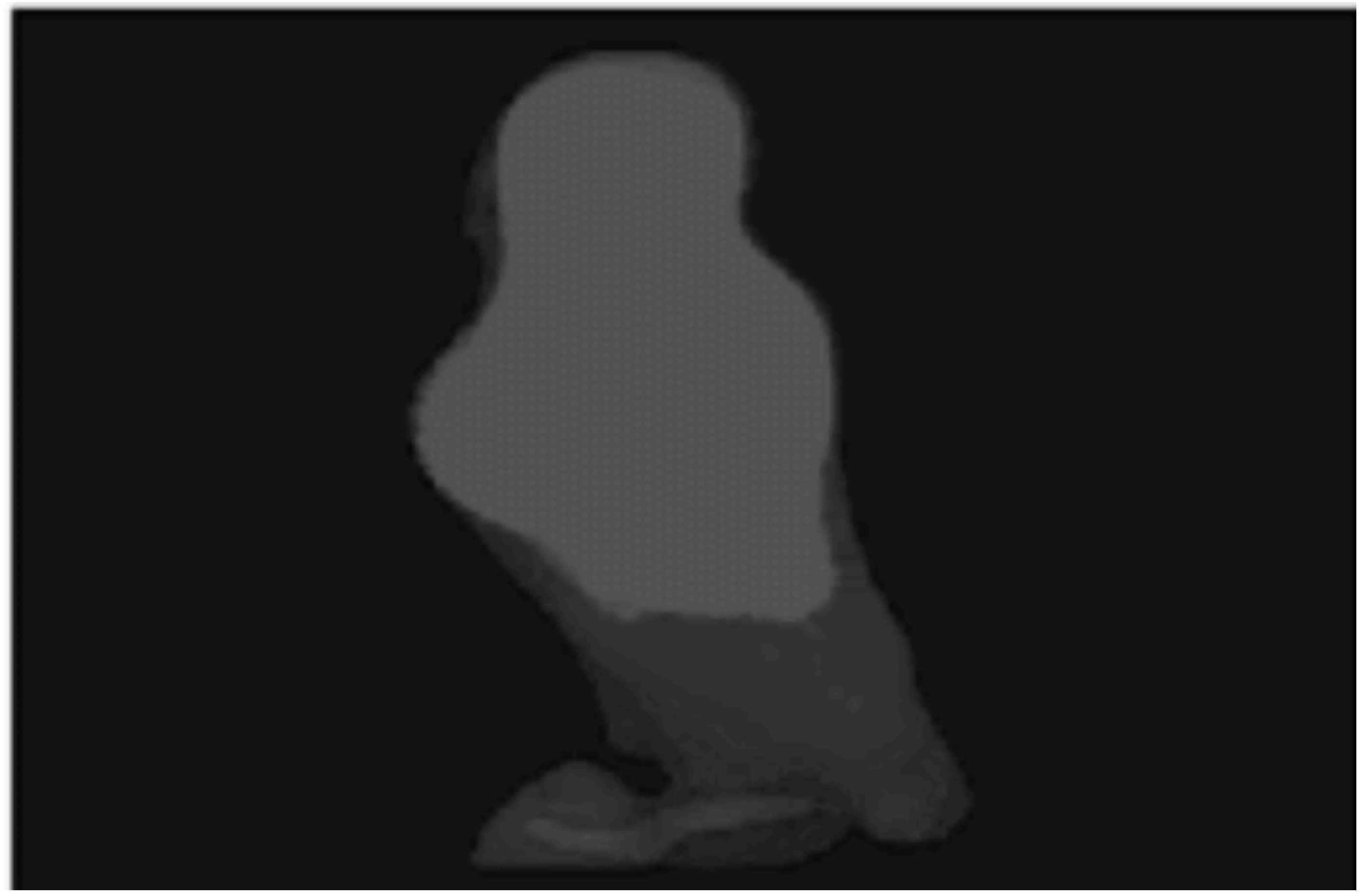
Average Pooling

- Max pooling: the strongest pattern signal in a window
- Average pooling: replace max with mean in max pooling
  - The average signal strength in a window

Max pooling

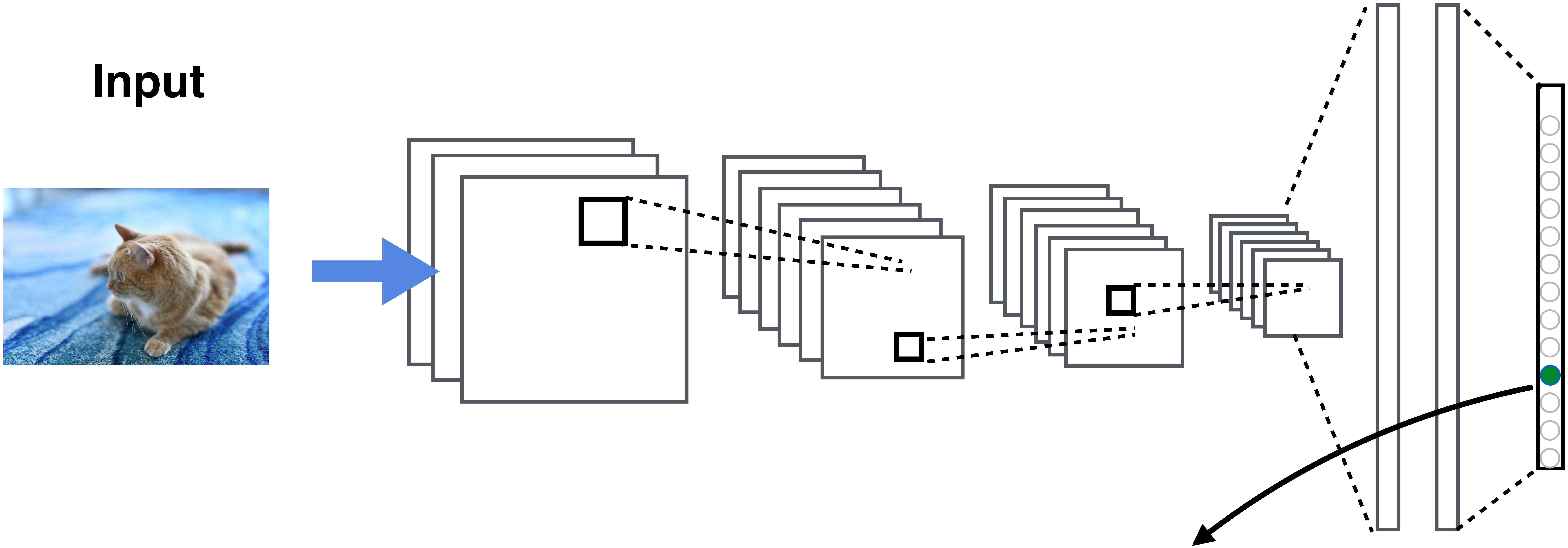


Average pooling



How to train a convolutional neural network?

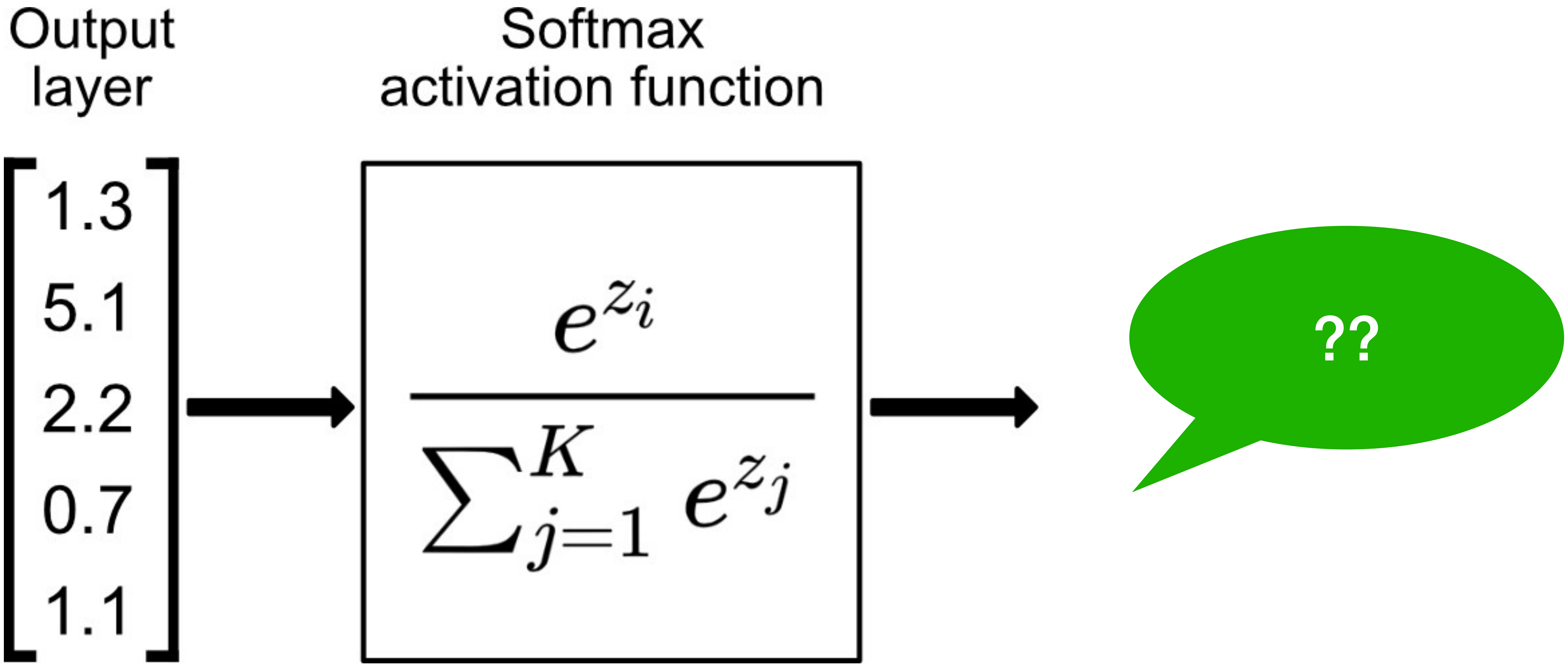
Input



$$p_i(\boldsymbol{x}) = \frac{\exp\left(f_i(\boldsymbol{x})\right)}{\sum_{j=1}^{N} \exp\left(f_j(\boldsymbol{x})\right)}, \quad \text{softmax}$$

Recall Softmax

Turns outputs f into probabilities (sum up to 1 across k classes)



Recall Softmax

Turns outputs f into probabilities (sum up to 1 across k classes)

Output layer	Softmax activation function	Probabilities
$\begin{bmatrix}1.3\\5.1\\2.2\\0.7\\1.1\end{bmatrix}$	$\longrightarrow \dfrac{e^{z_i}}{\sum_{j=1}^{K} e^{z_j}} \longrightarrow$	$\begin{array}{c}\begin{bmatrix}0.02\\0.90\\0.05\\0.01\\0.02\end{bmatrix}\end{array}\begin{array}{l}p_1\\p_2\\ \vdots\\ \\p_K\end{array}$

Normalized

logits = pre-softmax.

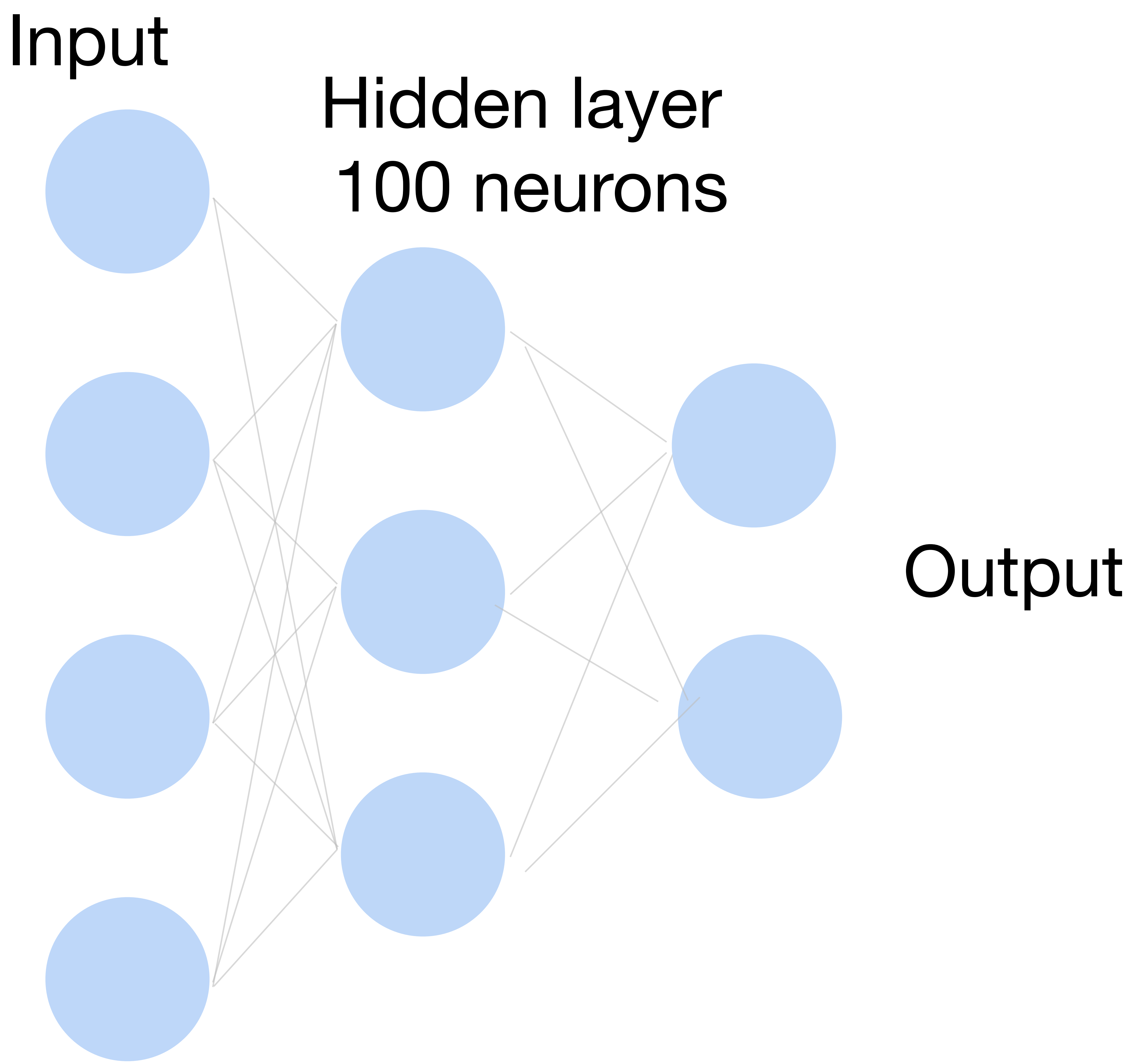
How to train a neural network?

Loss function: $$\frac{1}{|D|}\sum_i \ell(\mathbf{x}_i, y_i)$$

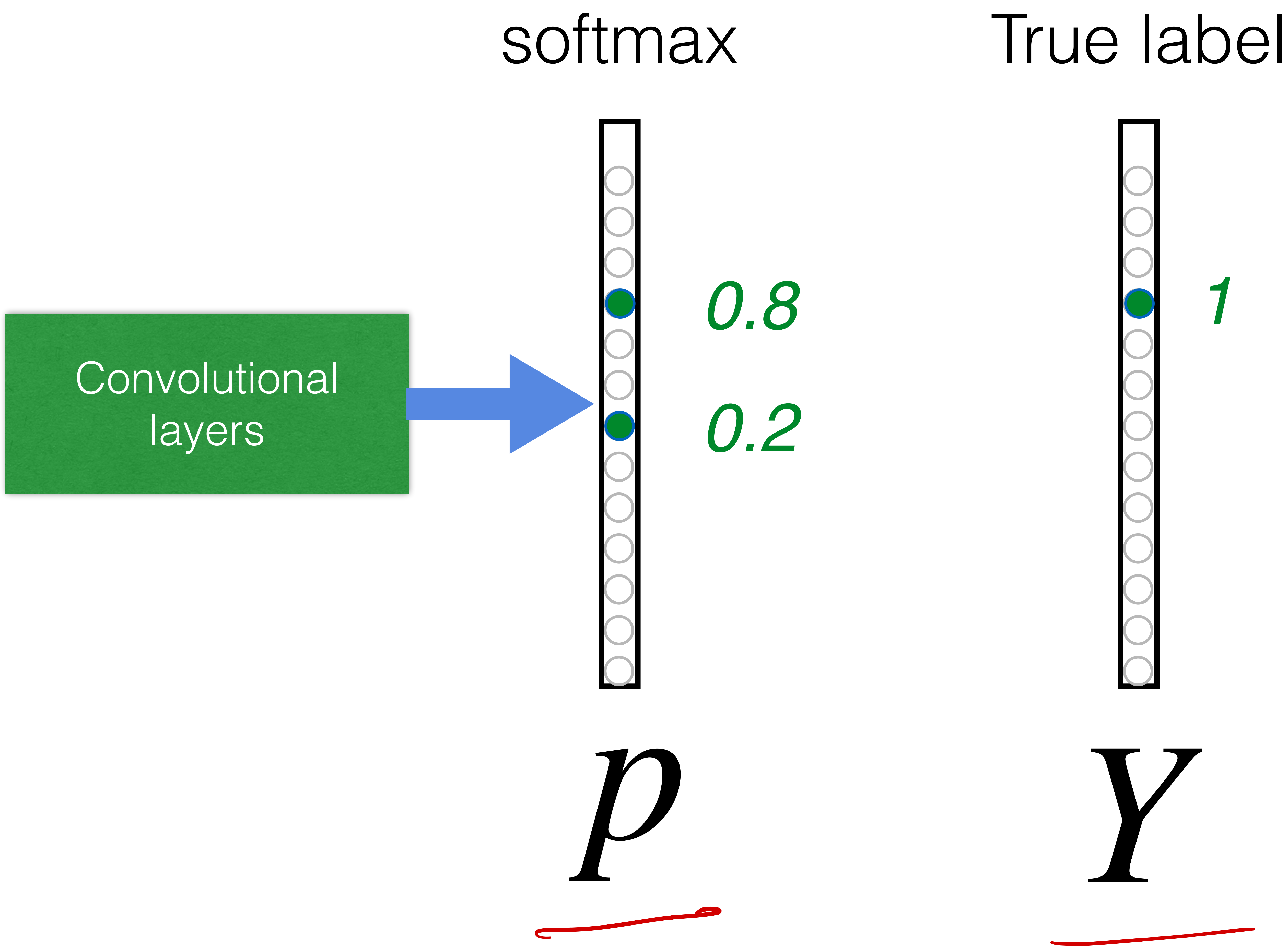
Per-sample loss:

$$\ell(\mathbf{x}, y) = \sum_{j=1}^{K} -y_j \log p_j$$

Also known as cross-entropy loss or softmax loss



Cross-Entropy Loss



$$L_{CE} = \sum_i - Y_i \log(p_i)$$

$$= -Y_3 \log(p_3)$$

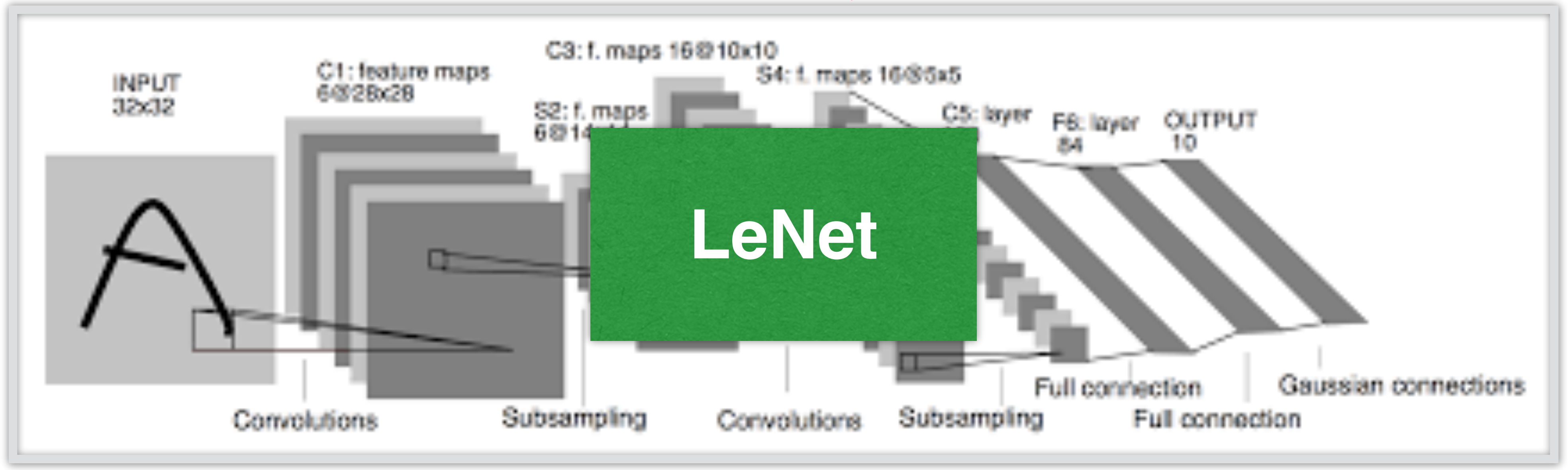
$$= -\log(0.8)$$

Goal: push **p** and **Y** to be identical

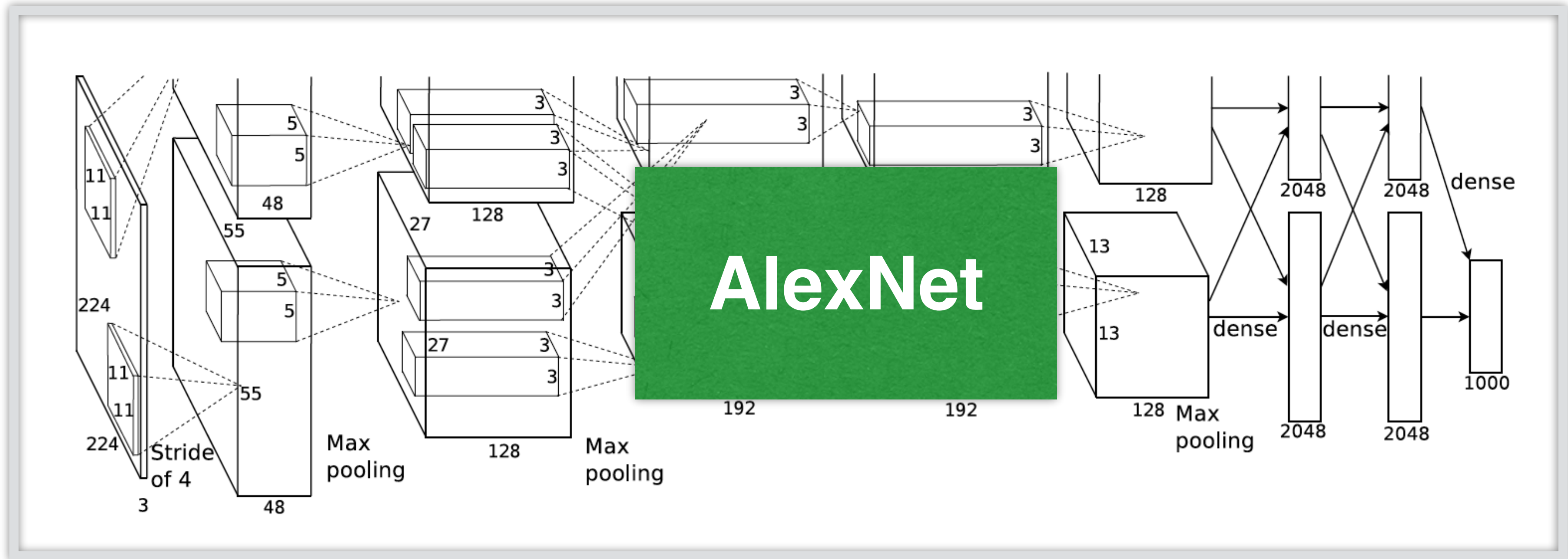
Convolutional Neural Networks Examples

Evolution of neural net architectures

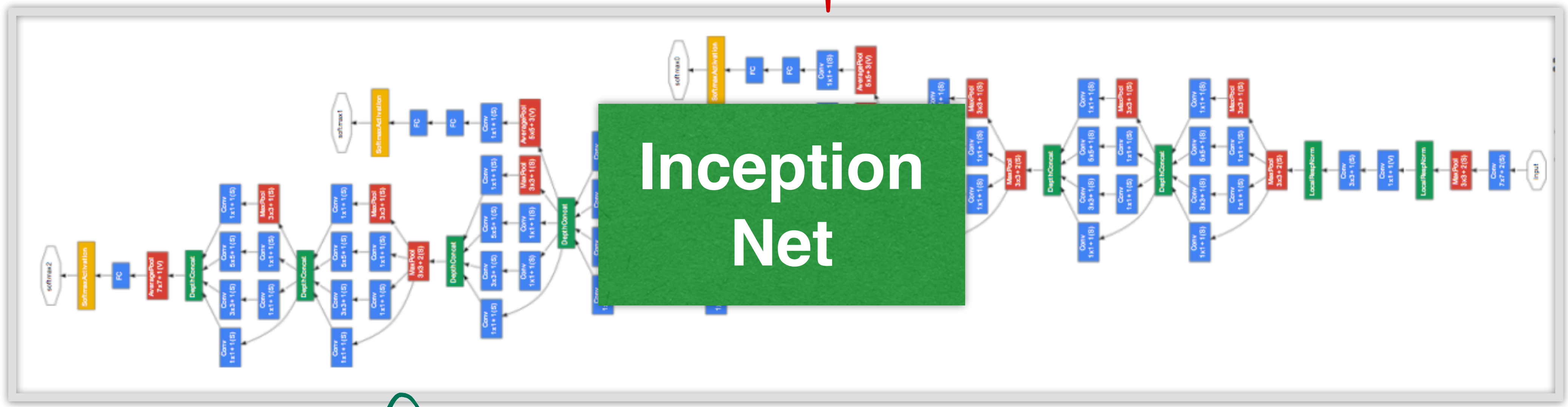
1989 Yann LeCun



2012



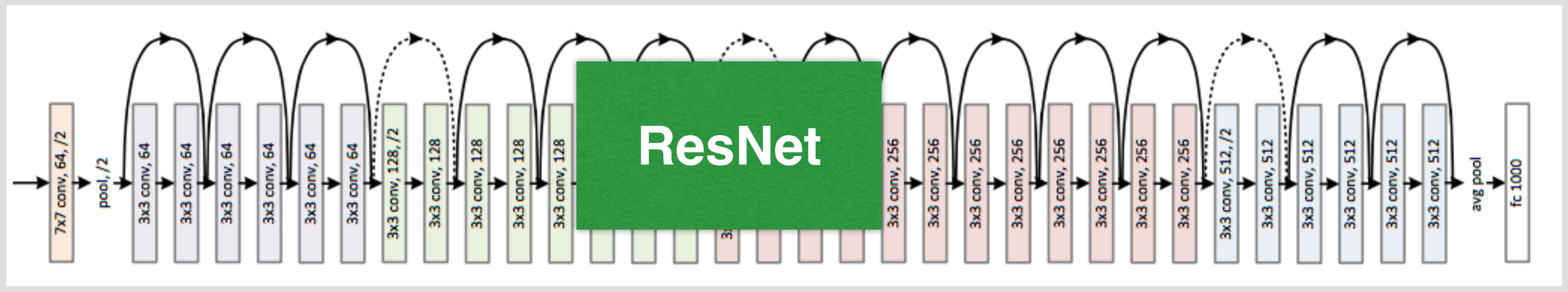
2014



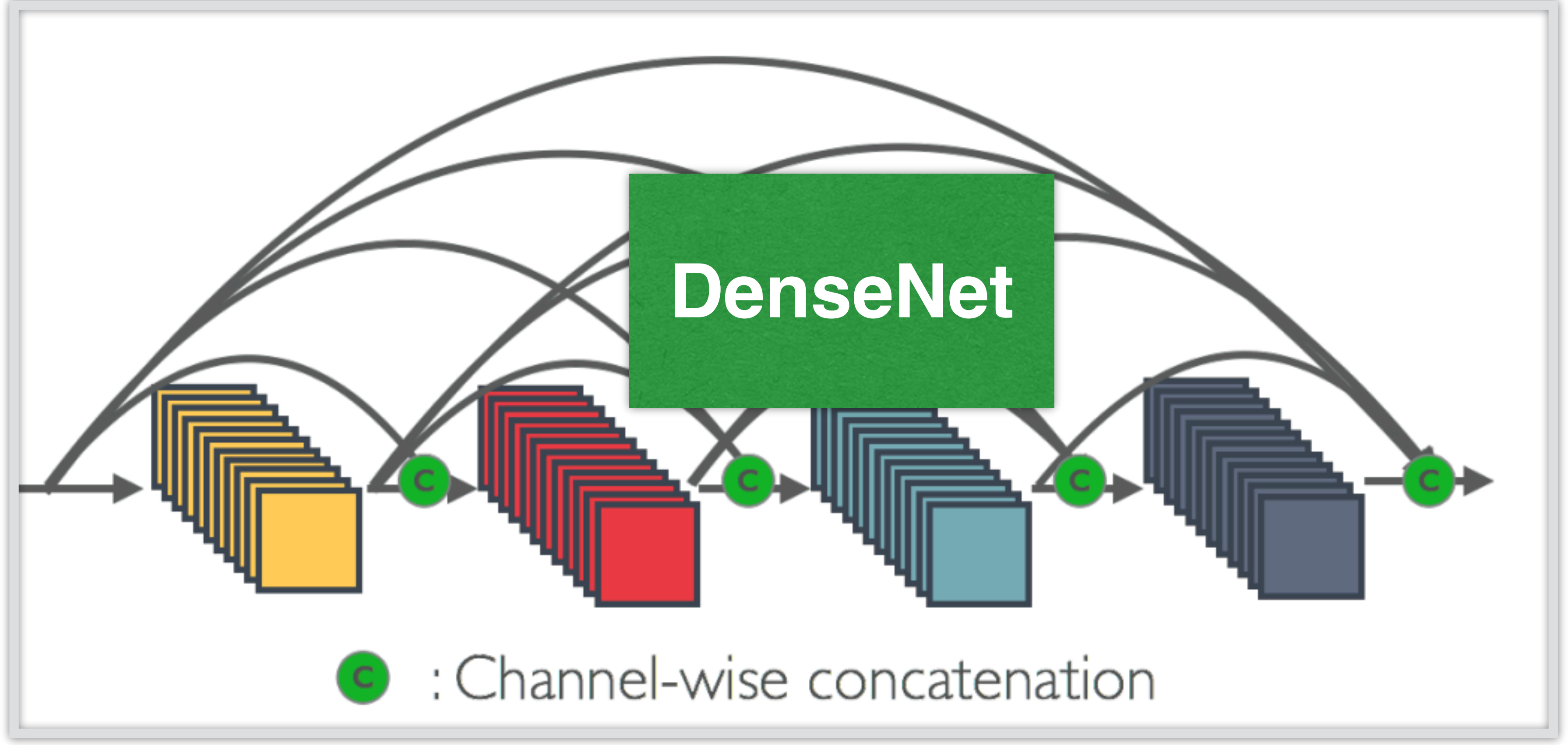
2014 VGG

GoogleLeNet

2015



2016



LeNet Architecture (first conv nets)

16x5x5

flatten into 400-dim vector.

$\sigma(\cdot) = \begin{cases} \text{Sigmoid (original)} \\ \text{ReLU (recent)} \end{cases}$

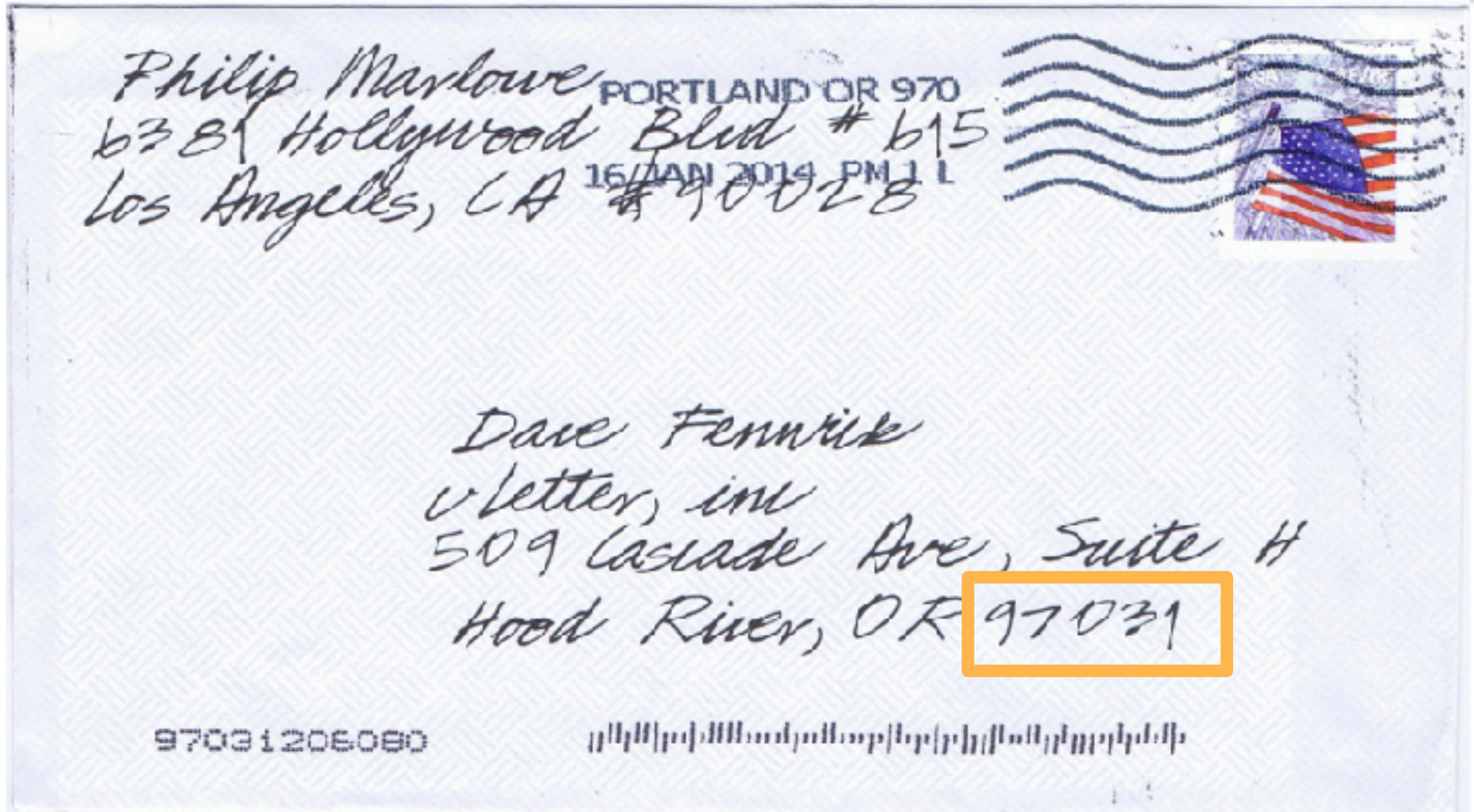
5x5 kernel

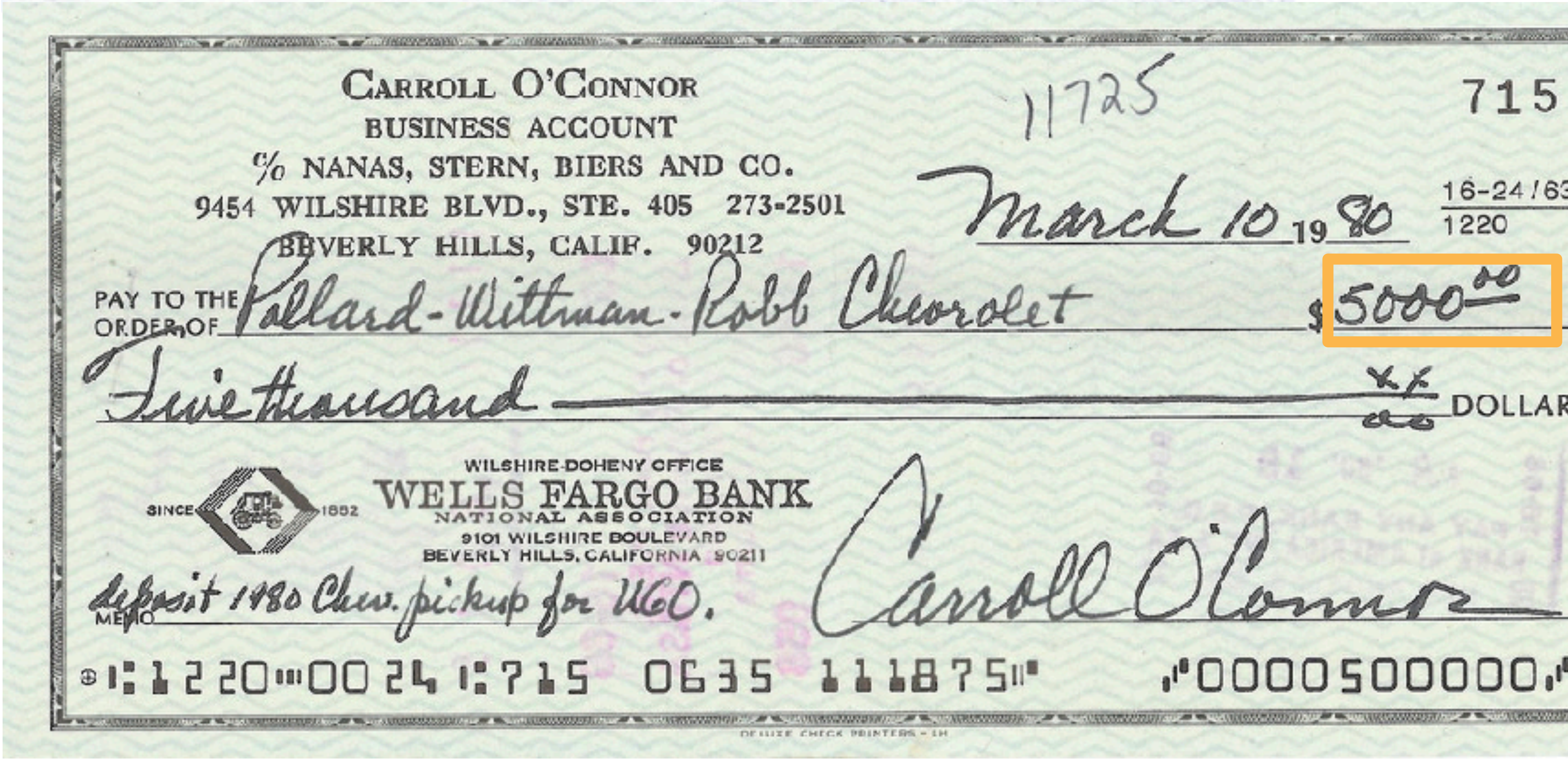
Co

↳ subsampling / pooling

Gradient-based learning applied to document recognition, by Y. LeCun, L. Bottou, Y. Bengio and P. Haffner

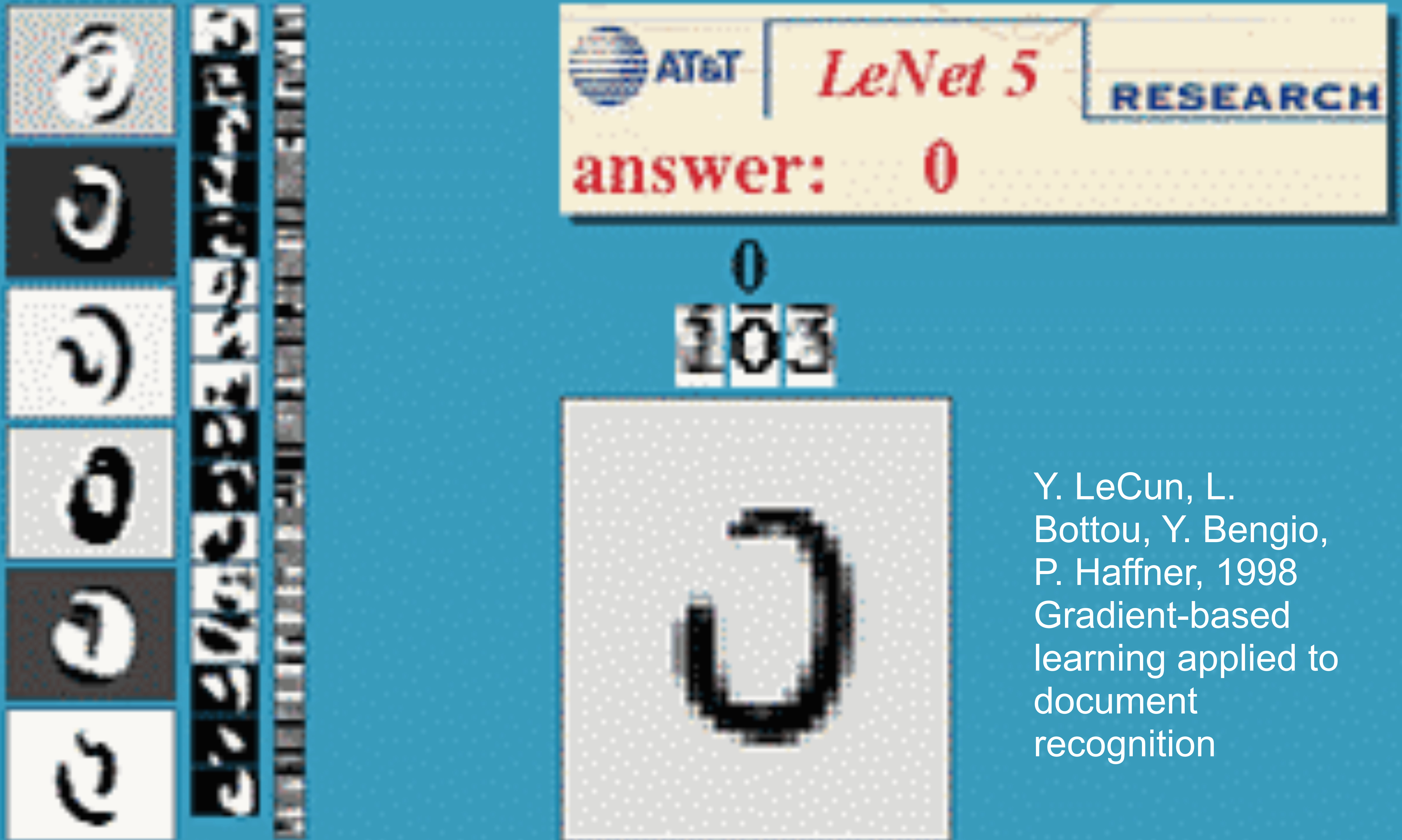
Handwritten Digit Recognition





MNIST

- Centered and scaled
- 50,000 training data
- 10,000 test data
- 28 x 28 images
- 10 classes



Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradient-based learning applied to document recognition

LeNet Architecture

Gradient-based learning applied to document recognition,
by Y. LeCun, L. Bottou, Y. Bengio and P. Haffner

LeNet in Pytorch

```
def __init__(self):
    super(LeNet5, self).__init__()
    # Convolution (In LeNet-5, 32x32 images are given as input. Hence padding of 2 is done below)
    self.conv1 = torch.nn.Conv2d(in_channels=1, out_channels=6, kernel_size=5, stride=1, padding=2, bias=True)
    # Max-pooling
    self.max_pool_1 = torch.nn.MaxPool2d(kernel_size=2)
    # Convolution
    self.conv2 = torch.nn.Conv2d(in_channels=6, out_channels=16, kernel_size=5, stride=1, padding=0, bias=True)
    # Max-pooling
    self.max_pool_2 = torch.nn.MaxPool2d(kernel_size=2)
    # Fully connected layer
    self.fc1 = torch.nn.Linear(16*5*5, 120)   # convert matrix with 16*5*5 (= 400) features to a matrix of 120 features (co
    self.fc2 = torch.nn.Linear(120, 84)       # convert matrix with 120 features to a matrix of 84 features (columns)
    self.fc3 = torch.nn.Linear(84, 10)        # convert matrix with 84 features to a matrix of 10 features (columns)
```

https://github.com/bollakarthikeya/LeNet-5-PyTorch/blob/master/lenet5_gpu.py

LeNet in Pytorch

```python
def forward(self, x):
    # convolve, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.conv1(x))
    # max-pooling with 2x2 grid
    x = self.max_pool_1(x)
    # convolve, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.conv2(x))
    # max-pooling with 2x2 grid
    x = self.max_pool_2(x)
    # first flatten 'max_pool_2_out' to contain 16*5*5 columns
    # read through https://stackoverflow.com/a/42482819/7551231
    x = x.view(-1, 16*5*5)
    # FC-1, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.fc1(x))
    # FC-2, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.fc2(x))
    # FC-3
    x = self.fc3(x)

    return x
```

Let’s walk through an example using PyTorch

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html

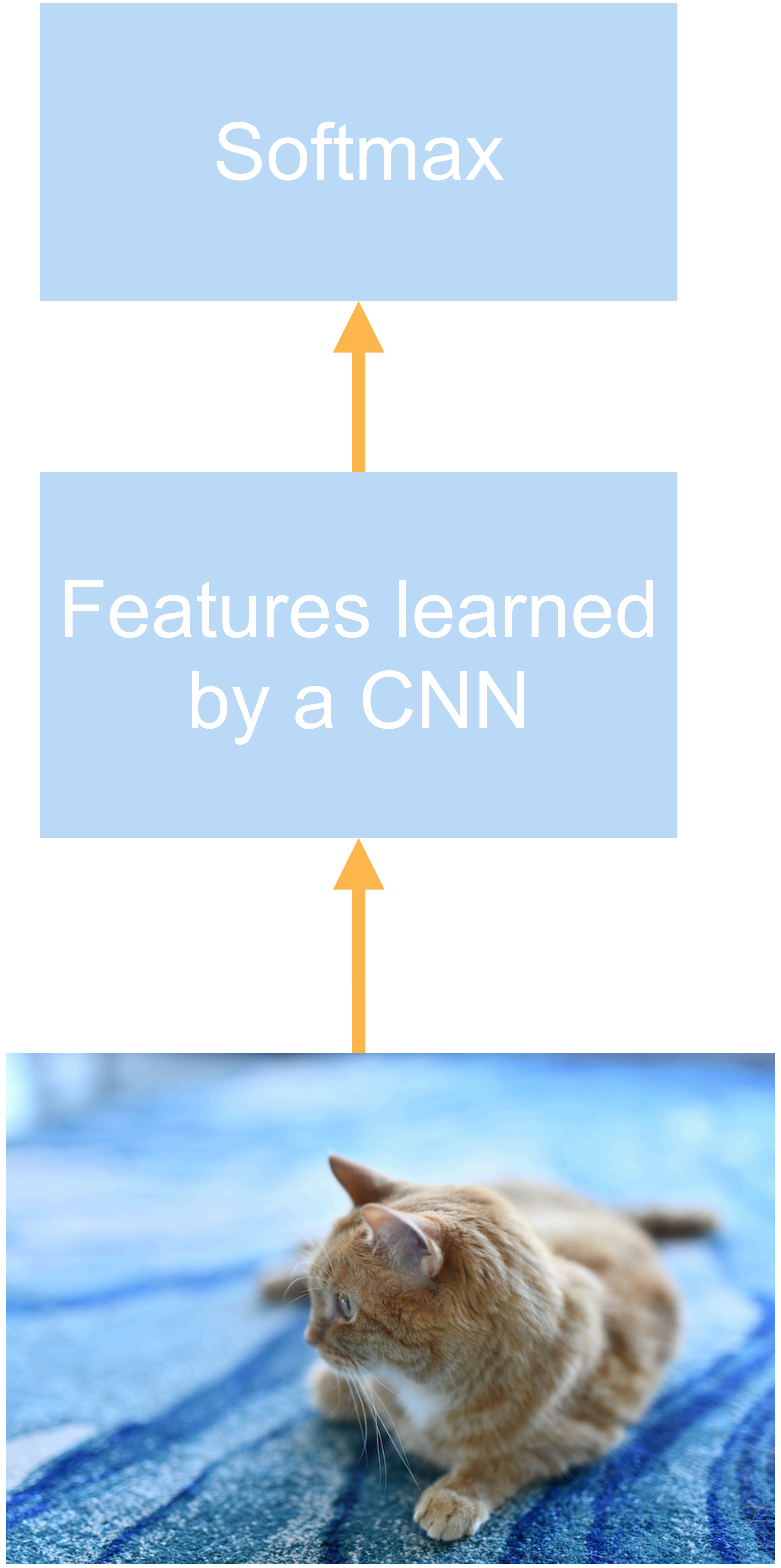
Deng et al. 2009

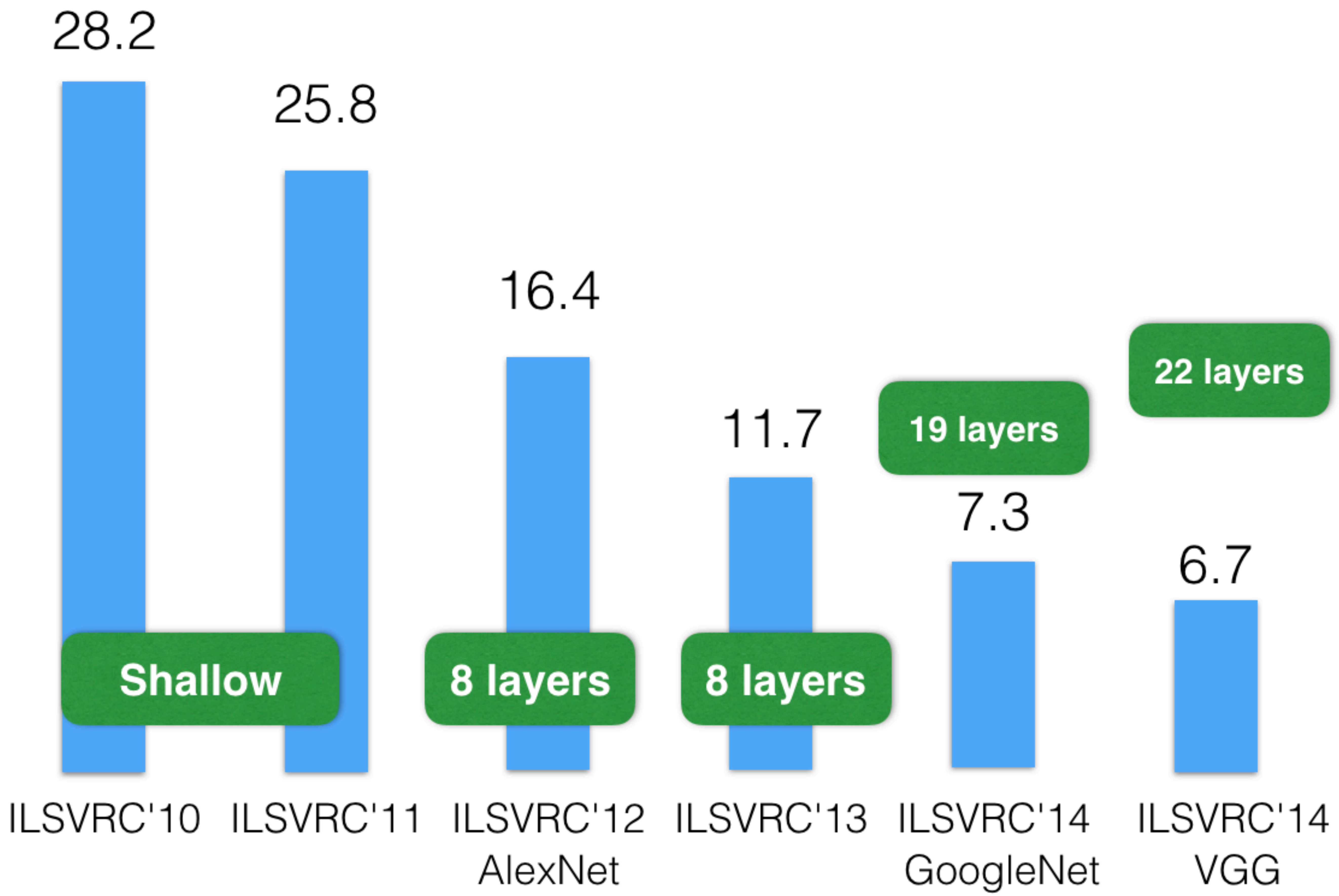
Alex Krizhesky, Ilya Sutskever, Geoff Hinton, 2012

AlexNet

AlexNet

- AlexNet won ImageNet competition in 2012
- Deeper and bigger LeNet
- Paradigm shift for computer vision



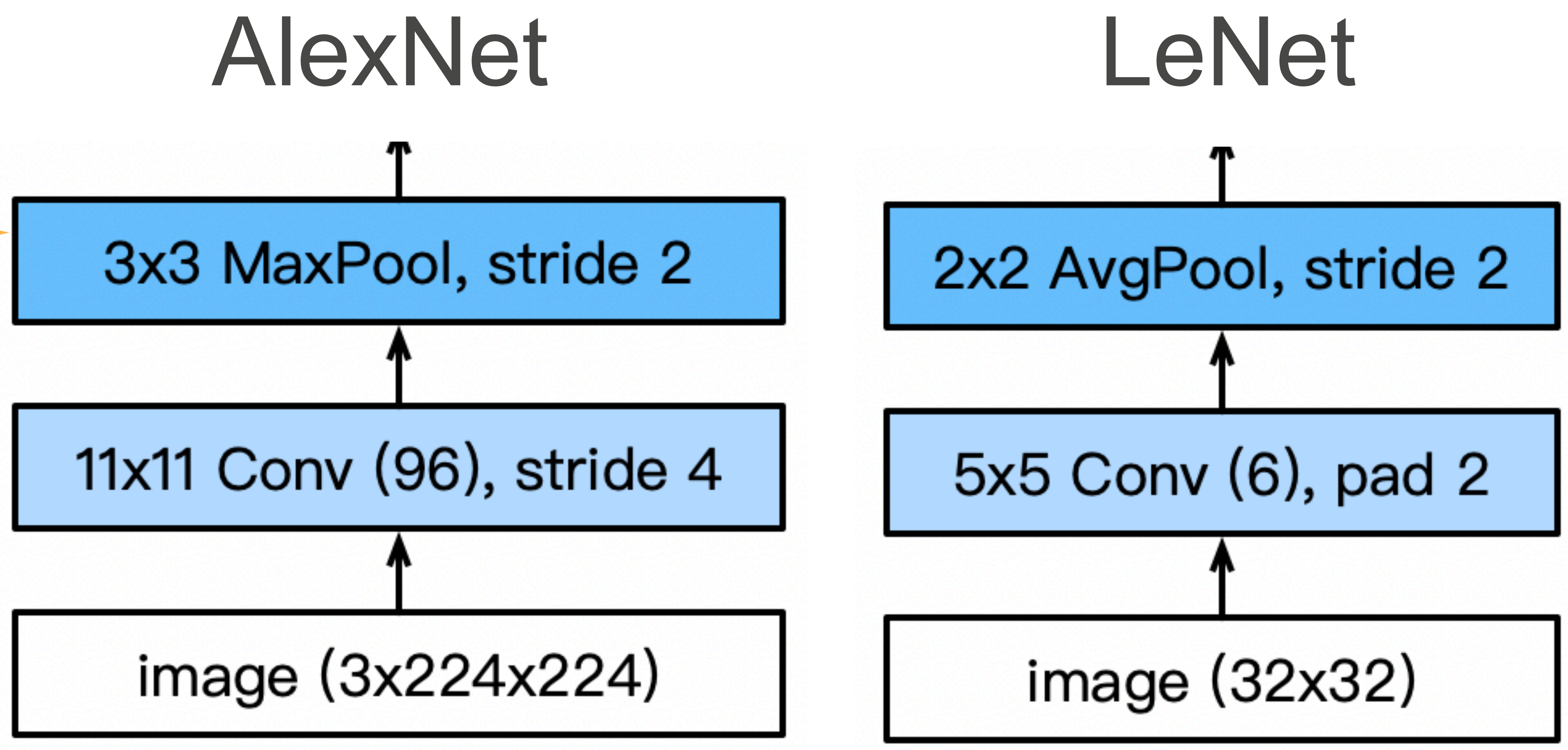


ImageNet Top-5 Classification Accuracy (%)

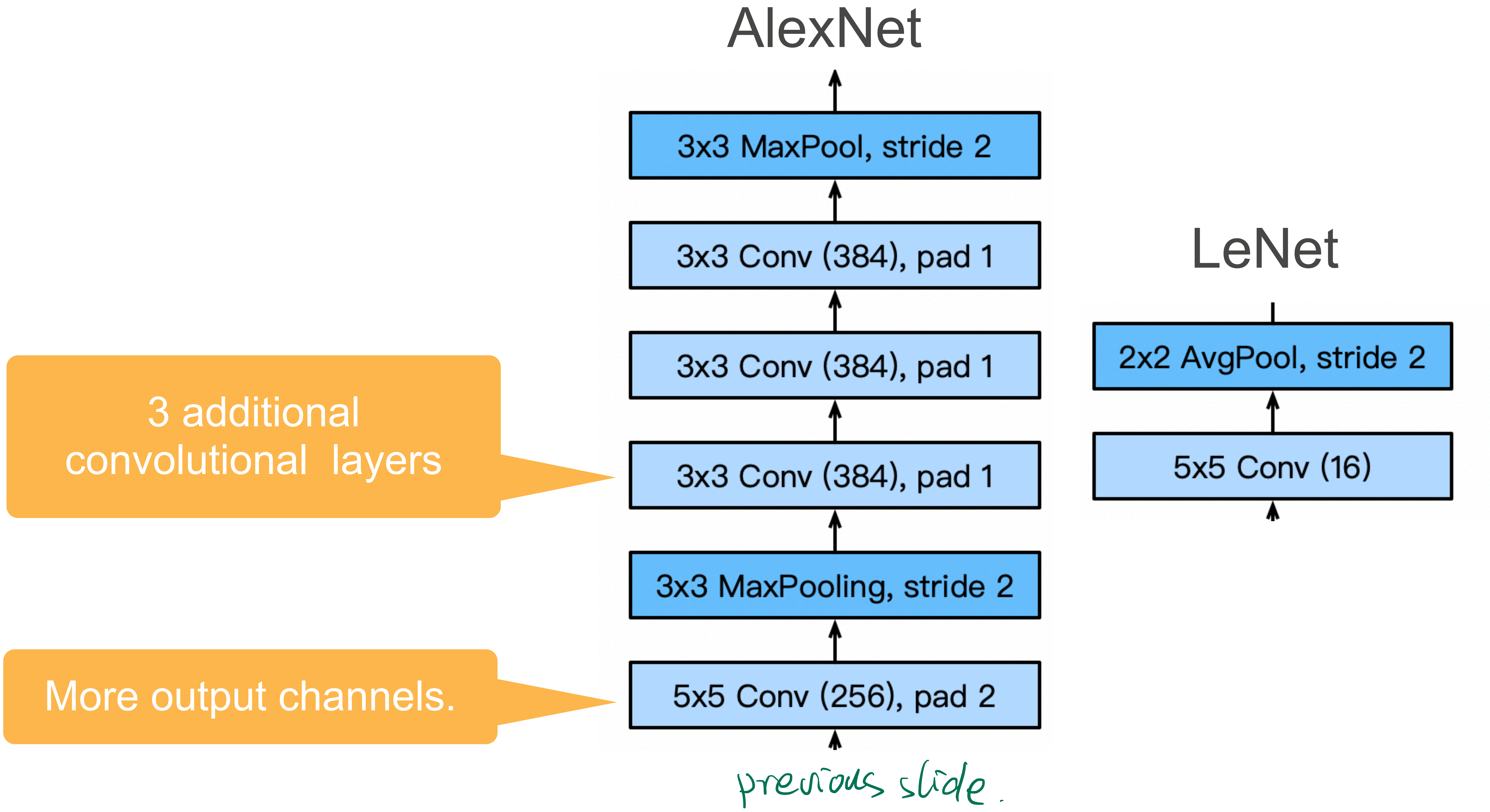
AlexNet Architecture

Larger pool size

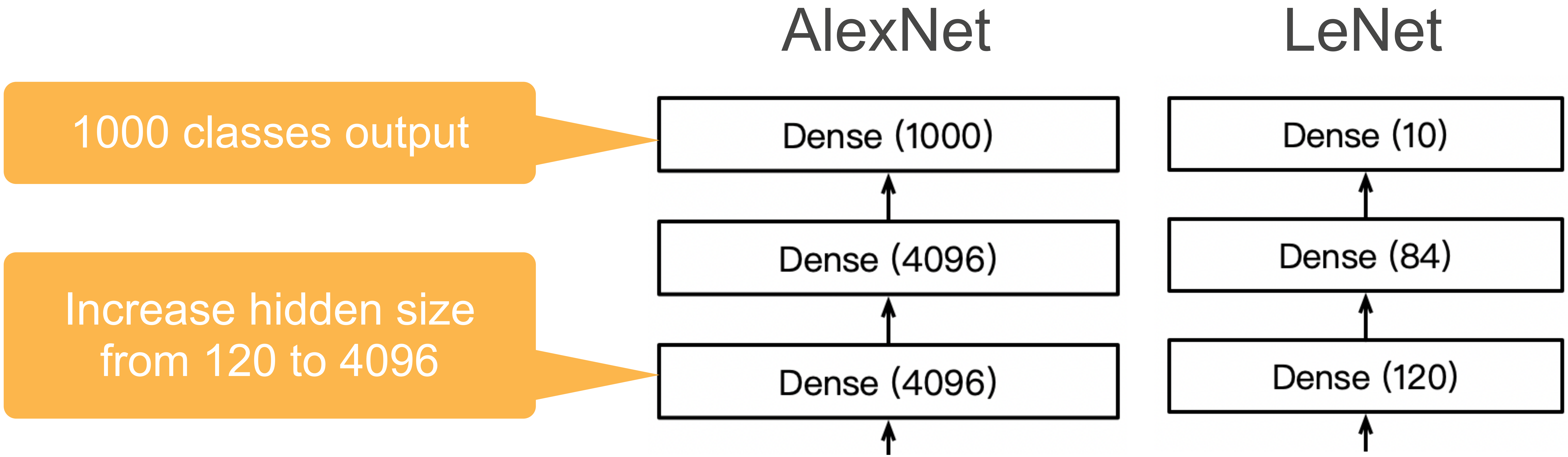
Larger kernel size, stride because of the increased image size, and more output channels.



AlexNet Architecture

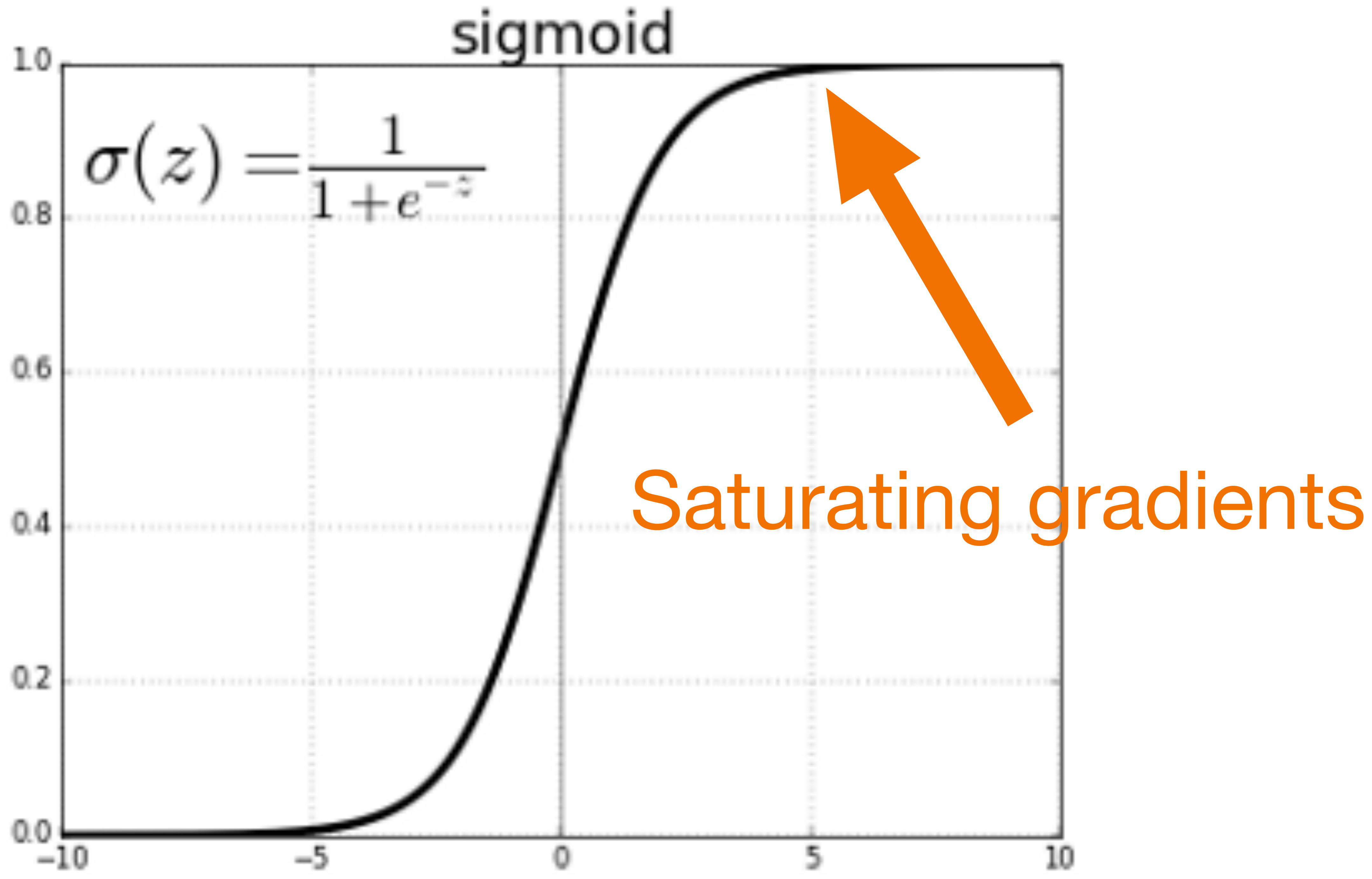


AlexNet Architecture



More Differences…

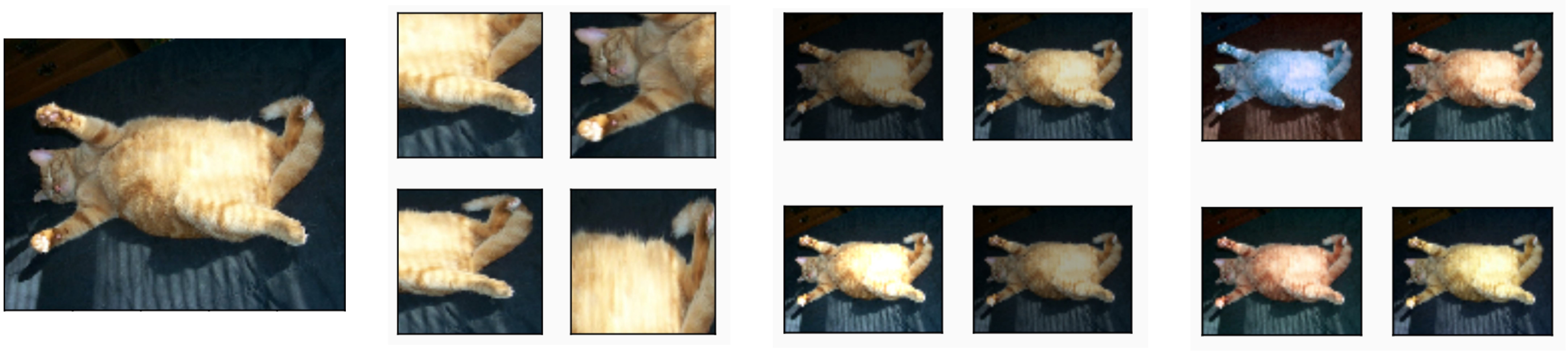
- Change activation function from sigmoid to ReLu (no more vanishing gradient)



More Differences…

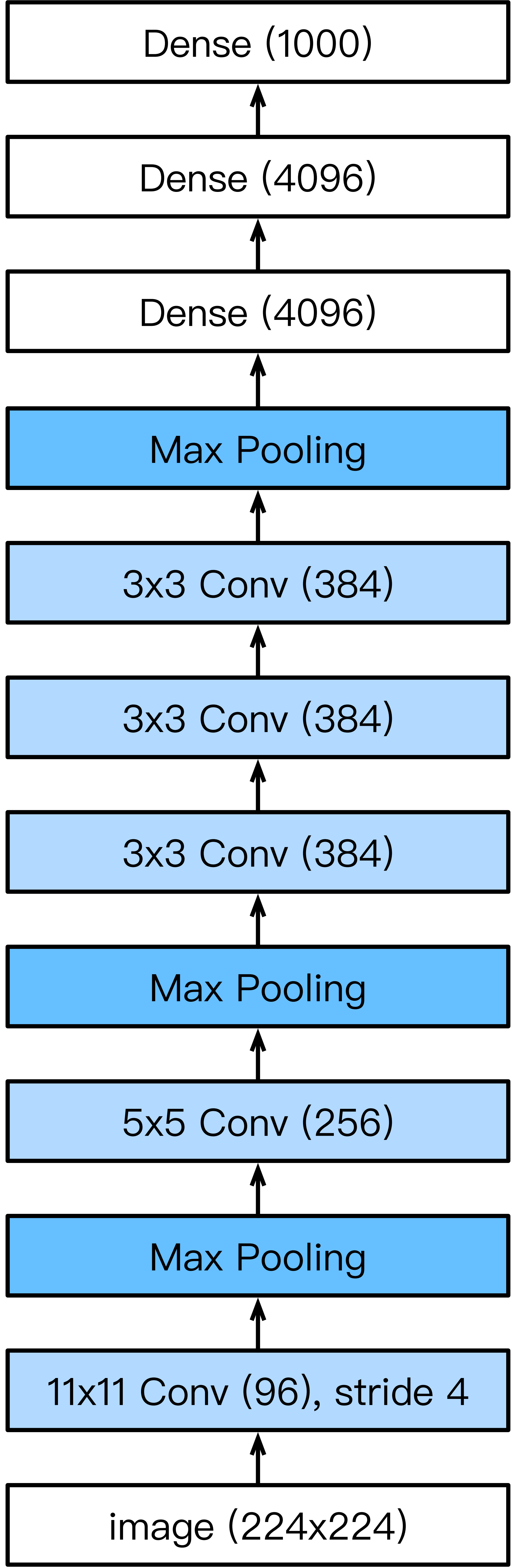
- Change activation function from sigmoid to ReLu (no more vanishing gradient)
- Data augmentation

- Dropout



Complexity

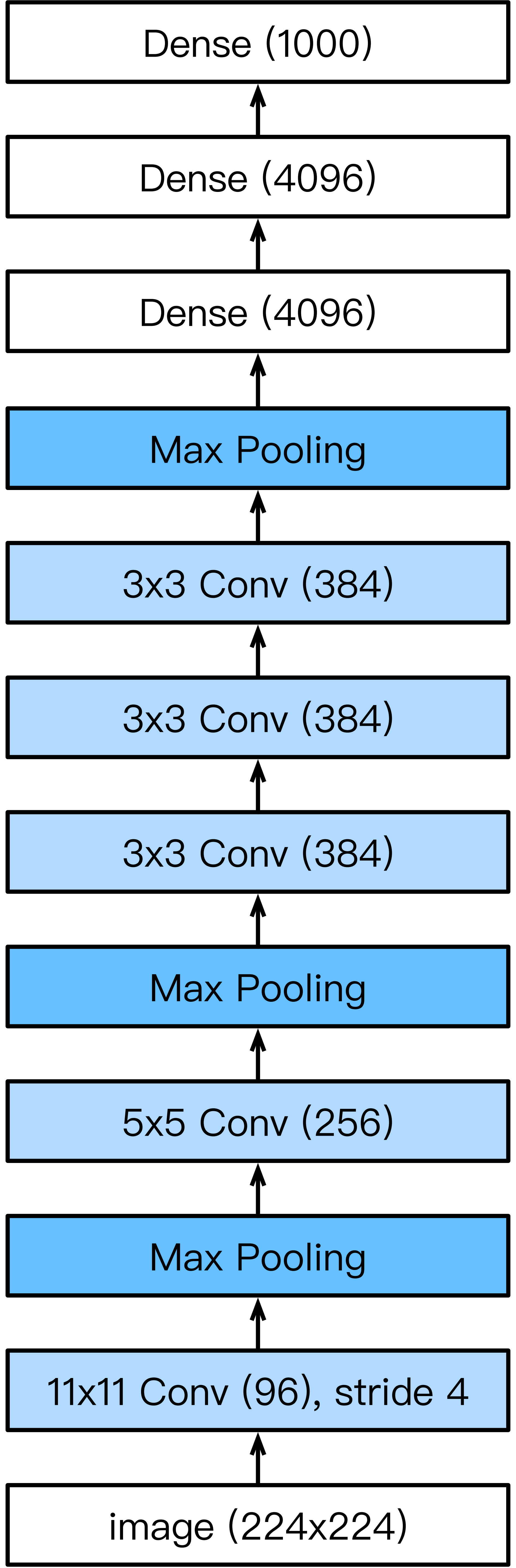
	#parameters	
	AlexNet	**LeNet**
Conv1	35K	150
Conv2	614K	2.4K
Conv3-5	3M	
Dense1	26M	0.048M
Dense2	16M	0.01M
Total	46M	0.06M
Increase	11x	1x



Complexity

	#parameters	
	AlexNet	LeNet
Conv1	$\boxed{35K}$	150
Conv2	614K	2.4K
Conv3-5	3M	
Dense1	26M	0.048M
Dense2	16M	0.01M
Total	46M	0.06M
Increase	11x	1x

11x11x3x96=35k



Which of the following are true about AlexNet? Select all that apply.

A. AlexNet contains 8 layers. The first five are convolutional layers.

B.The last three layers are fully connected layers.

C.some of the convolutional layers are followed by max-pooling (layers).

D. AlexNet achieved excellent performance in the 2012 ImageNet challenge.

Krizhevsky, A., Sutskever, I., & Hinton, G. E. (2012). Imagenet classification with deep convolutional neural networks. *Advances in neural information processing systems* (pp. 1097–1105).

Which of the following are true about AlexNet? Select all that apply.

A. AlexNet contains 8 layers. The first five are convolutional layers.

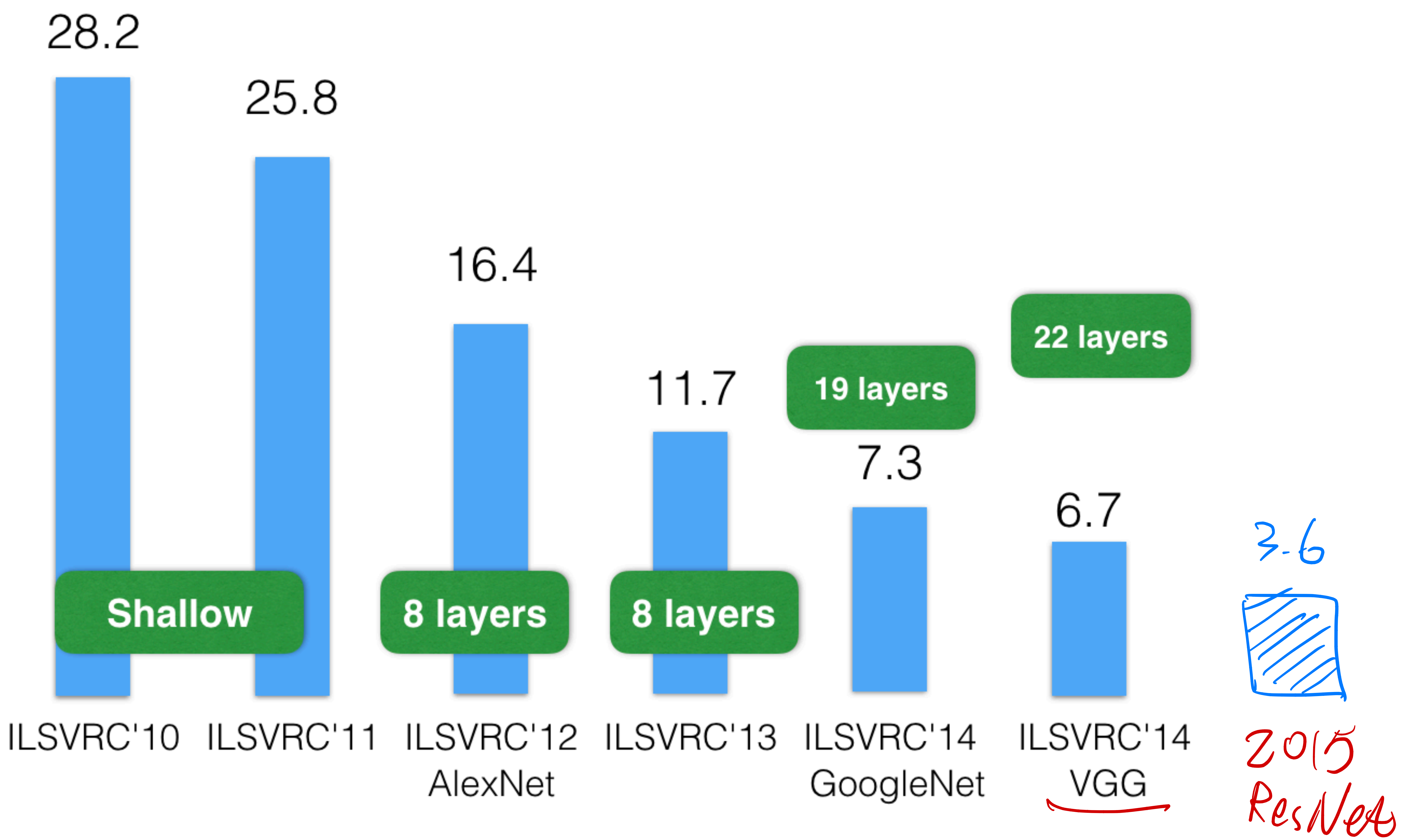
B.The last three layers are fully connected layers.

C.some of the convolutional layers are followed by max-pooling (layers).

D. AlexNet achieved excellent performance in the 2012 ImageNet challenge.

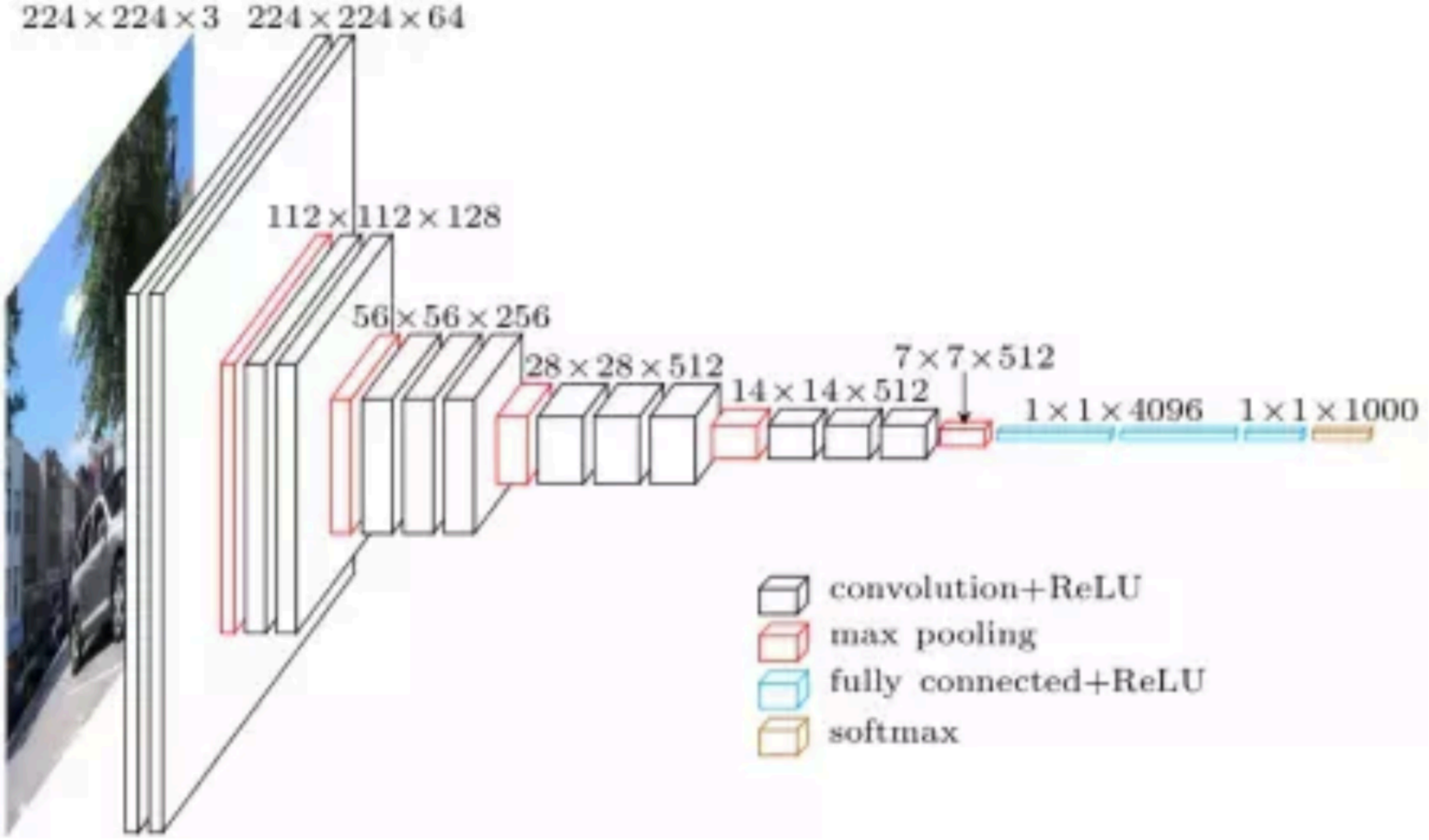
All options are true!

Krizhevsky, A., Sutskever, I., & Hinton, G. E. (2012). Imagenet classification with deep convolutional neural networks. *Advances in neural information processing systems* (pp. 1097–1105).



ImageNet Top-5 Classification Accuracy (%)

VGG



224 × 224 × 3
224 × 224 × 64
112 × 112 × 128
56 × 56 × 256
28 × 28 × 512
14 × 14 × 512
7 × 7 × 512
1 × 1 × 4096
1 × 1 × 1000
convolution+ReLU
max pooling
fully connected+ReLU
softmax

Progress

- LeNet (1995)
  - 2 convolution + pooling layers
  - 2 hidden dense layers
- AlexNet
  - Bigger and deeper LeNet
  - ReLu, preprocessing
- VGG
  - Bigger and deeper AlexNet (repeated VGG blocks)

Which of the following statement is True for the success of deep models?

- Better design of the neural networks
- Large scale training dataset
- Available computing power
- All of the above

Which of the following statement is True for the success of deep models?

- Better design of the neural networks
- Large scale training dataset
- Available computing power
- All of the above

What we’ve learned today

- Brief review of convolutional computations
- Convolutional Neural Networks
    - LeNet (first conv nets)
    - AlexNet
- PyTorch demo

Acknowledgement:

Some of the slides in these lectures have been adapted/borrowed from materials developed by Yin Li (https://happyharrycn.github.io/CS540-Fall20/schedule/), Alex Smola and Mu Li:
https://courses.d2l.ai/berkeley-stat-157/index.html