



CS540 Introduction to Artificial Intelligence

Convolutional Neural Networks (II)

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November 4, 2021

Slides created by Sharon Li [modified by Yudong Chen]

Outline

- Brief review of convolutional computations
- Convolutional Neural Networks
 - LeNet (first conv nets)
 - AlexNet

Review: 2-D Convolution

Input

0	1	2
3	4	5
6	7	8

*

Kernel

0	1
2	3

=

Output

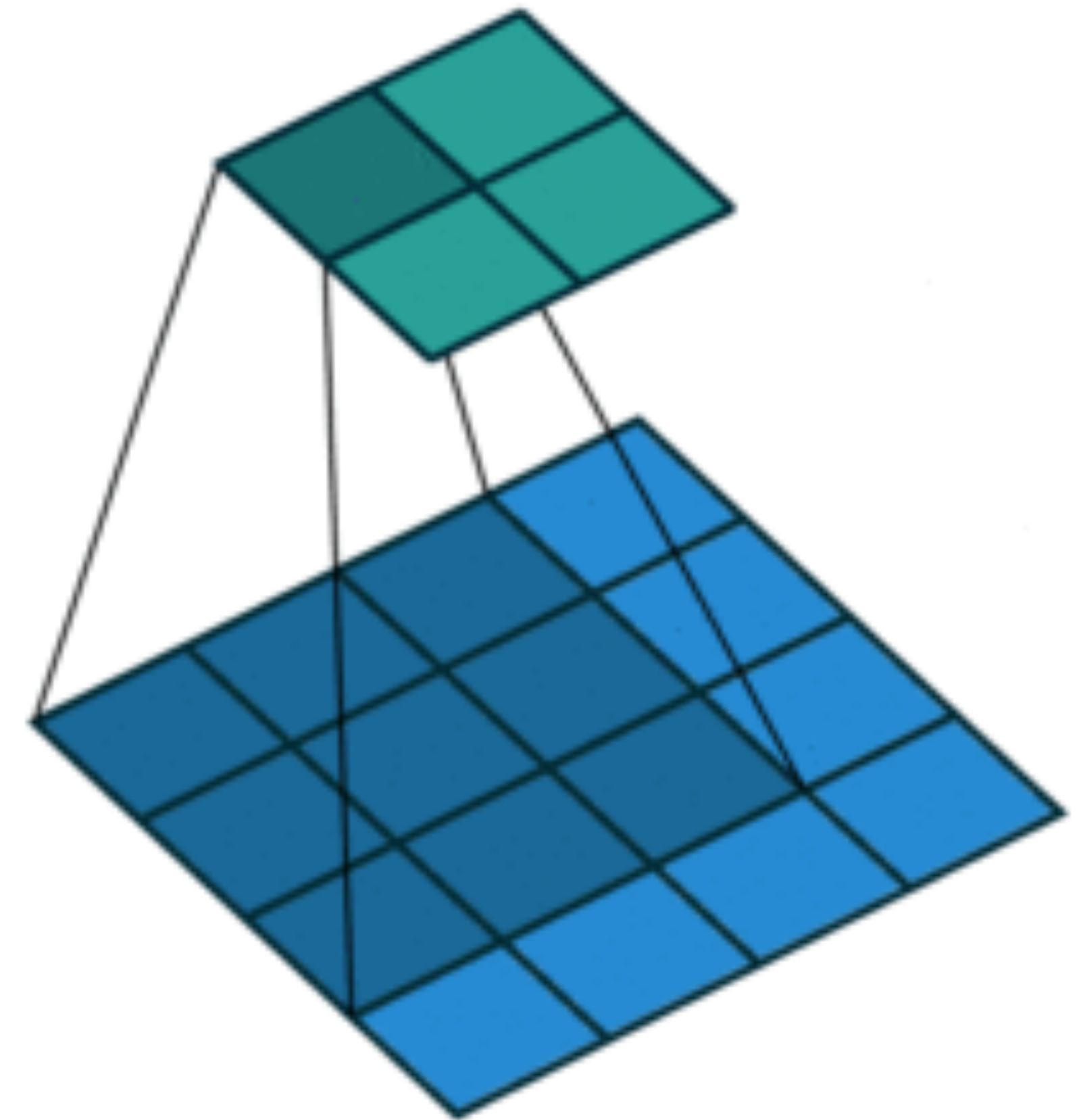
19	25
37	43

$$0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3 = 19,$$

$$1 \times 0 + 2 \times 1 + 4 \times 2 + 5 \times 3 = 25,$$

$$3 \times 0 + 4 \times 1 + 6 \times 2 + 7 \times 3 = 37,$$

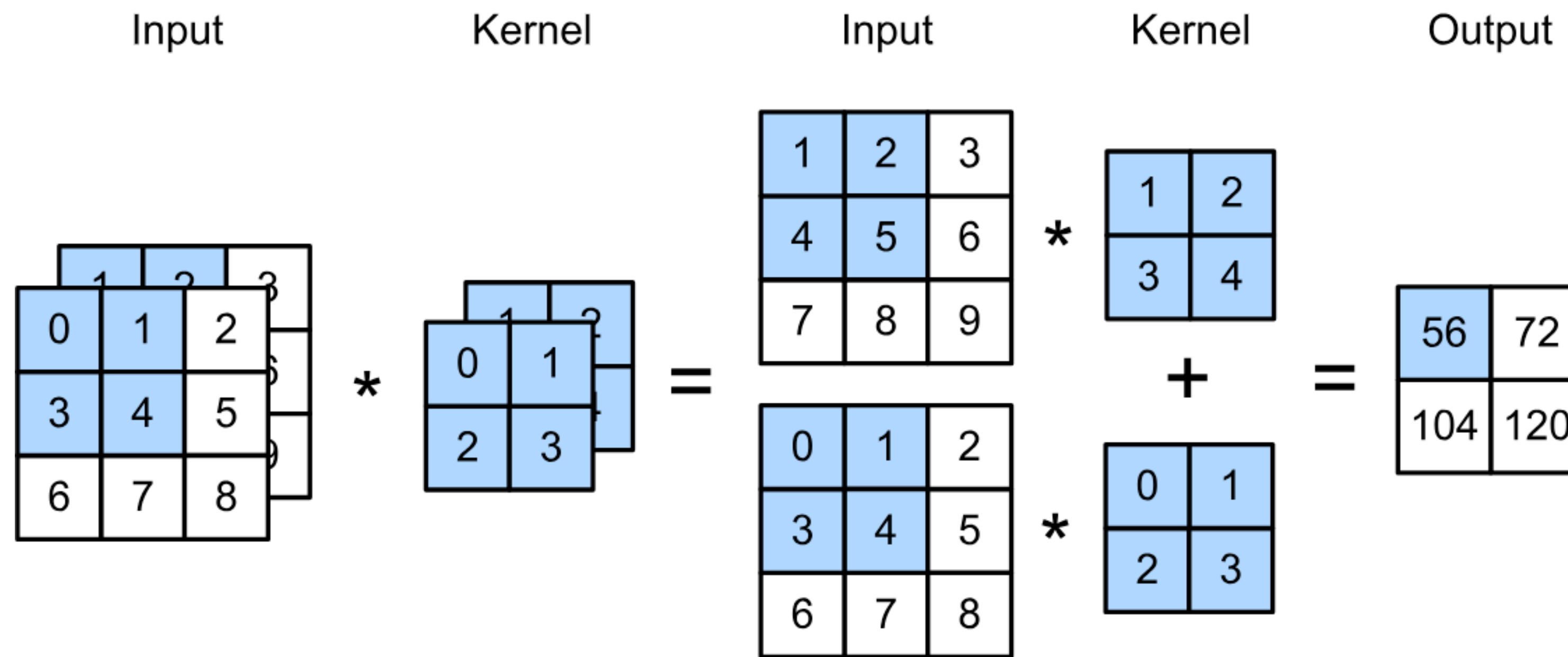
$$4 \times 0 + 5 \times 1 + 7 \times 2 + 8 \times 3 = 43.$$



(vdumoulin@ Github)

Review: Multiple Input Channels

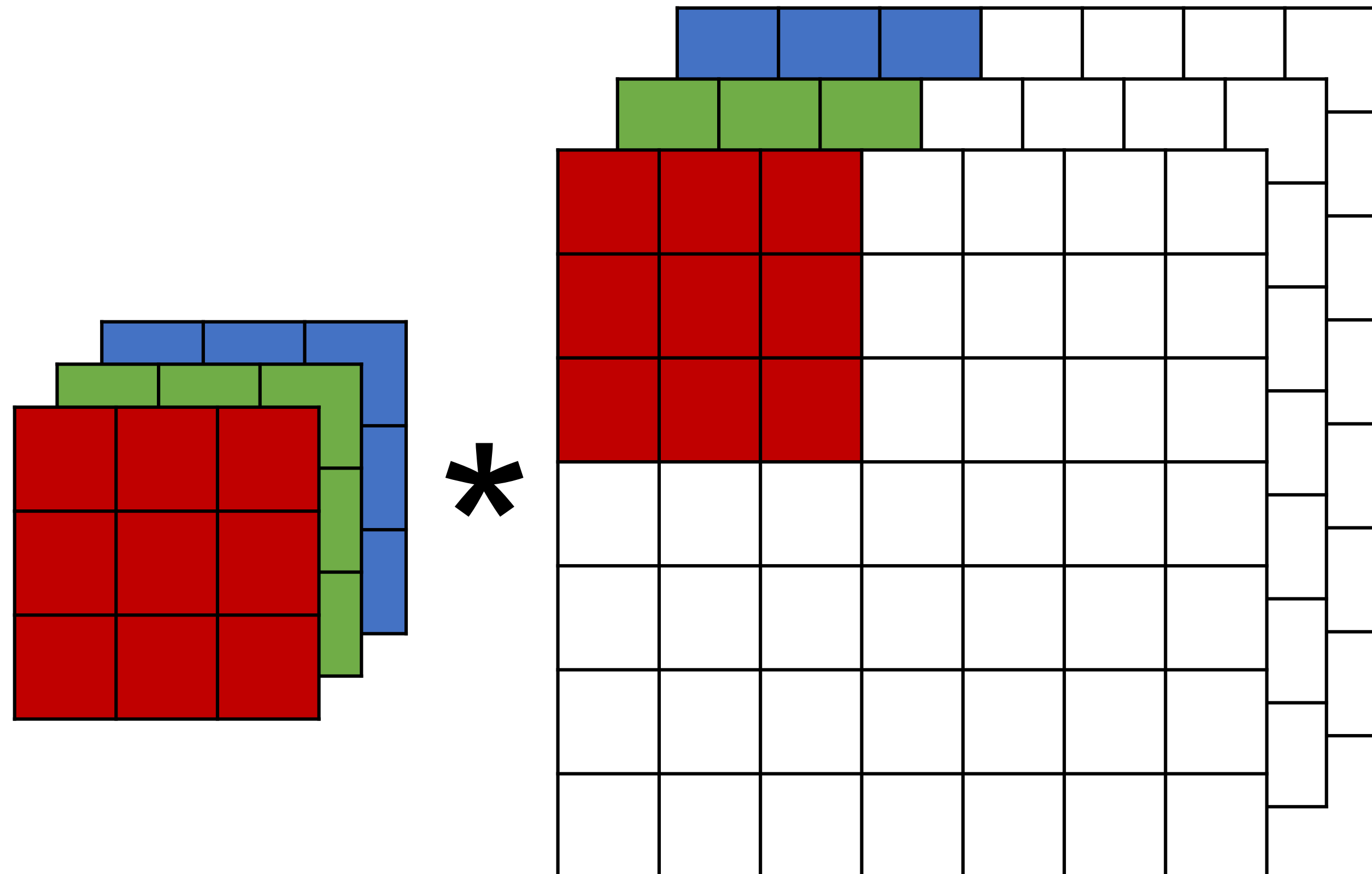
- Input and kernel can be 3D, e.g., an RGB image have 3 channels
- Have a kernel for each channel, and then sum results over channels



$$(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4) \\ + (0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3) \\ = 56$$

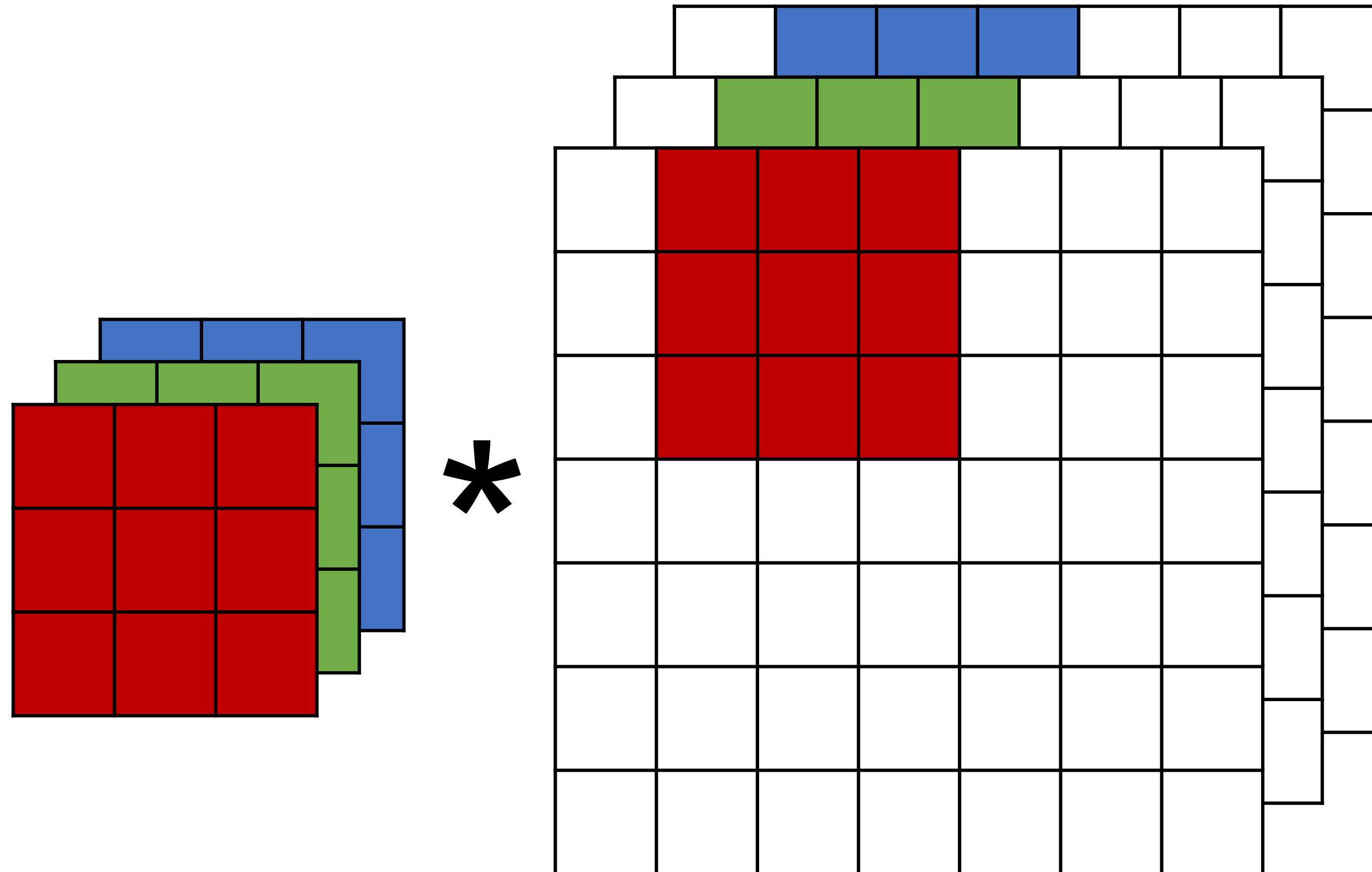
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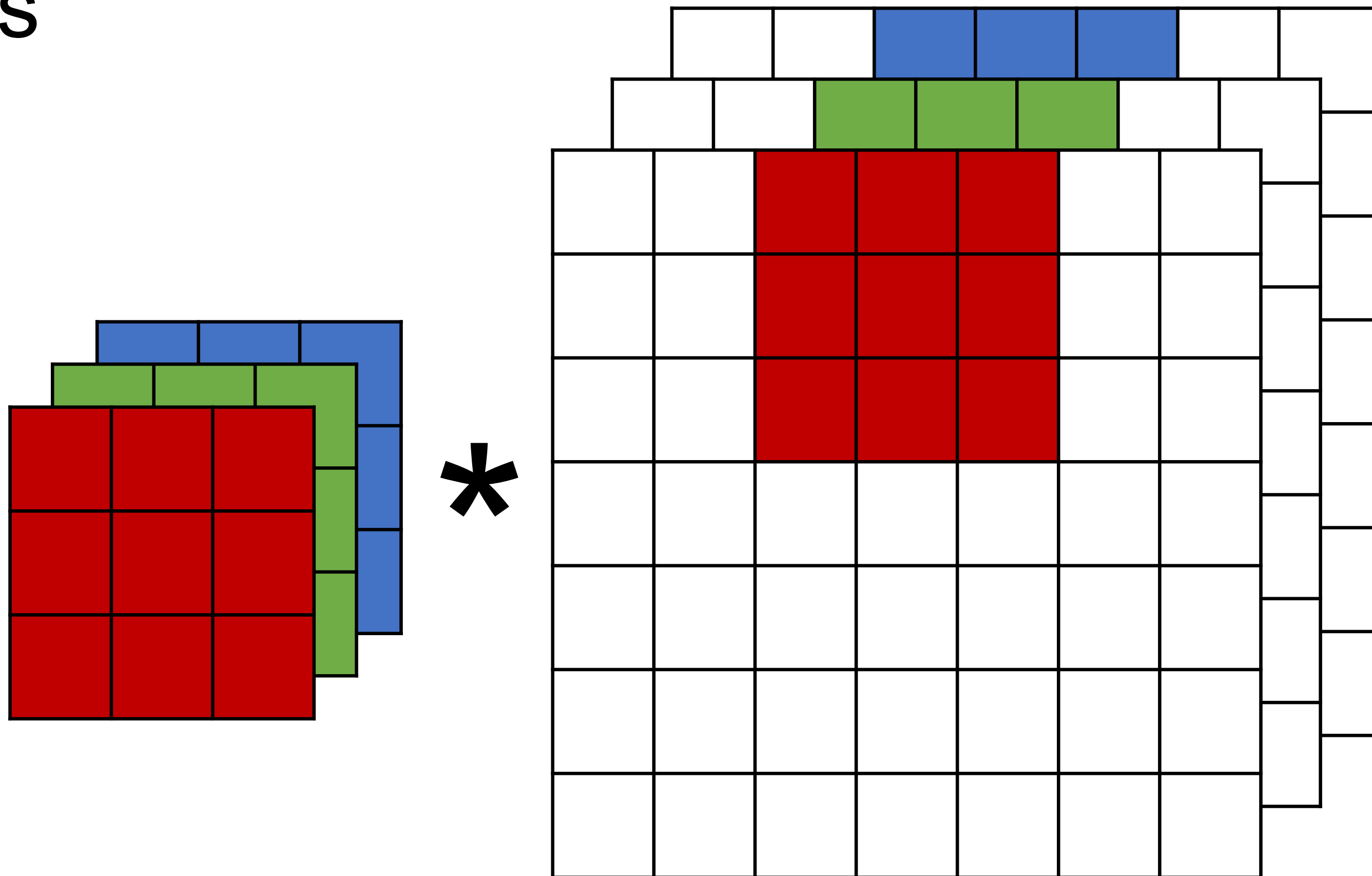
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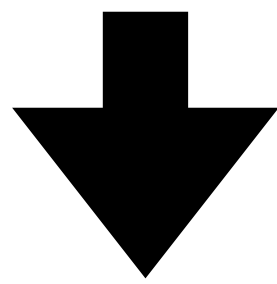
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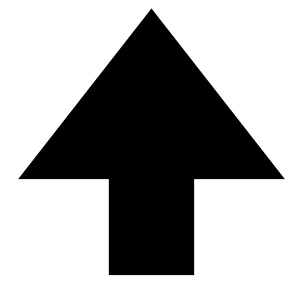


Output shape

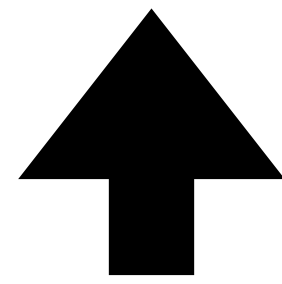
Kernel/filter size



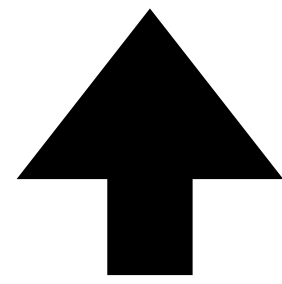
$$\lfloor (n_h - k_h + p_h + s_h) / s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w) / s_w \rfloor$$



Input size



Pad



Stride

Consider a convolution layer with 16 filters. Each filter has a size of $11 \times 11 \times 3$, a stride of 2×2 . Given an input image of size $22 \times 22 \times 3$, if we don't allow a filter to fall outside of the input, what is the output size?

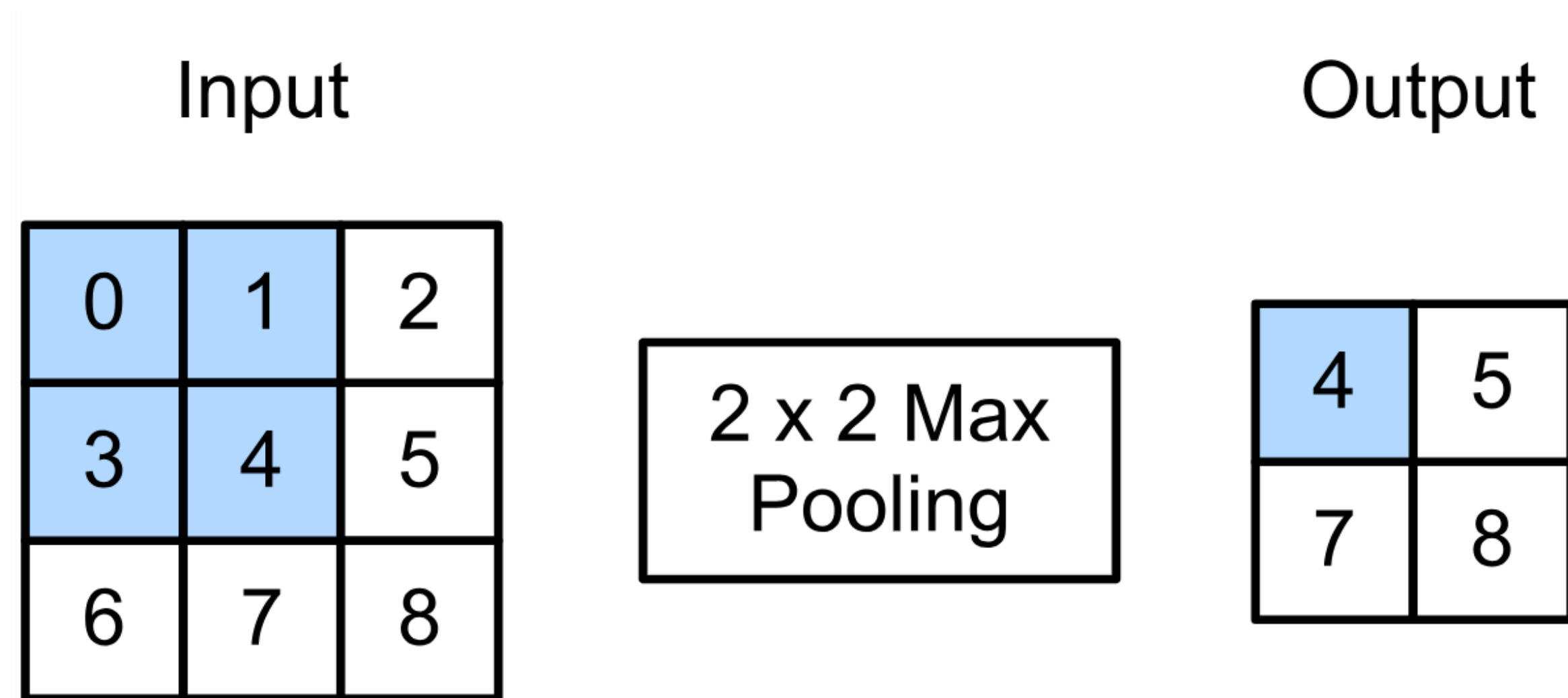
- $11 \times 11 \times 16$
- $6 \times 6 \times 16$
- $7 \times 7 \times 16$
- $5 \times 5 \times 16$



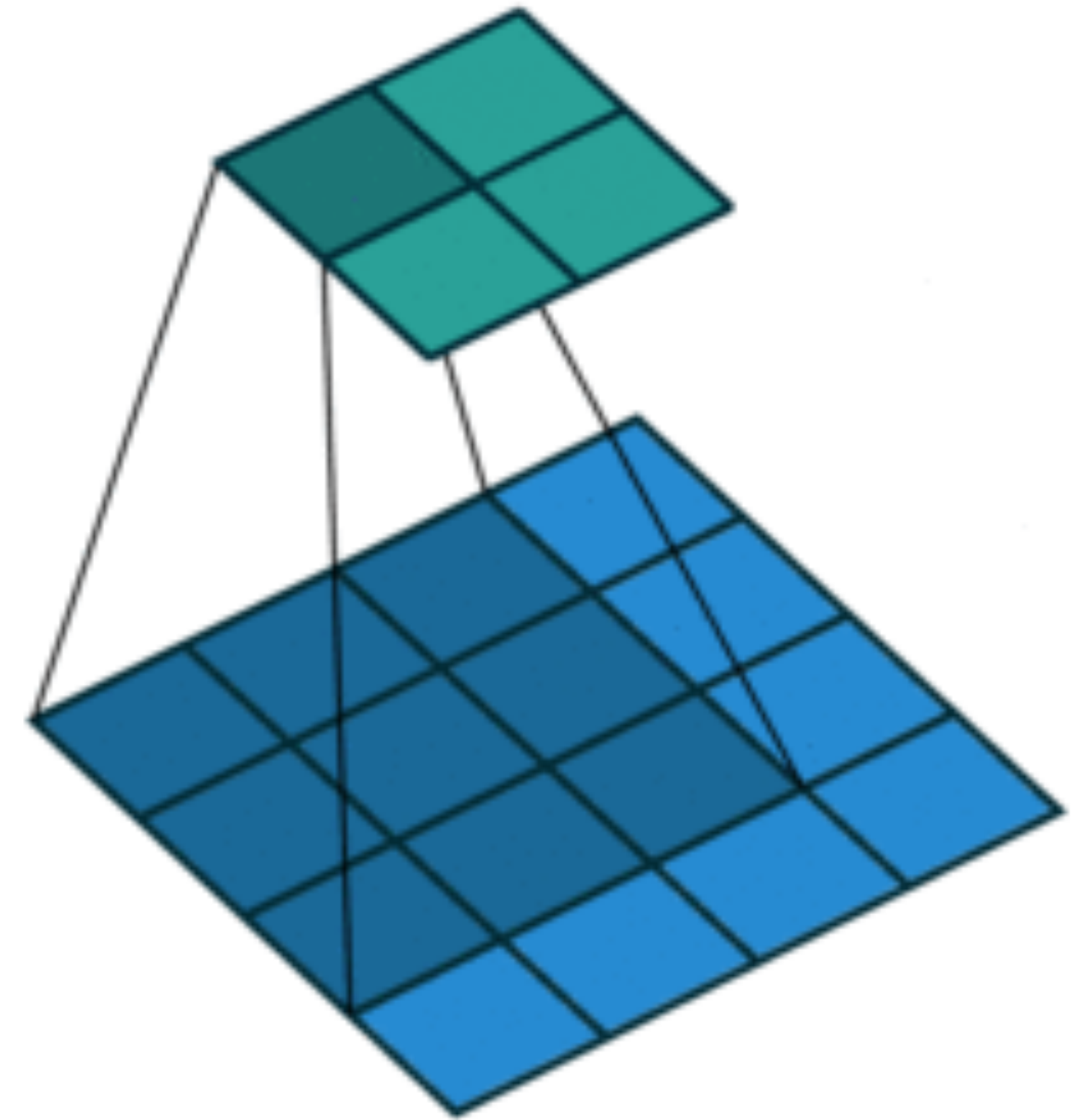
Pooling Layer

2-D Max Pooling

- Returns the maximal value in the sliding window



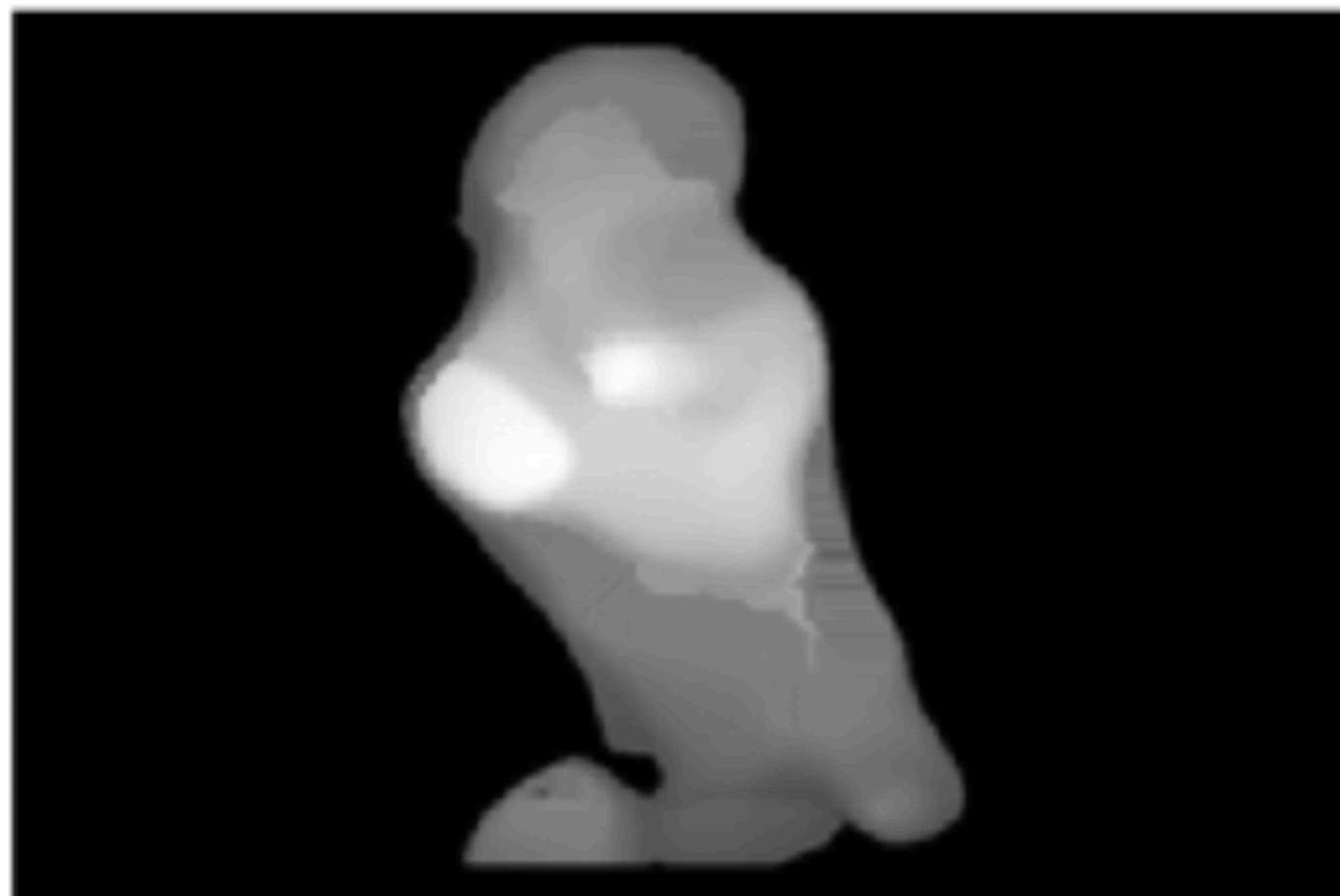
$$\max(0, 1, 3, 4) = 4$$



Average Pooling

- Max pooling: the strongest pattern signal in a window
- Average pooling: replace max with mean in max pooling
- The average signal strength in a window

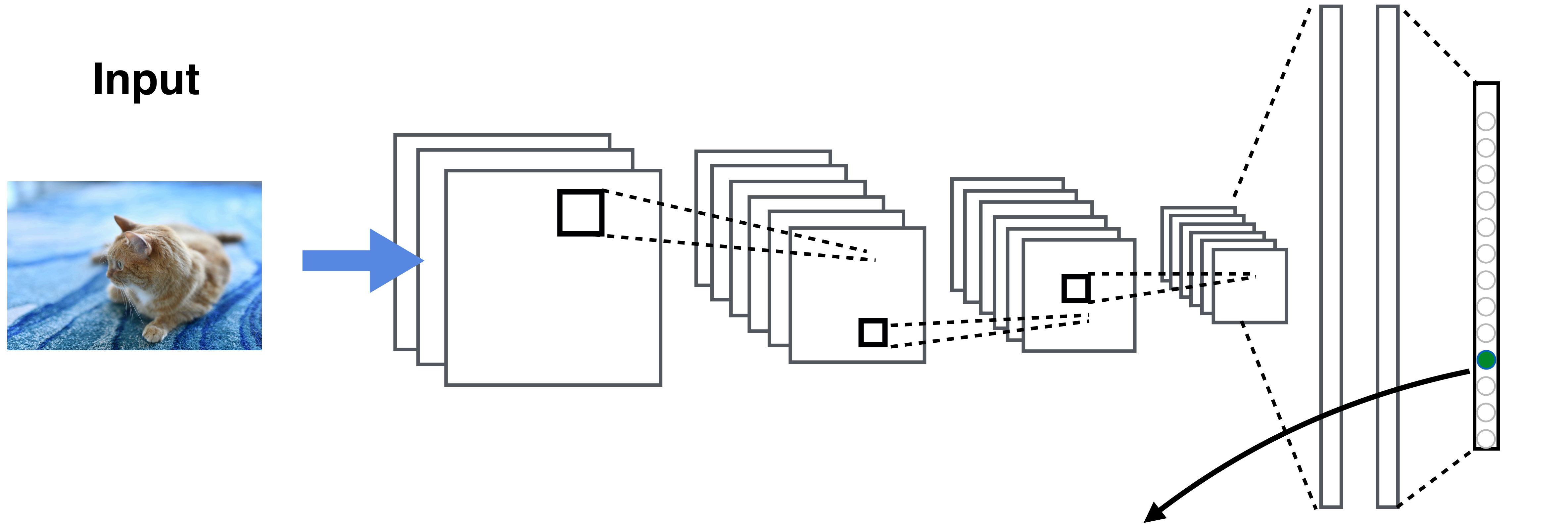
Max pooling



Average pooling



How to train a convolutional neural network?



$$p_i(\mathbf{x}) = \frac{\exp(f_i(\mathbf{x}))}{\sum_{j=1}^N \exp(f_j(\mathbf{x}))}, \text{ softmax}$$

Recall Softmax

Turns outputs f into probabilities (sum up to 1 across k classes)

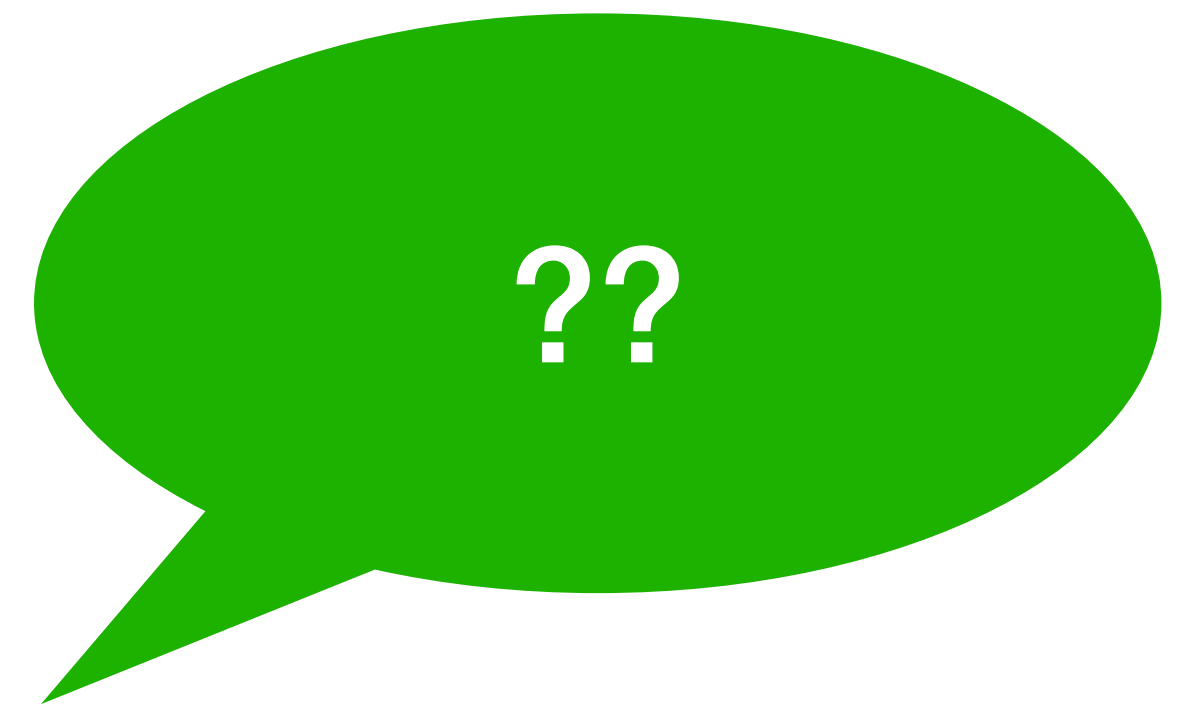
Output
layer

$$\begin{bmatrix} 1.3 \\ 5.1 \\ 2.2 \\ 0.7 \\ 1.1 \end{bmatrix}$$



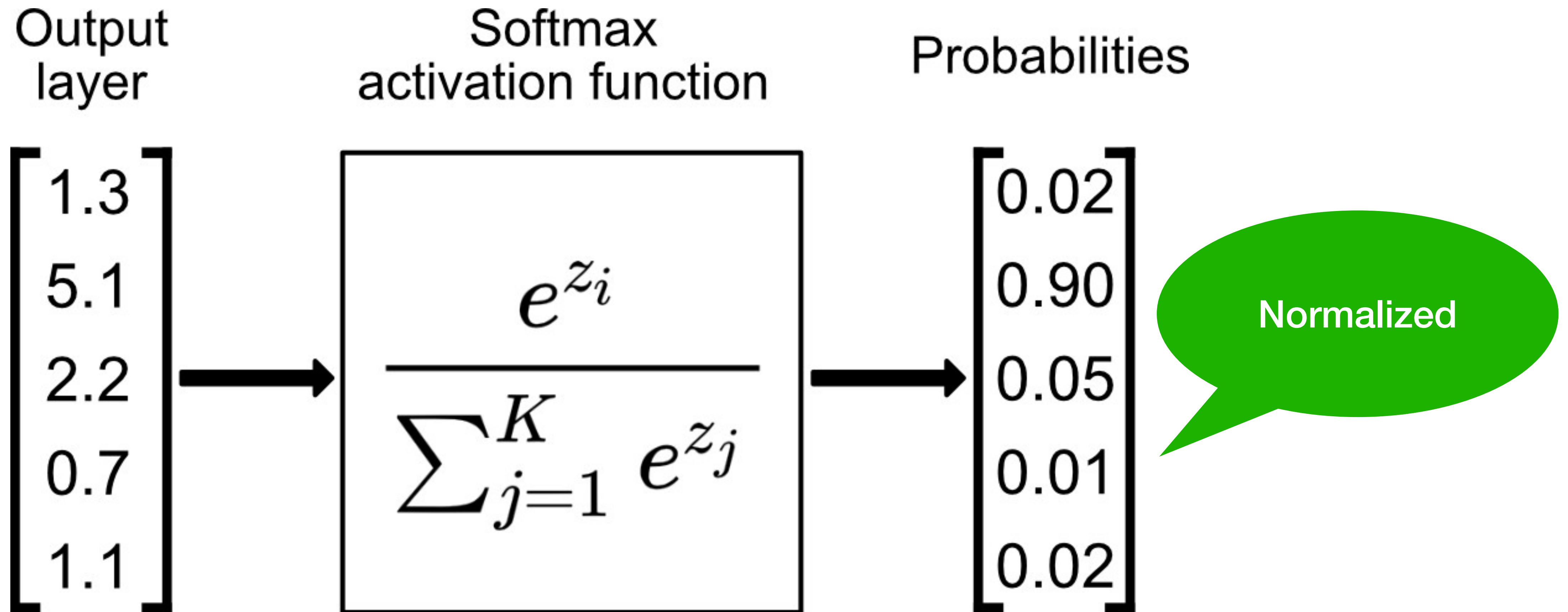
Softmax
activation function

$$\frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}}$$



Recall Softmax

Turns outputs f into probabilities (sum up to 1 across k classes)

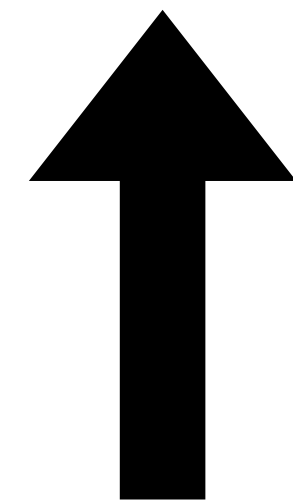


How to train a neural network?

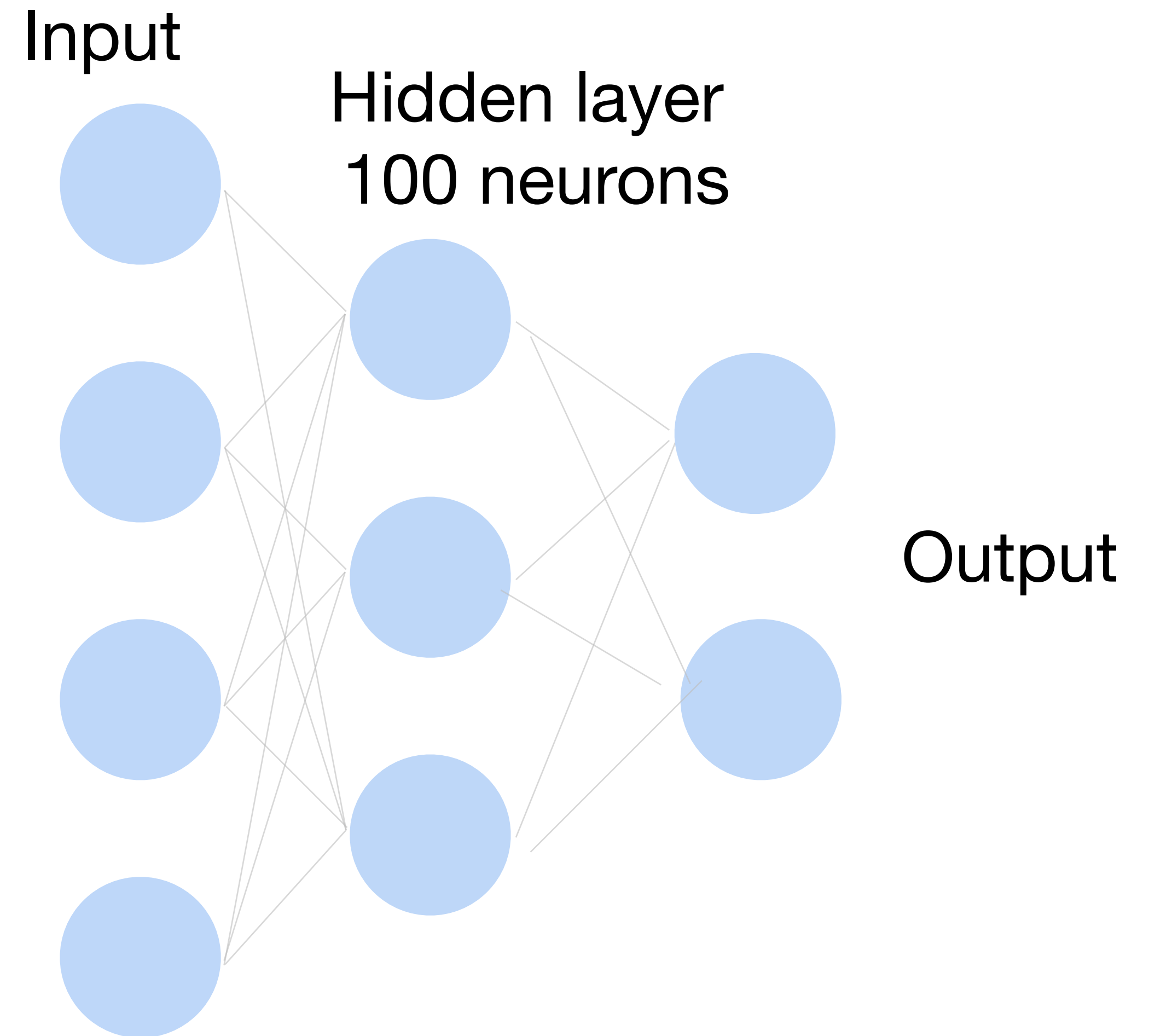
Loss function: $\frac{1}{|D|} \sum_i \ell(\mathbf{x}_i, y_i)$

Per-sample loss:

$$\ell(\mathbf{x}, y) = \sum_{j=1}^K -y_j \log p_j$$



Also known as **cross-entropy loss**
or **softmax loss**

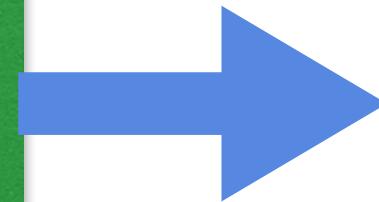


Cross-Entropy Loss

softmax

True label

Convolutional
layers



0.8

0.2

p



1

Y

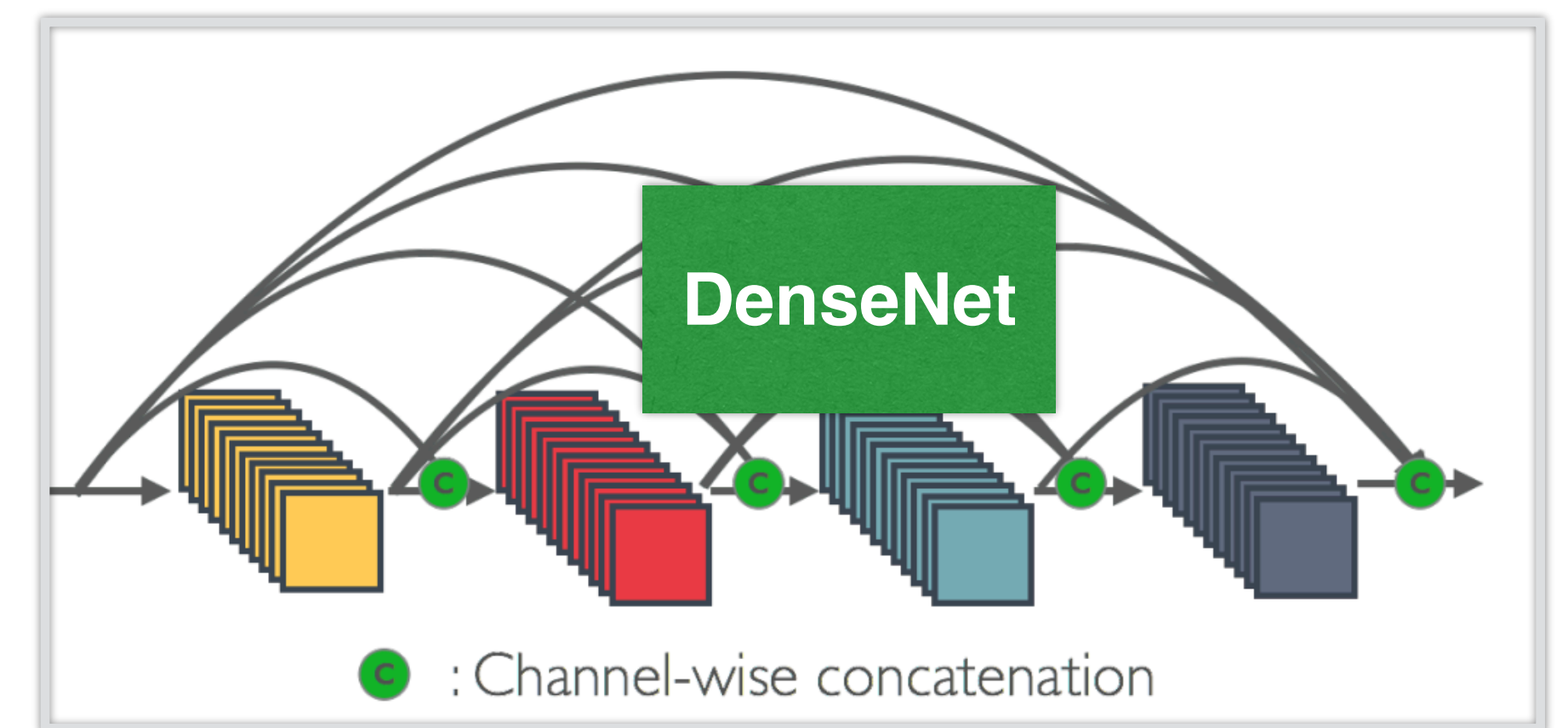
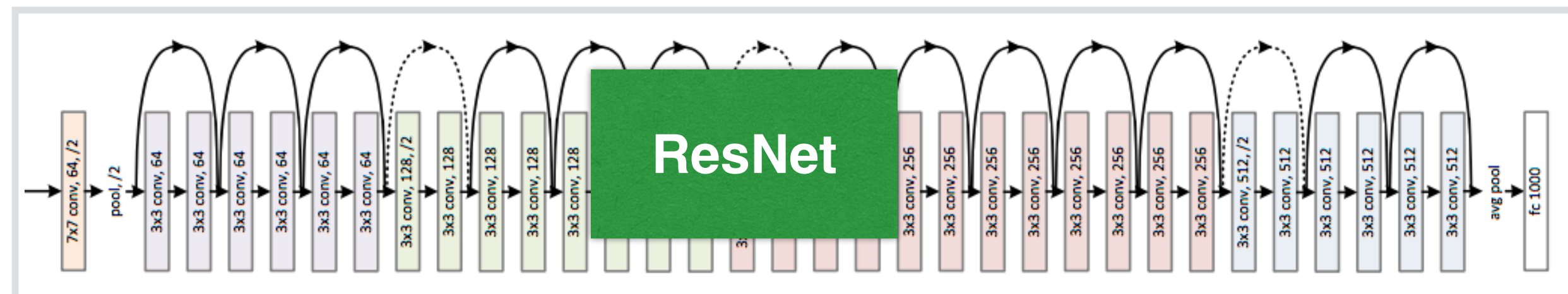
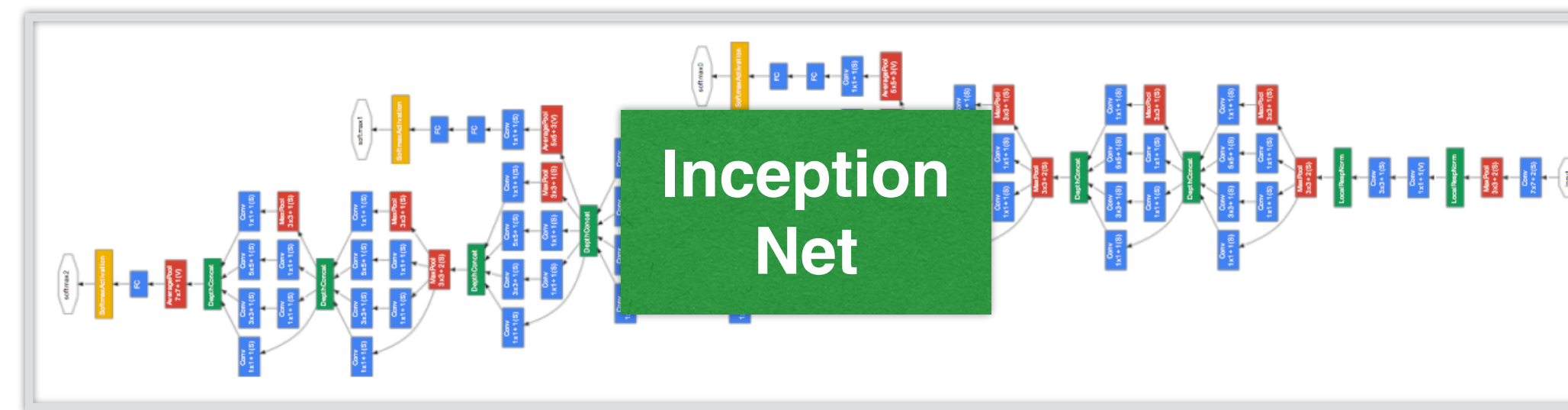
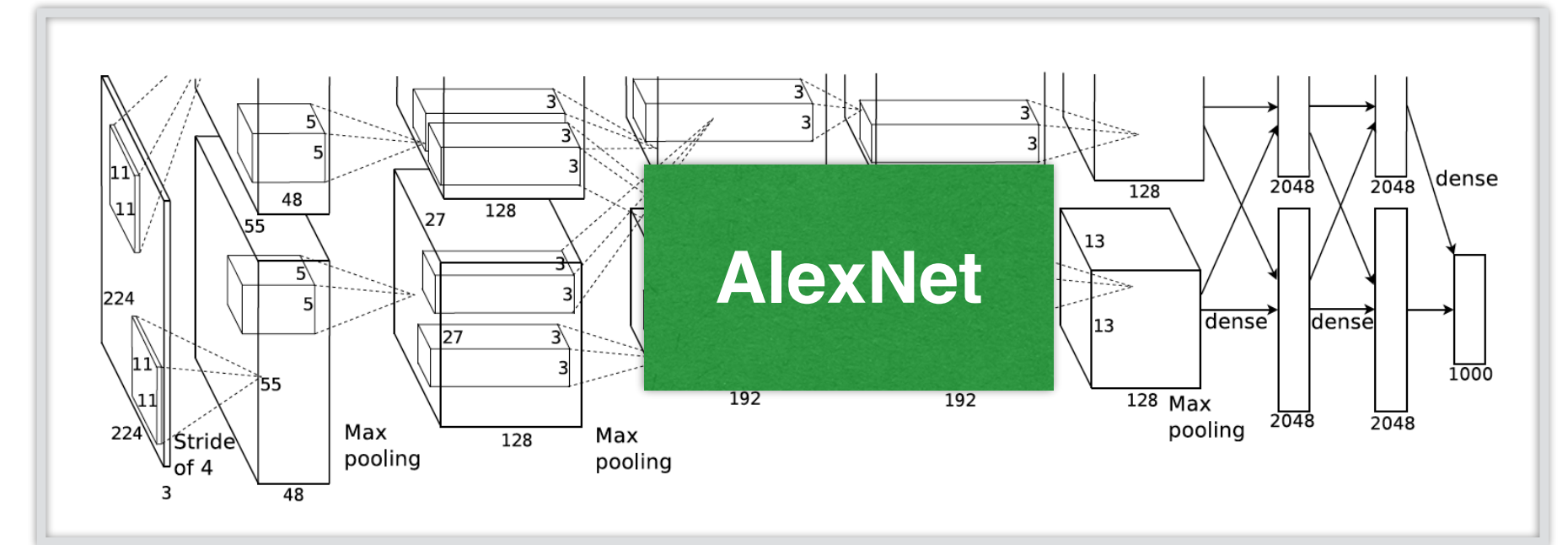
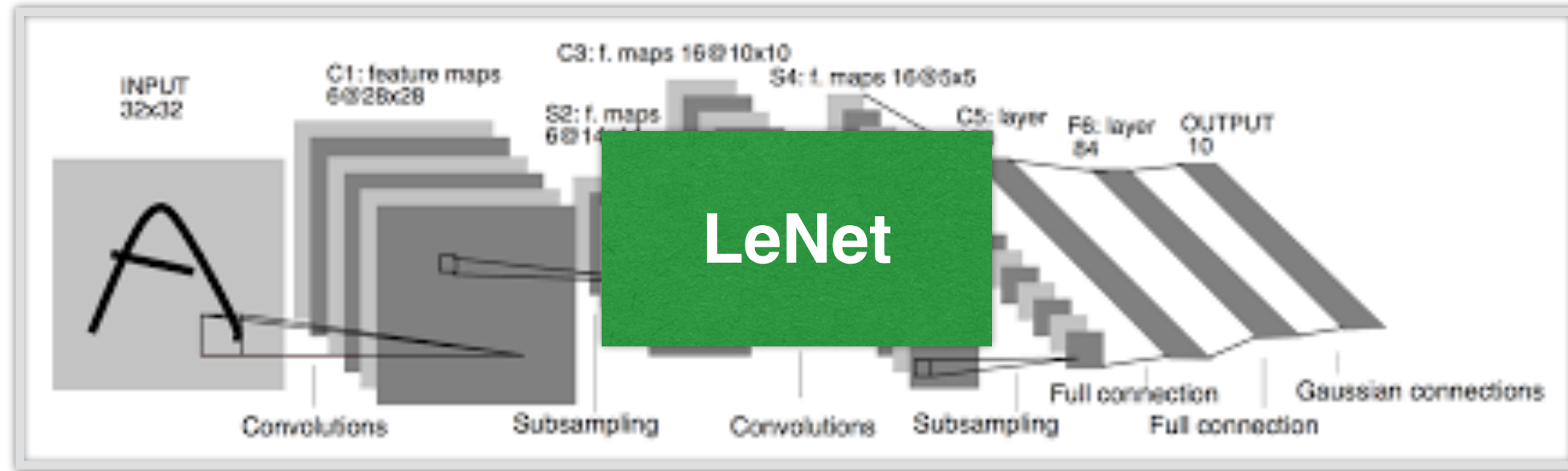
$$L_{CE} = \sum_i -Y_i \log(p_i)$$
$$= -\log(0.8)$$

Goal: push \mathbf{p} and \mathbf{Y} to be identical

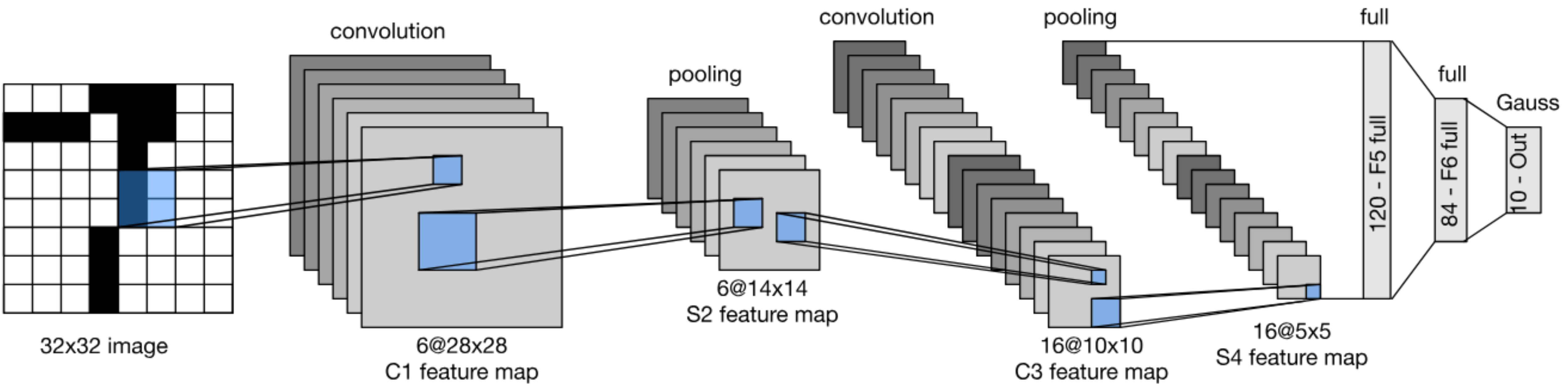
Convolutional Neural Networks

Examples

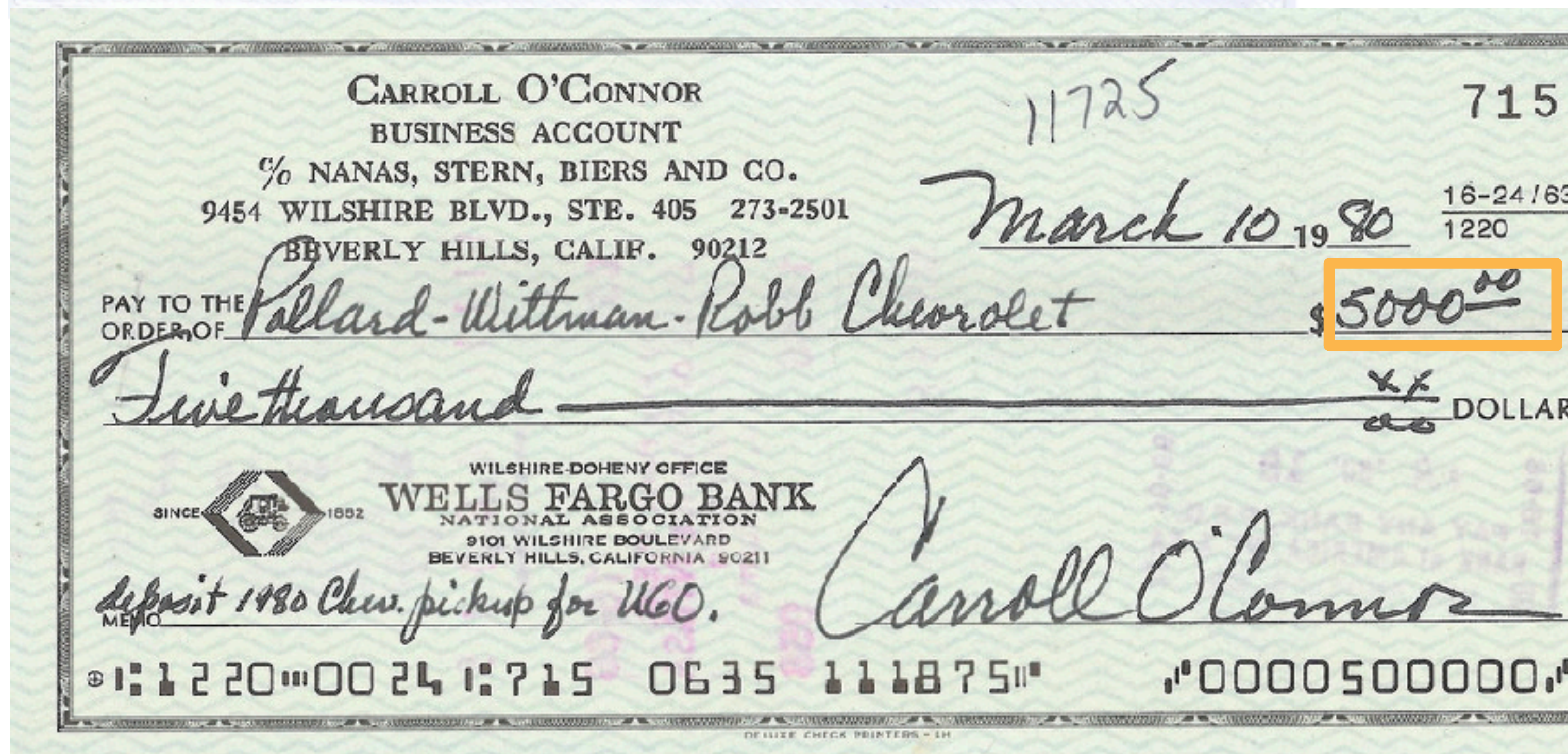
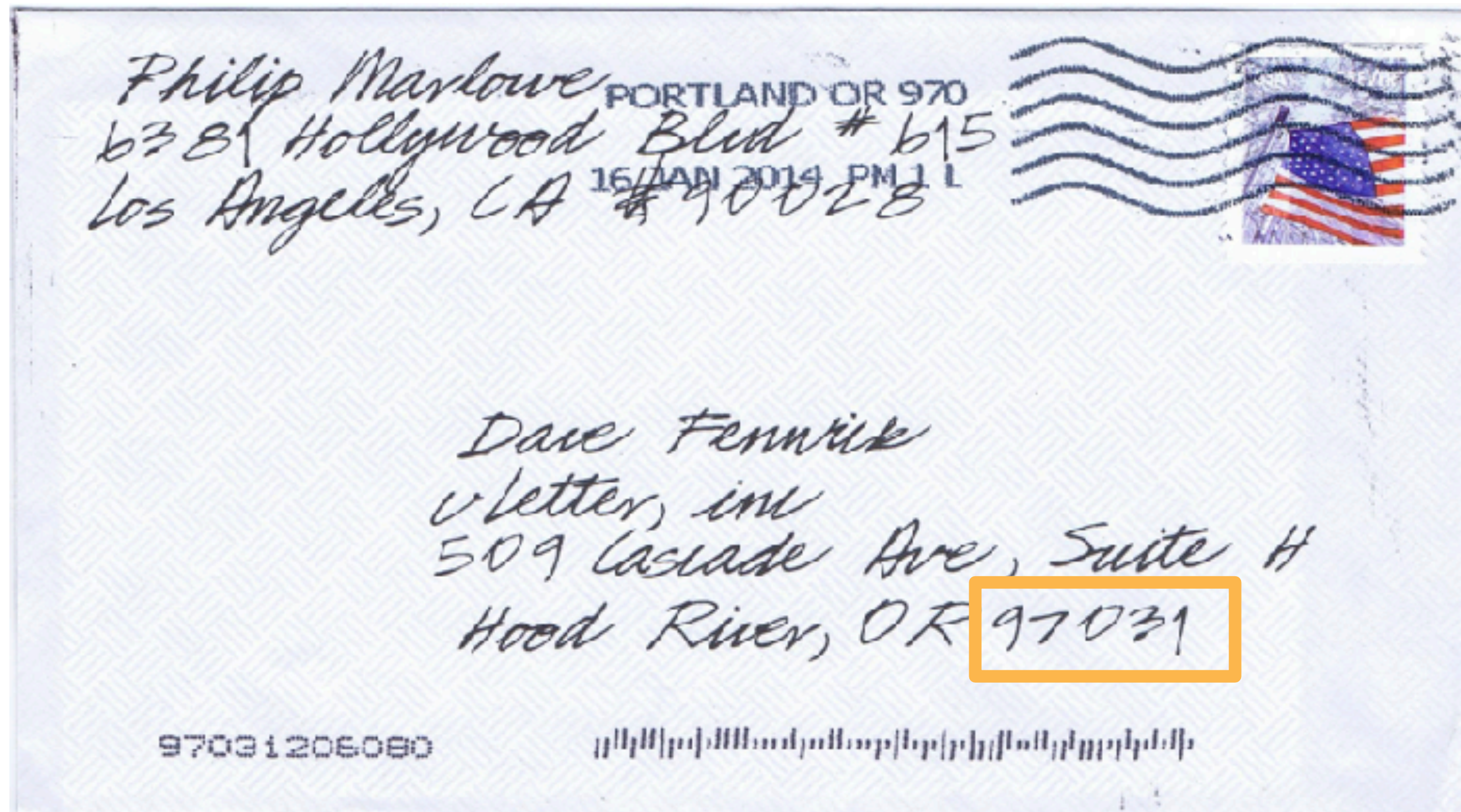
Evolution of neural net architectures



LeNet Architecture (first conv nets)



Handwritten Digit Recognition



MNIST

- Centered and scaled
- 50,000 training data
- 10,000 test data
- 28 x 28 images
- 10 classes





AT&T

LeNet 5

RESEARCH

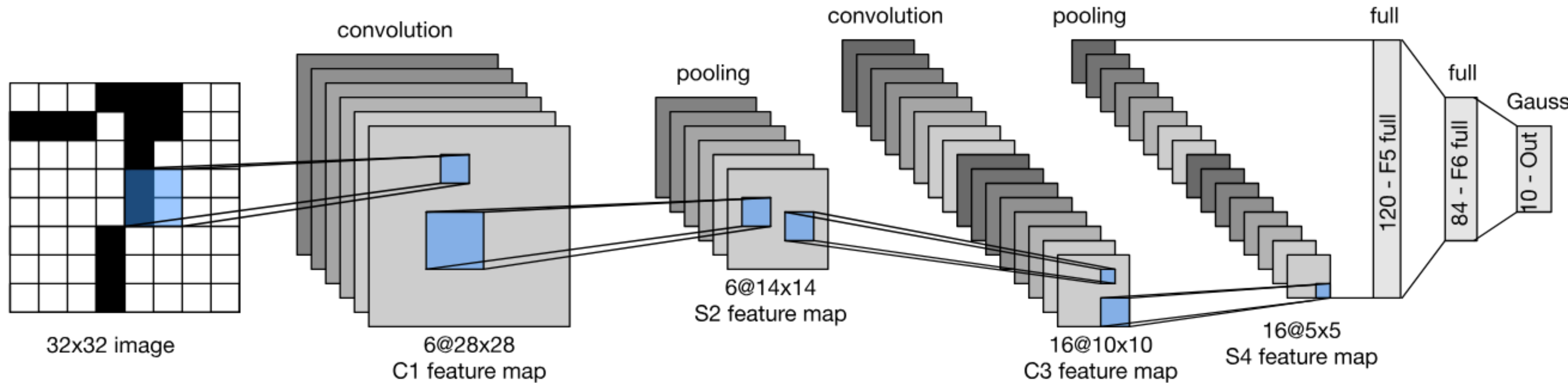
answer: 0

0
103



Y. LeCun, L.
Bottou, Y. Bengio,
P. Haffner, 1998
Gradient-based
learning applied to
document
recognition

LeNet Architecture



LeNet in Pytorch

```
def __init__(self):
    super(LeNet5, self).__init__()
    # Convolution (In LeNet-5, 32x32 images are given as input. Hence padding of 2 is done below)
    self.conv1 = torch.nn.Conv2d(in_channels=1, out_channels=6, kernel_size=5, stride=1, padding=2, bias=True)
    # Max-pooling
    self.max_pool_1 = torch.nn.MaxPool2d(kernel_size=2)
    # Convolution
    self.conv2 = torch.nn.Conv2d(in_channels=6, out_channels=16, kernel_size=5, stride=1, padding=0, bias=True)
    # Max-pooling
    self.max_pool_2 = torch.nn.MaxPool2d(kernel_size=2)
    # Fully connected layer
    self.fc1 = torch.nn.Linear(16*5*5, 120) # convert matrix with 16*5*5 (= 400) features to a matrix of 120 features (columns)
    self.fc2 = torch.nn.Linear(120, 84) # convert matrix with 120 features to a matrix of 84 features (columns)
    self.fc3 = torch.nn.Linear(84, 10) # convert matrix with 84 features to a matrix of 10 features (columns)
```



```
def forward(self, x):
    # convolve, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.conv1(x))
    # max-pooling with 2x2 grid
    x = self.max_pool_1(x)
    # convolve, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.conv2(x))
    # max-pooling with 2x2 grid
    x = self.max_pool_2(x)
    # first flatten 'max_pool_2_out' to contain 16*5*5 columns
    # read through https://stackoverflow.com/a/42482819/7551231
    x = x.view(-1, 16*5*5)
    # FC-1, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.fc1(x))
    # FC-2, then perform ReLU non-linearity
    x = torch.nn.functional.relu(self.fc2(x))
    # FC-3
    x = self.fc3(x)

    return x
```

LeNet in Pytorch

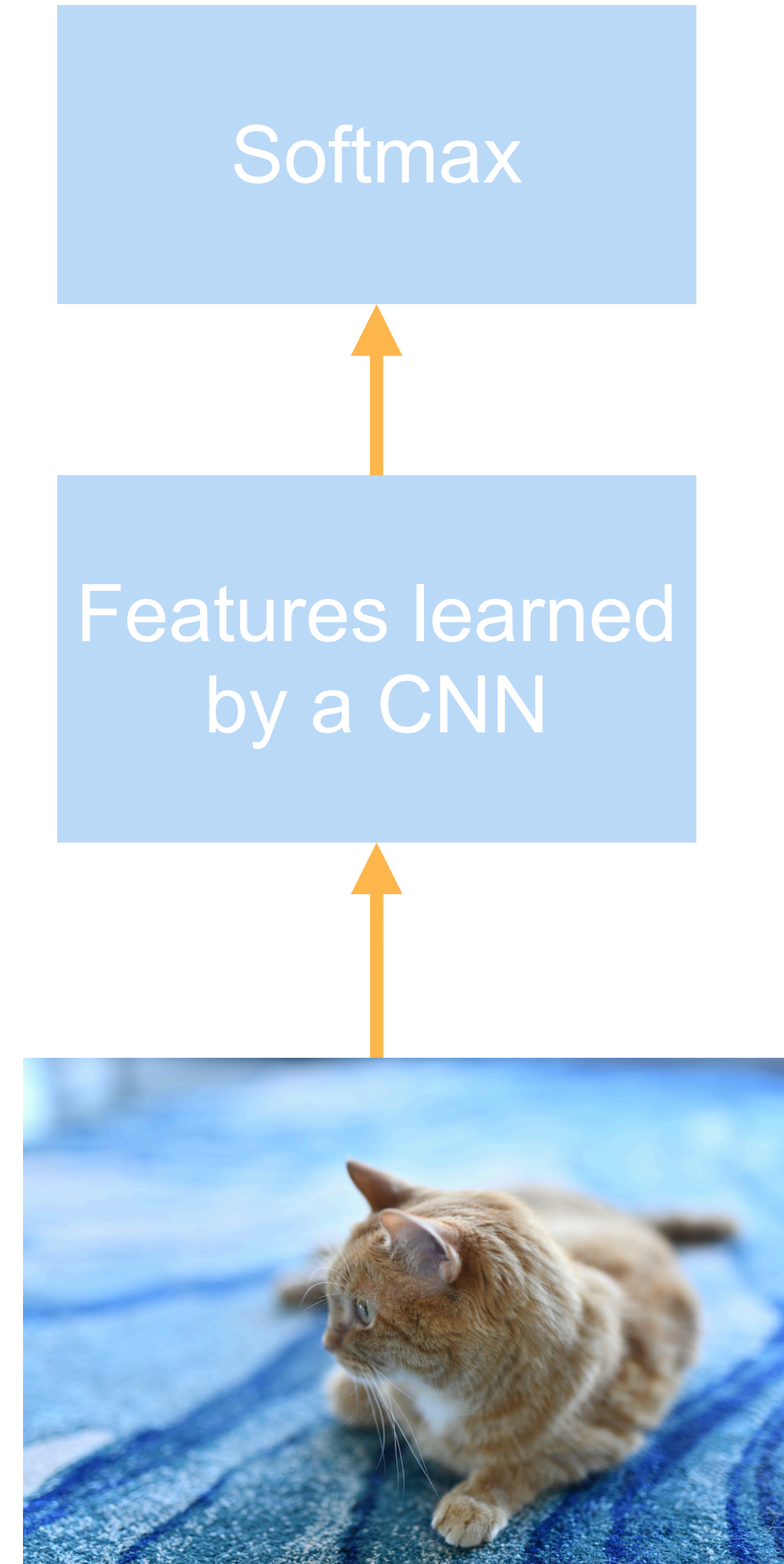
Let's walk through an example using PyTorch

https://pytorch.org/tutorials/beginner/blitz/cifar10_tutorial.html



AlexNet

- AlexNet won ImageNet competition in 2012
- Deeper and bigger LeNet
- Paradigm shift for computer vision

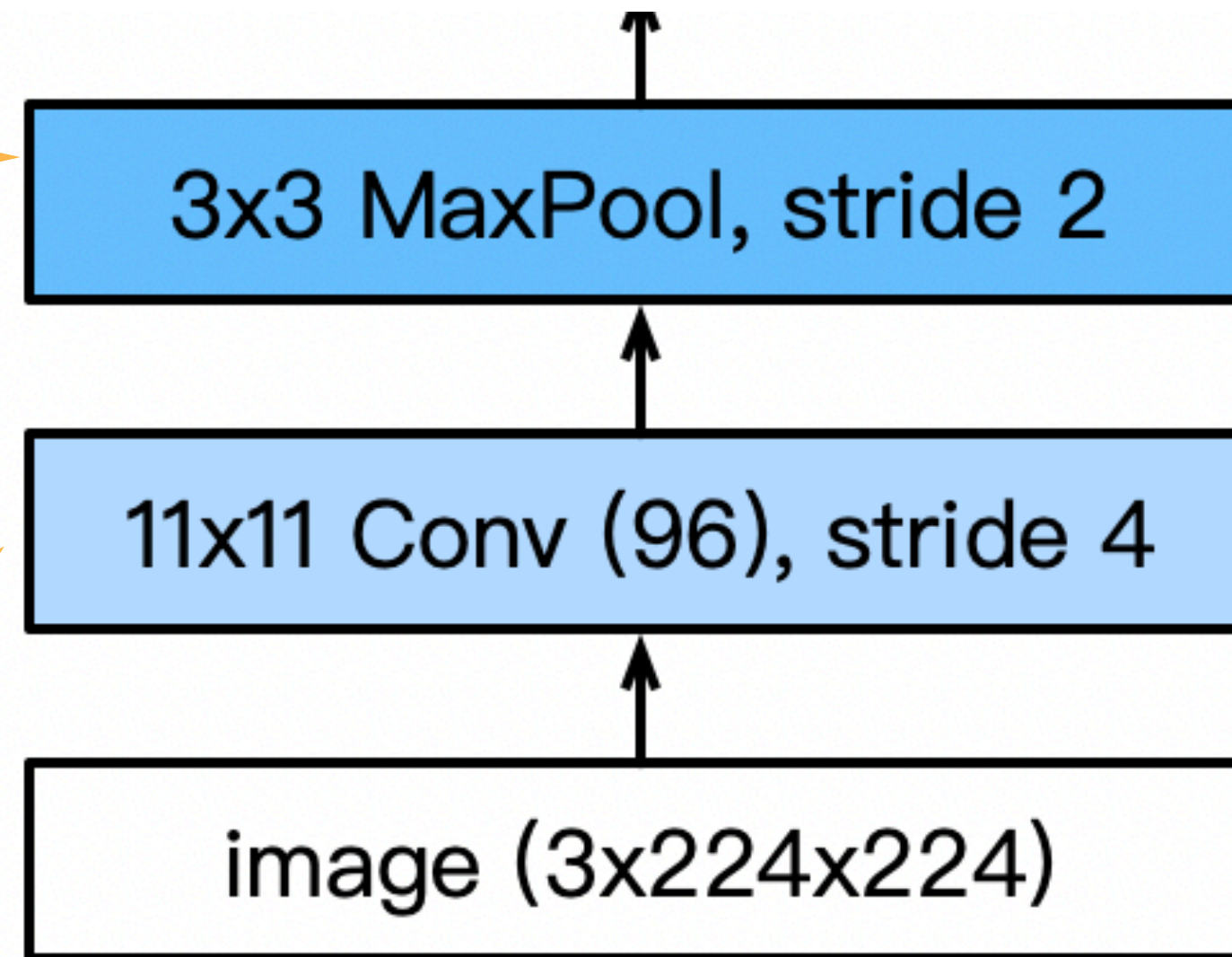


AlexNet Architecture

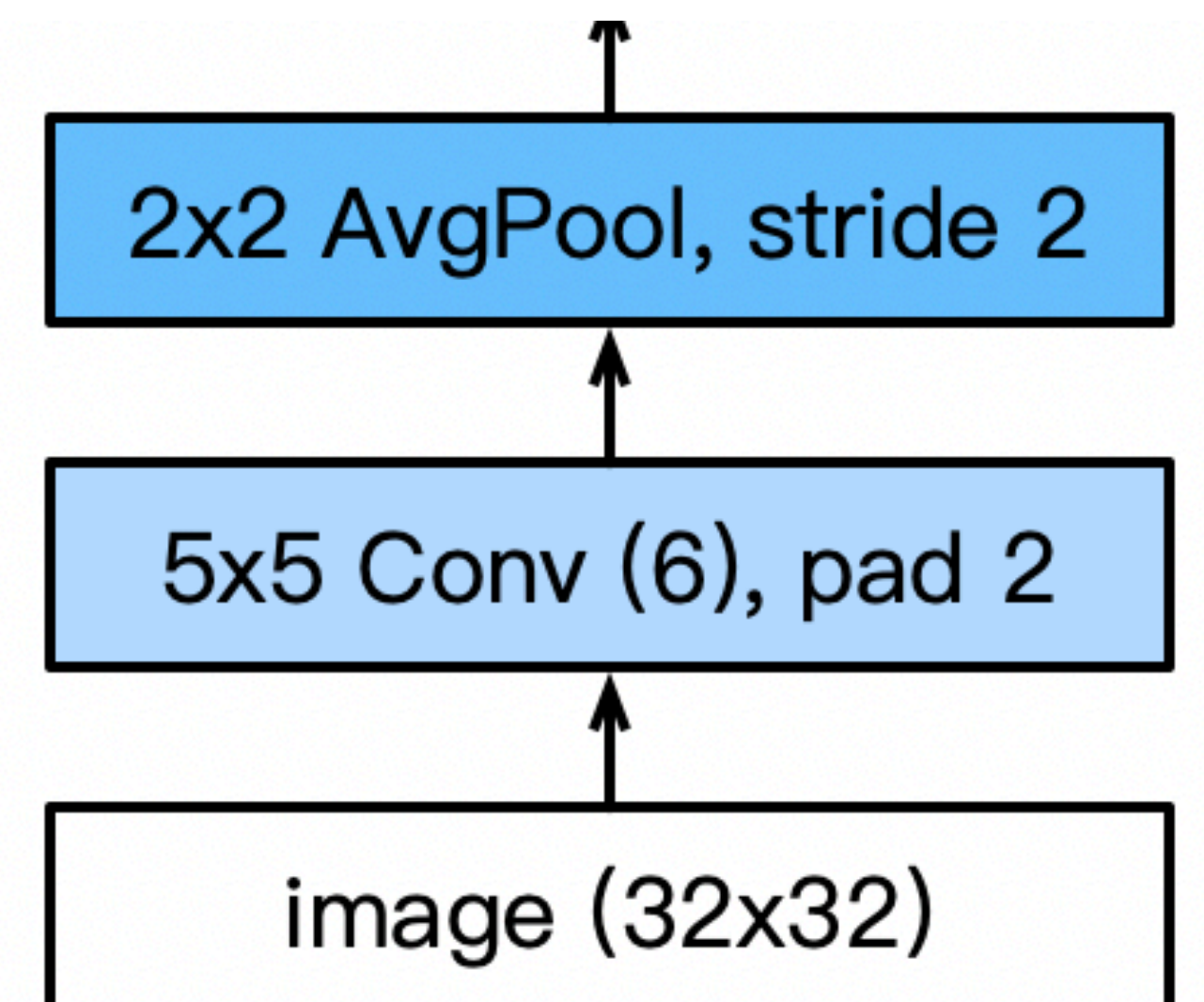
Larger pool size

Larger kernel size, stride because of the increased image size, and more output channels.

AlexNet

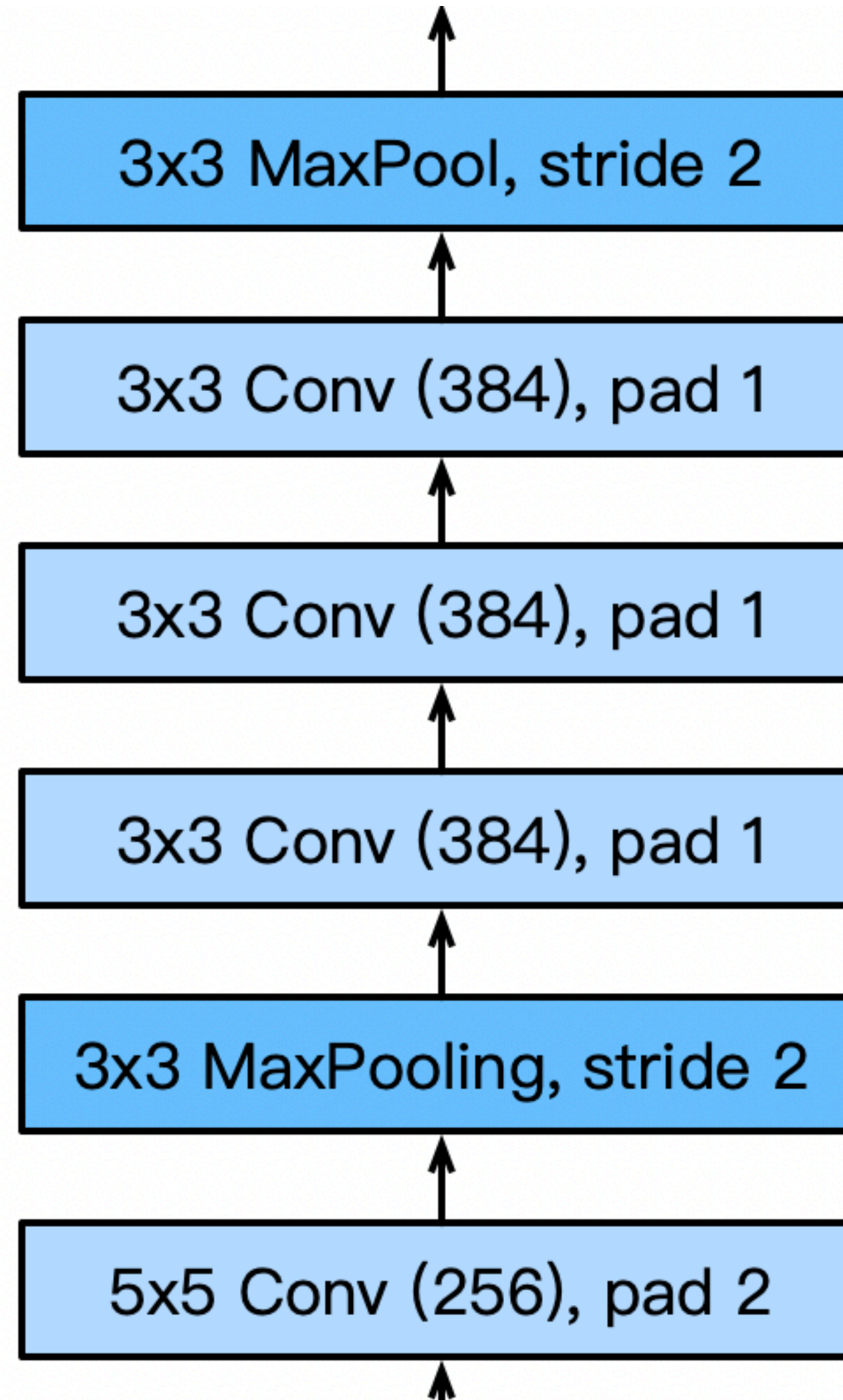


LeNet

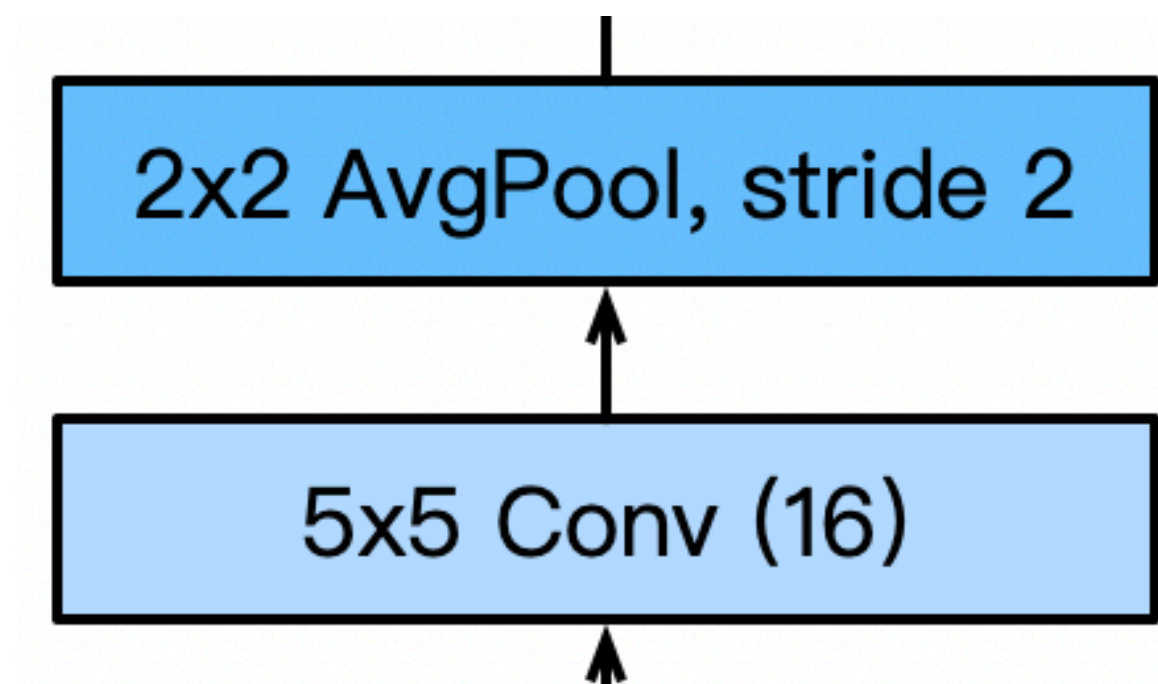


AlexNet Architecture

AlexNet



LeNet



3 additional convolutional layers

More output channels.

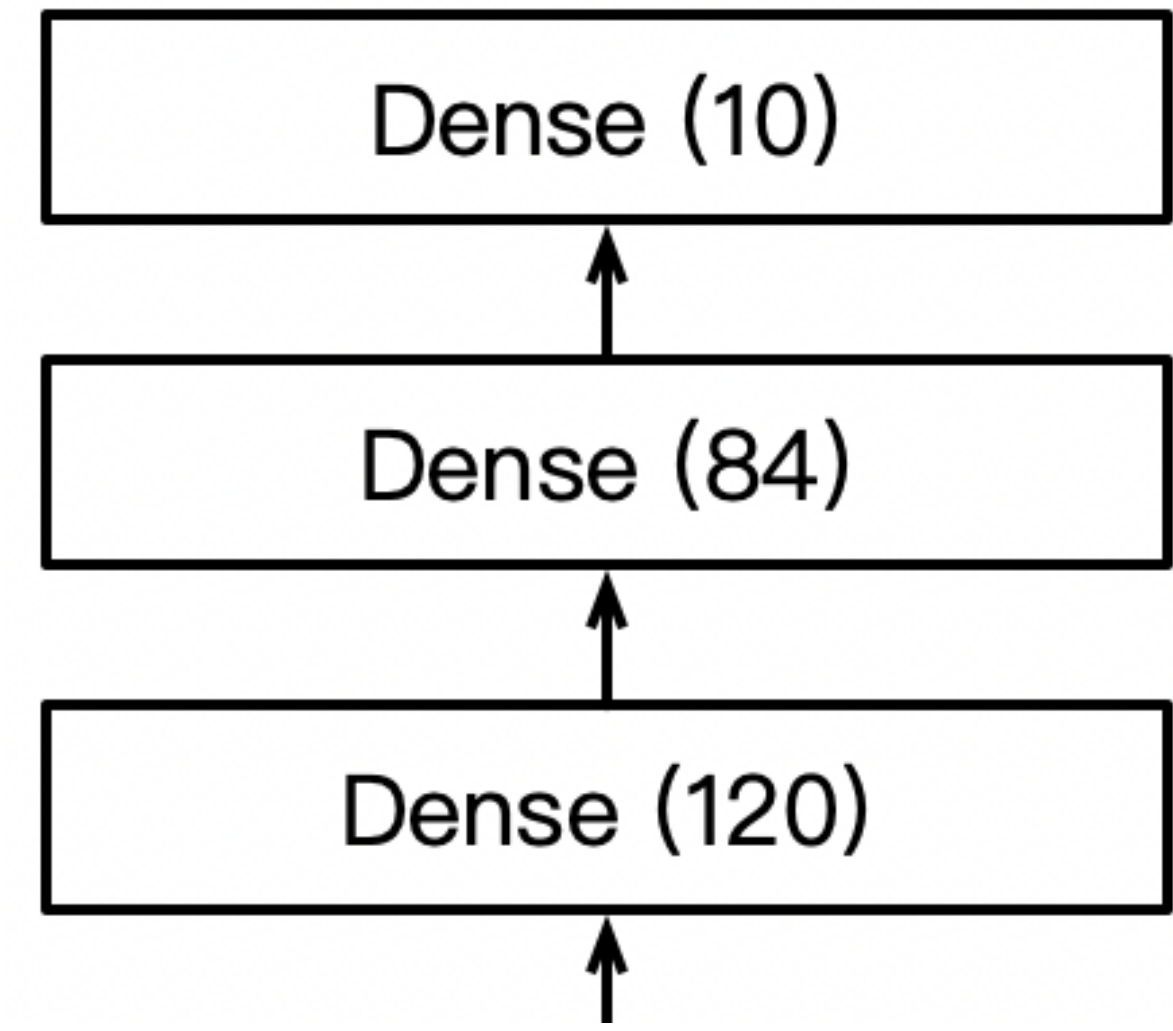
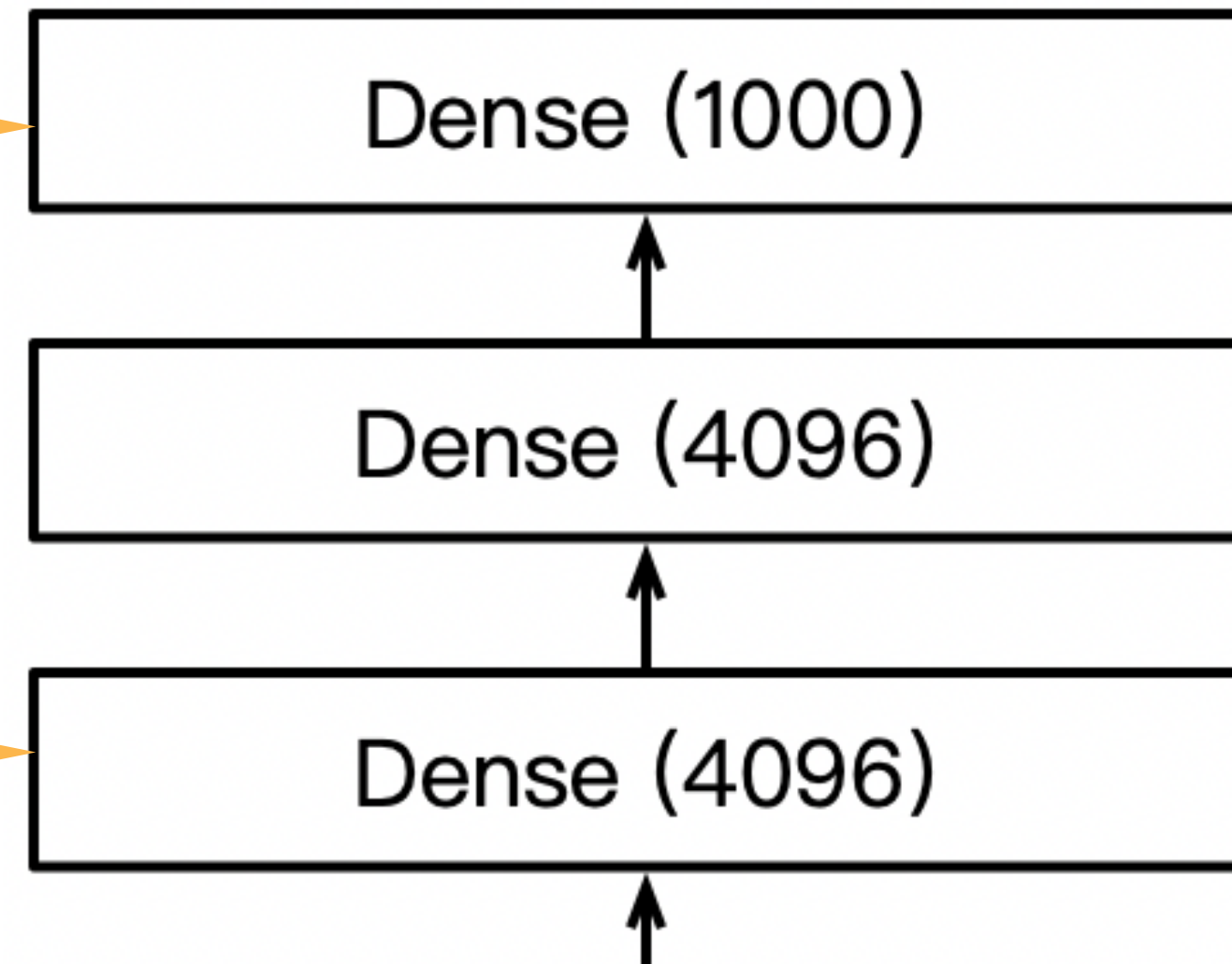
AlexNet Architecture

AlexNet

LeNet

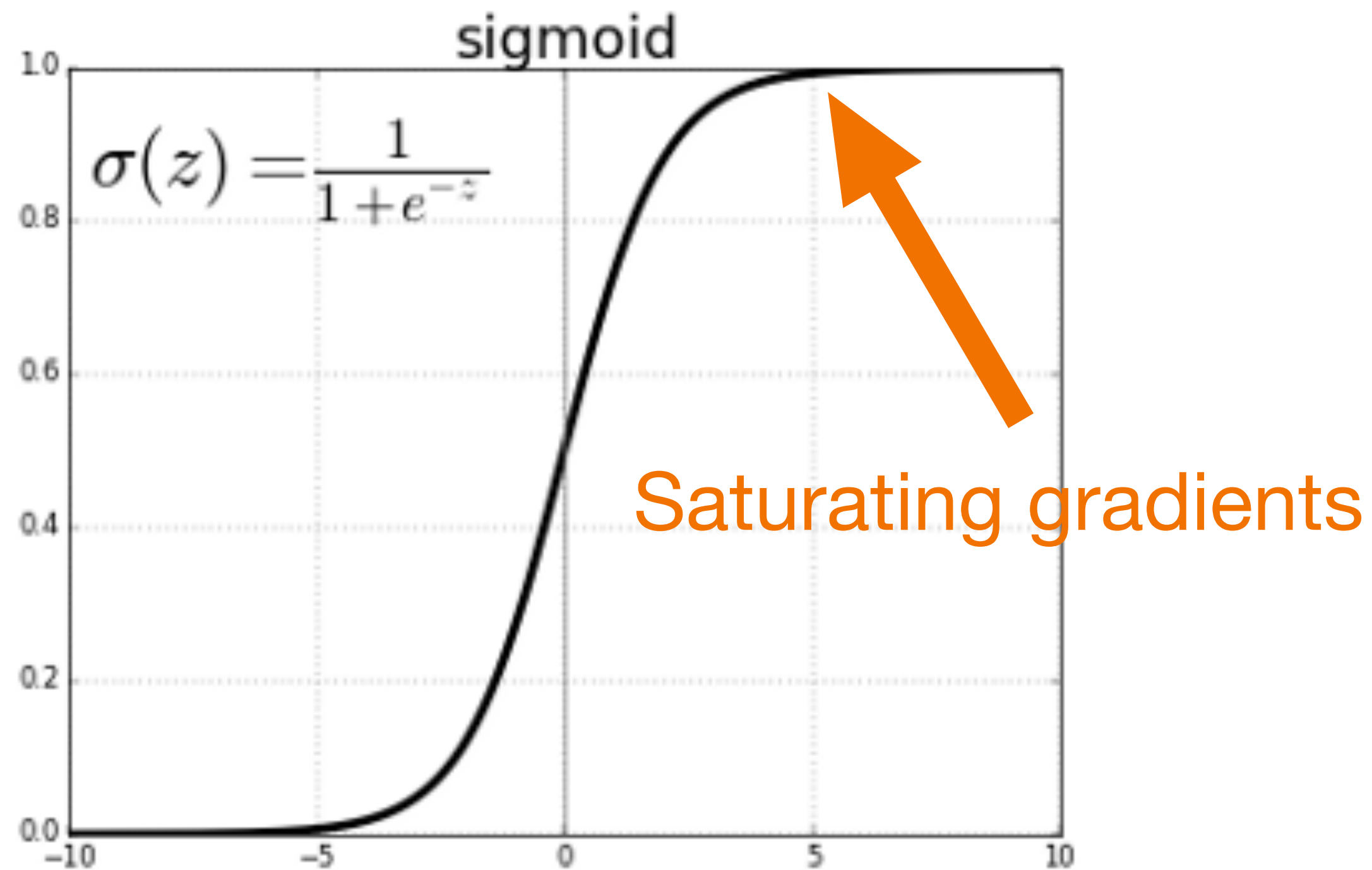
1000 classes output

Increase hidden size from 120 to 4096



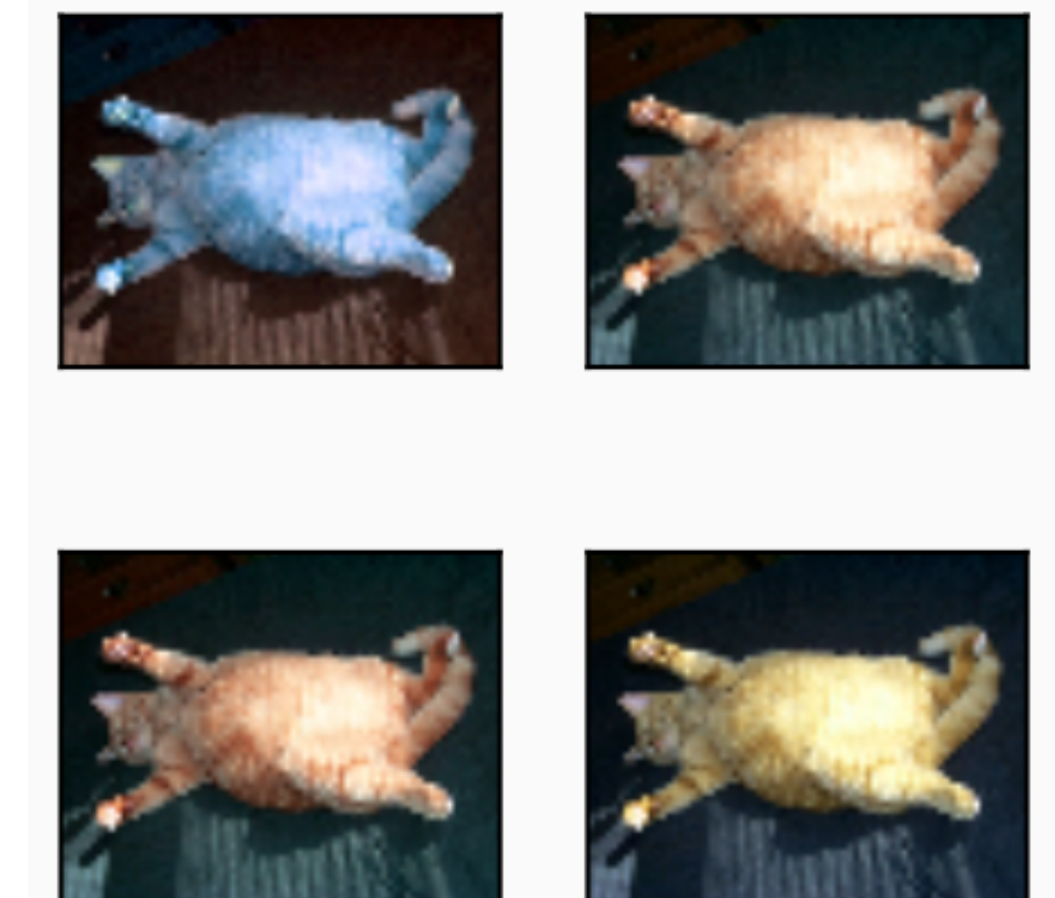
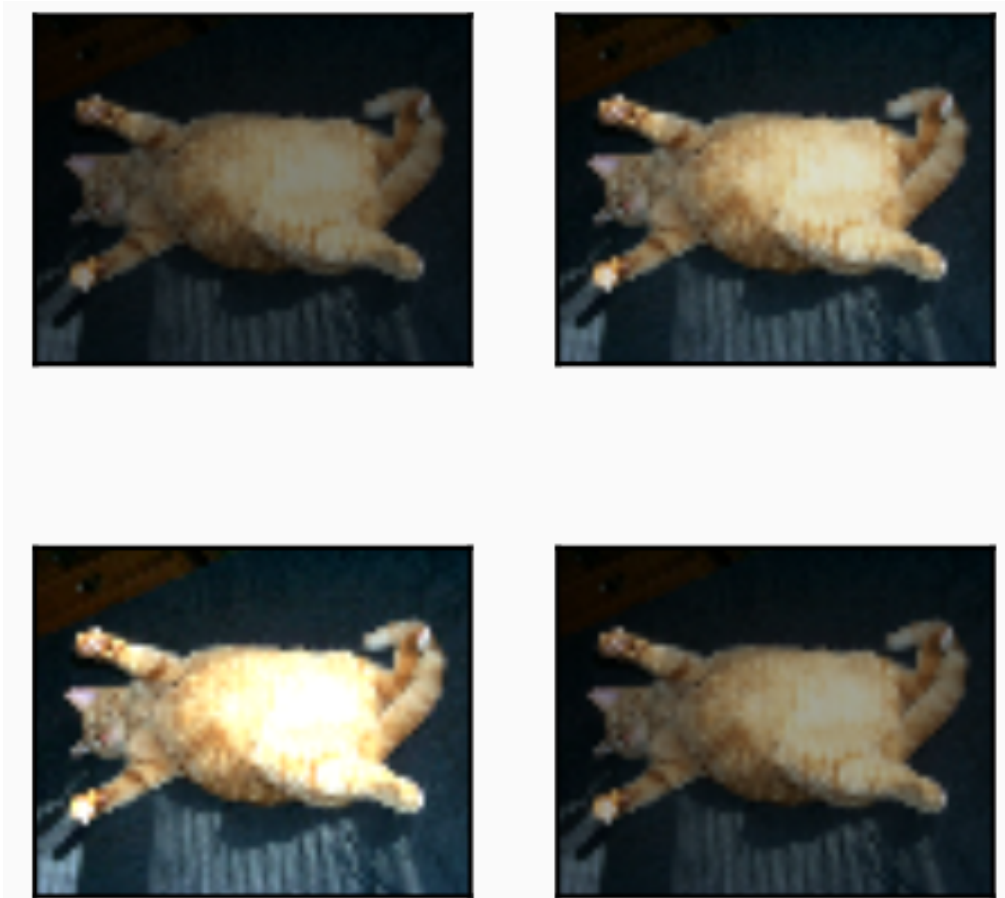
More Differences...

- Change activation function from sigmoid to ReLu (no more vanishing gradient)



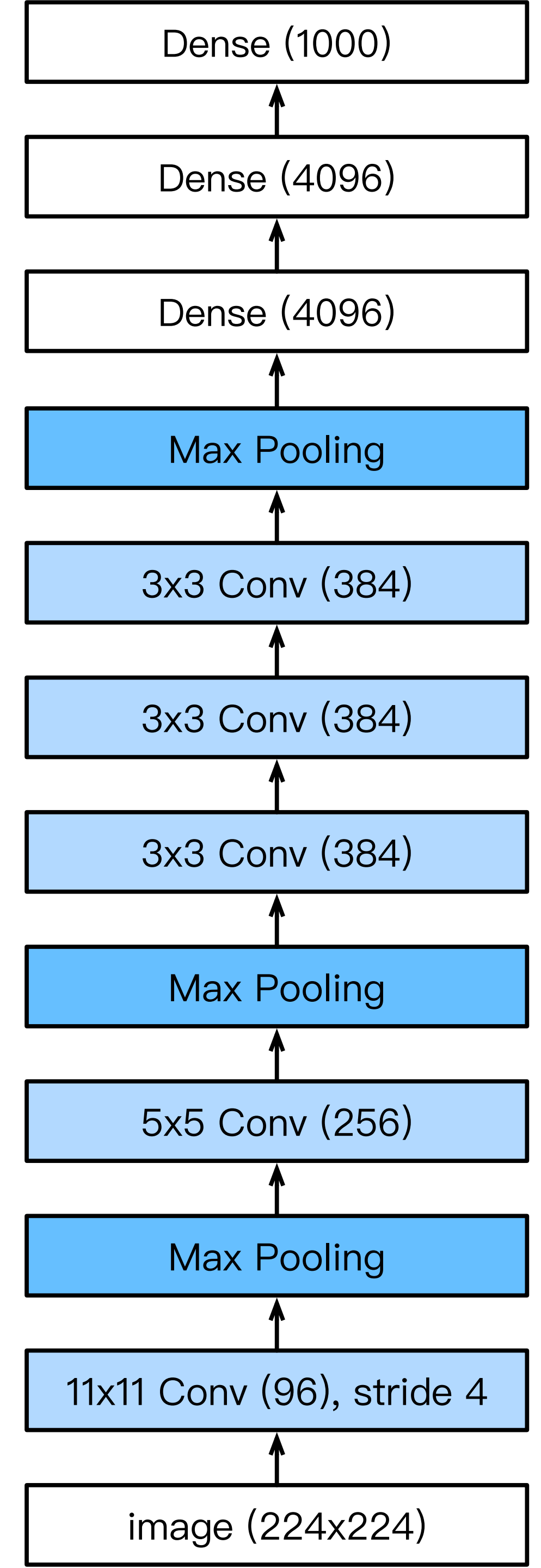
More Differences...

- Change activation function from sigmoid to ReLu (no more vanishing gradient)
- Data augmentation



Complexity

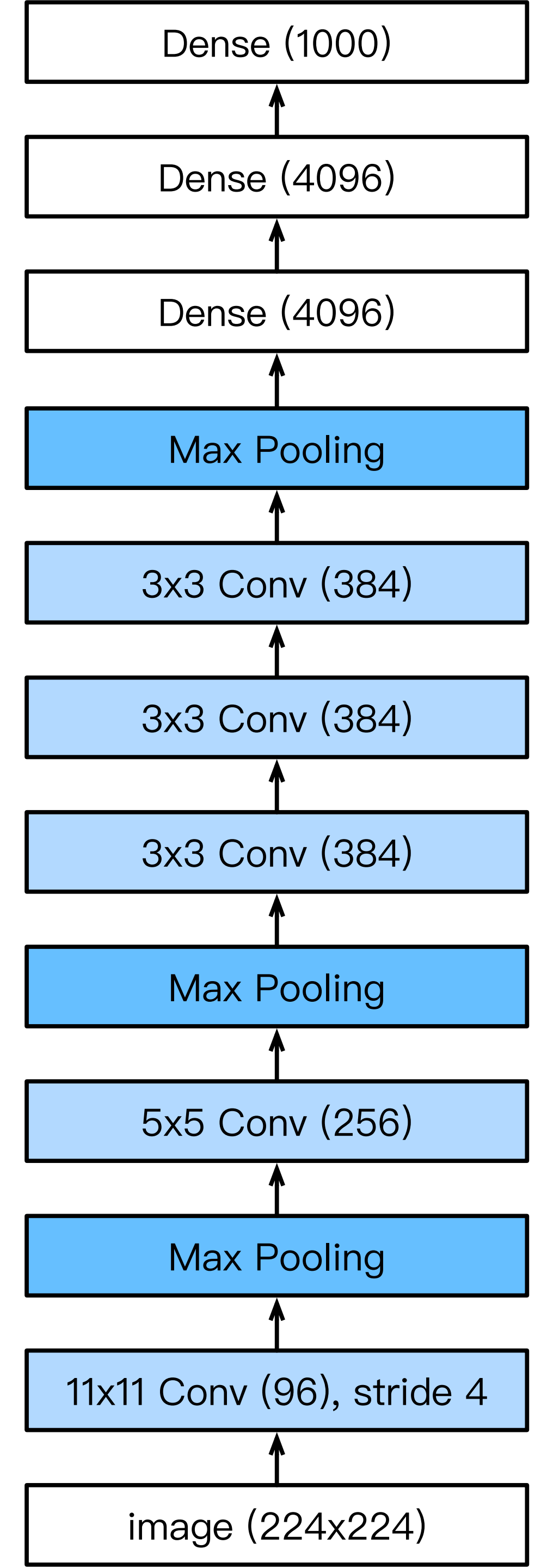
	#parameters	
	AlexNet	LeNet
Conv1	35K	150
Conv2	614K	2.4K
Conv3-5	3M	
Dense1	26M	0.048M
Dense2	16M	0.01M
Total	46M	0.06M
Increase	11x	1x

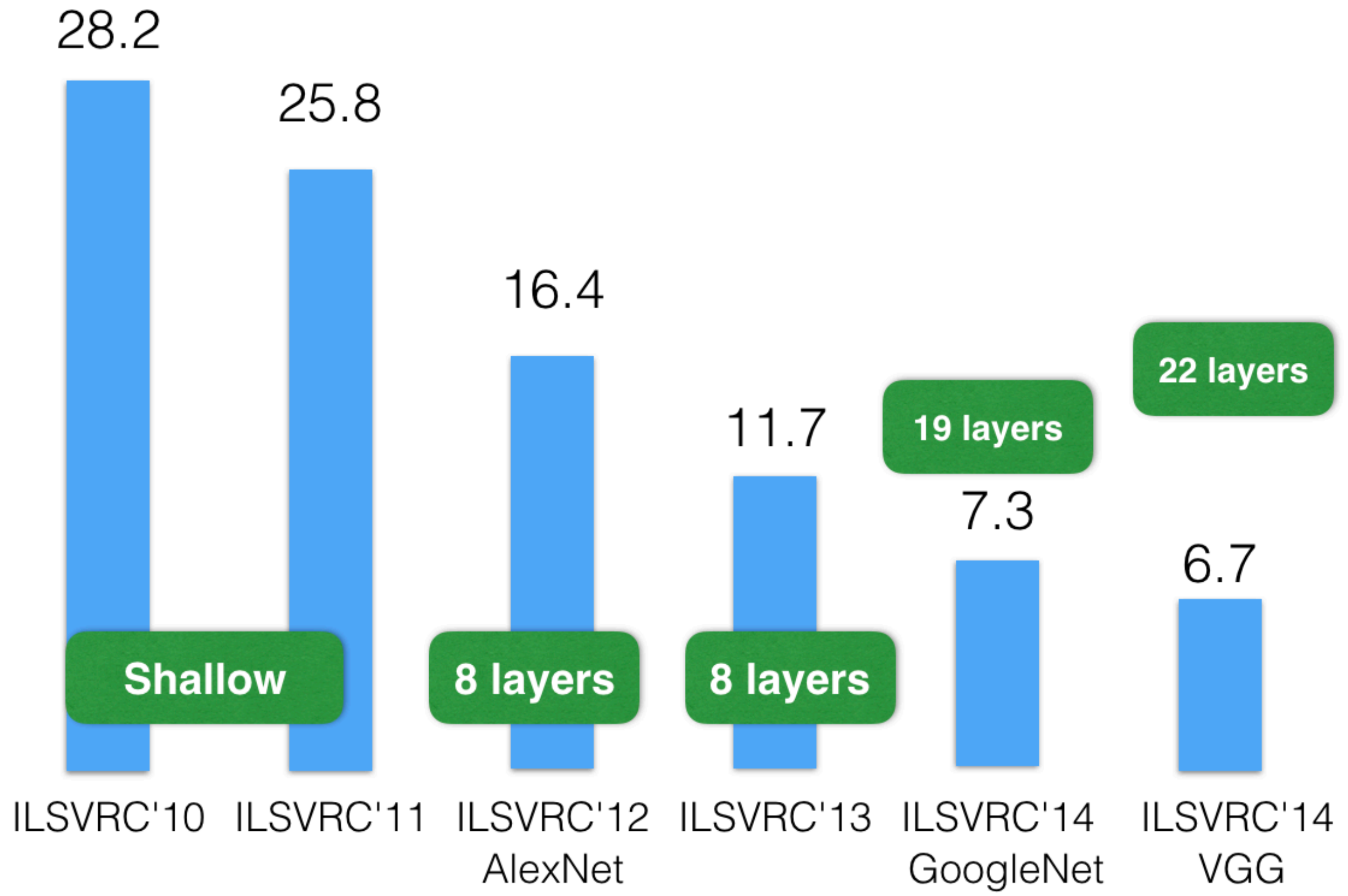


Complexity

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Total	46M	0.06M
Increase	11x	1x

$$11 \times 11 \times 3 \times 96 = 35k$$





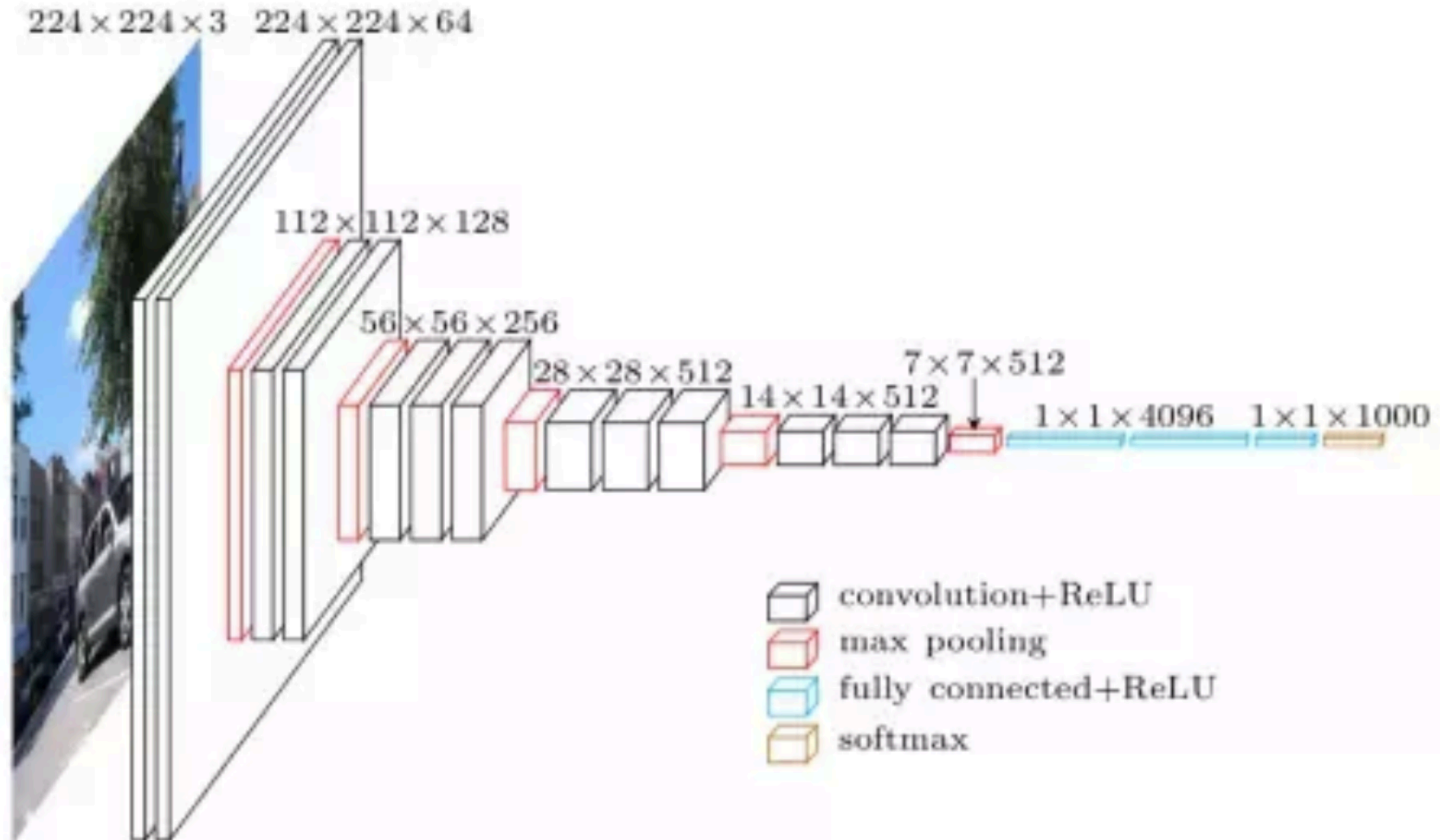
ImageNet Top-5 Classification Accuracy (%)

Which of the following are true about AlexNet? Select all that apply.

- A. AlexNet contains 8 layers. The first five are convolutional layers.
- B. The last three layers are fully connected layers.
- C. some of the convolutional layers are followed by **max-pooling** (layers).
- D. AlexNet achieved excellent performance in the 2012 ImageNet challenge.



VGG



Progress

- LeNet (1995)
 - 2 convolution + pooling layers
 - 2 hidden dense layers
- AlexNet
 - Bigger and deeper LeNet
 - ReLu, preprocessing
- VGG
 - Bigger and deeper AlexNet (repeated VGG blocks)

Which of the following statement is True for the success of deep models?

- Better design of the neural networks
- Large scale training dataset
- Available computing power
- All of the above

What we've learned today

- Brief review of convolutional computations
- Convolutional Neural Networks
 - LeNet (first conv nets)
 - AlexNet
- PyTorch demo



Acknowledgement:

Some of the slides in these lectures have been adapted/borrowed from materials developed by Yin Li (<https://happyharrycn.github.io/CS540-Fall20/schedule/>), Alex Smola and Mu Li:

<https://courses.d21.ai/berkeley-stat-157/index.html>