CS 540 Introduction to Artificial Intelligence
Search II: Informed Search

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Nov 16, 2021

Announcements

- HW8 released today, due next Tuesday
- Annotated slides
- Grading Info
- Class roadmap

Today: informed search

Thursday: advanced search.

Next week: Games.

Outline

- Uninformed vs Informed Search
  - Review of uninformed strategies, adding heuristics
- A* Search
  - Heuristic properties, stopping rules, analysis
- Extensions: Beyond A*
  - Iterative deepening, beam search

Breadth-First Search

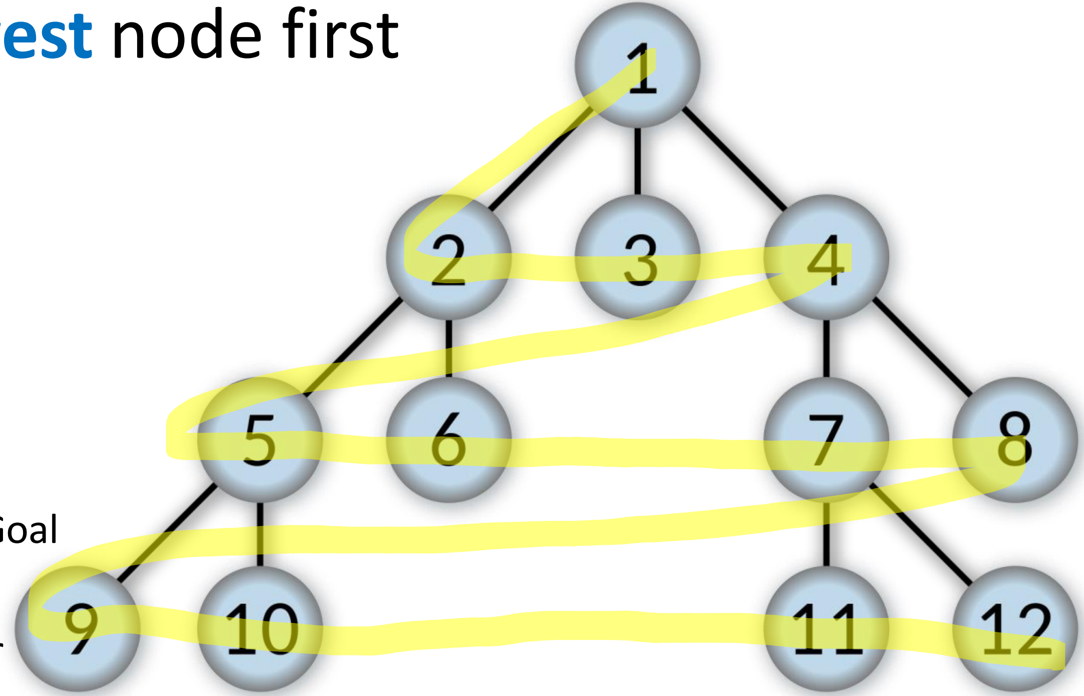
Recall: expand **shallowest** node first

- Data structure: queue
- **Properties**:
  - Complete
  - Optimal (if edge cost 1)
  - Time $O(b^d)$

Depth of Goal

Branching Factor

  - Space $O(b^d)$



Wiki

Uniform Cost Search

Recall: expand **least-cost** node first

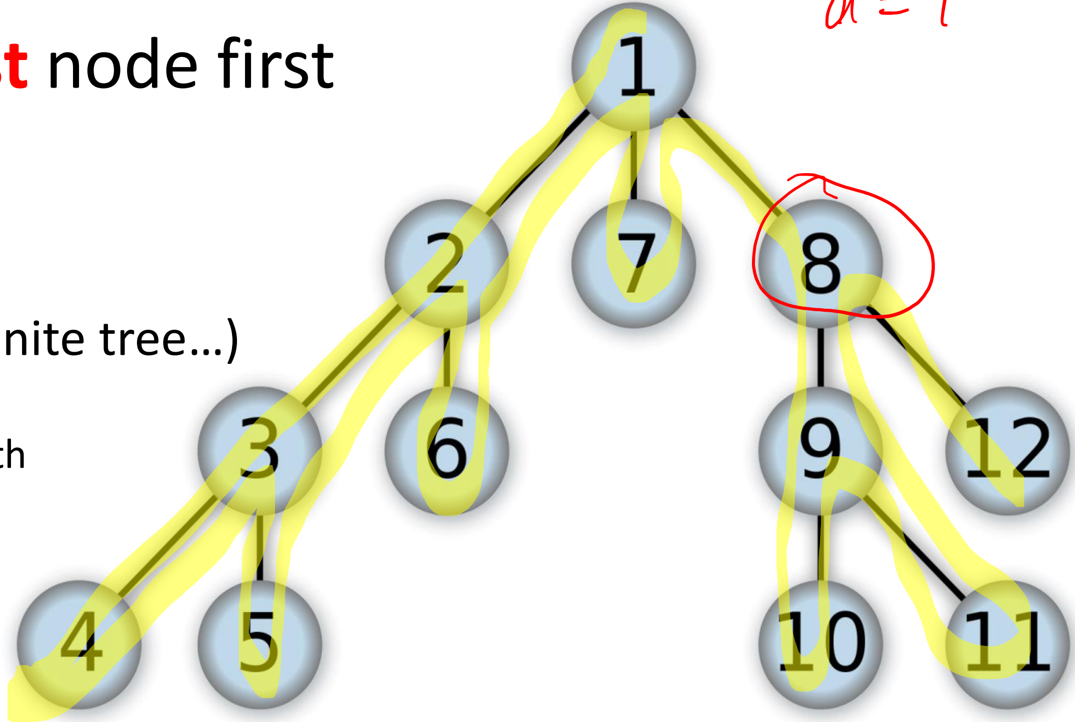
- Generalization of BFS
- Data structure: priority queue
- **Properties**:
  - Complete
  - Optimal (if weight lower bounded by ε)
  - Time $O(b^{C^*/\varepsilon})$
  - Space $O(b^{C^*/\varepsilon})$

Optimal goal path cost

Depth-First Search

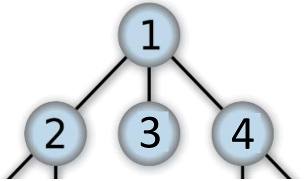
Recall: expand **deepest** node first

- Data structure: stack
- **Properties**:
  - Incomplete (stuck in infinite tree…)
  - Suboptimal
  - Time $O(b^m)$ ← Max Depth
  - Space $O(bm)$



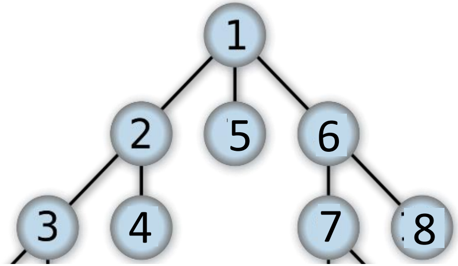
Wiki

Iterative Deepening DFS

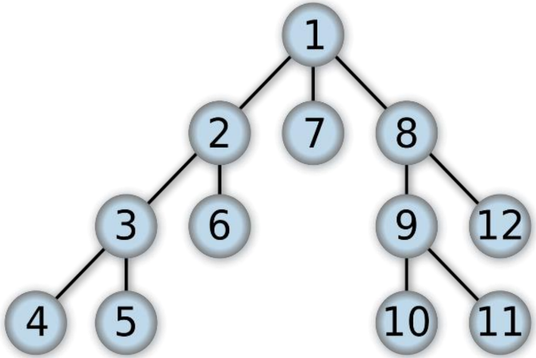


Repeated limited DFS

- Search like BFS, fringe like DFS
- **Properties**:
  - Complete
  - Optimal (if edge cost 1)
  - Time O(b^d)
  - Space O(bd)



A good option!



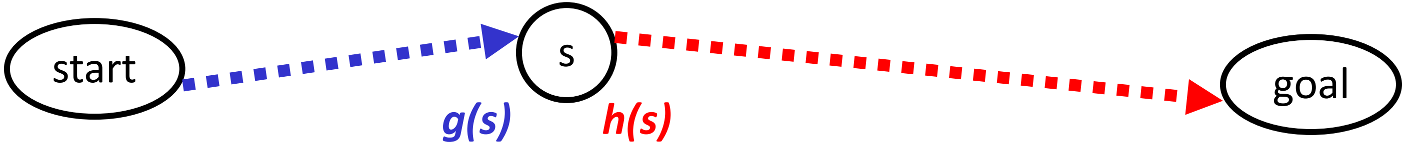
Uninformed vs Informed Search

Uninformed search (all of what we saw). Know:

- Path cost ***g***(*s*) from start to node *s*
- Successors.

Informed search. Know:

- All uninformed search properties, plus
- Heuristic **h(s)** from s to goal

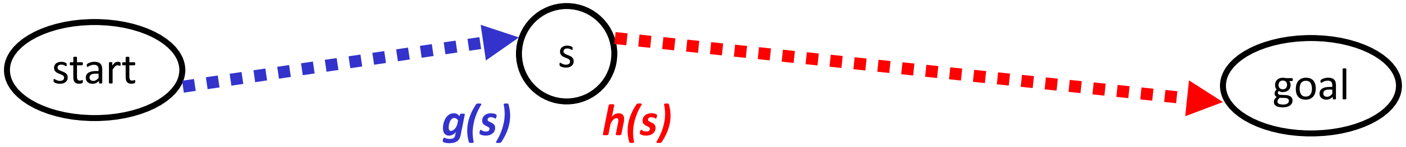


Informed Search

Informed search. Know:

- All uninformed search properties, plus
- Heuristic ***h(s)*** from *s* to goal

hope: h(s) ≈ true cost from s to goal.

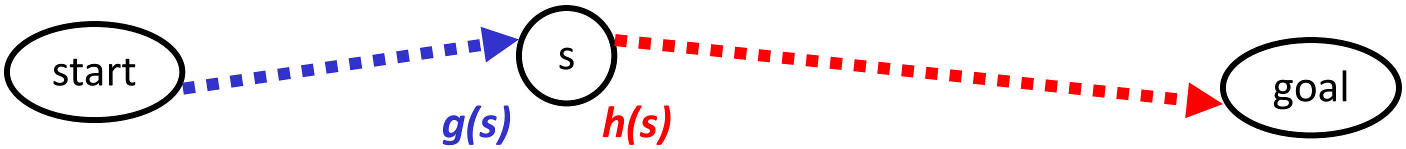


- Use side information to **speed up search.**

Using the Heuristic

Back to uniform-cost search

- We had the priority queue
- Expand the node with the smallest ***g*(*s*)**
  - ***g*(*s*)** "first-half-cost"



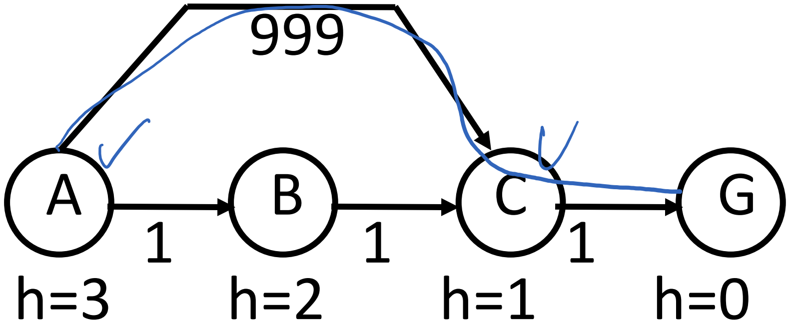
- Now let's use the heuristic ("second-half-cost")
  - Several possible approaches: let's see what works

Attempt 1: Best-First Greedy

One approach: just use ***h*(*s*)** alone

- Specifically, expand node with smallest ***h*(*s*)**
- This isn't a good idea. Why?

B: 2
C: 1

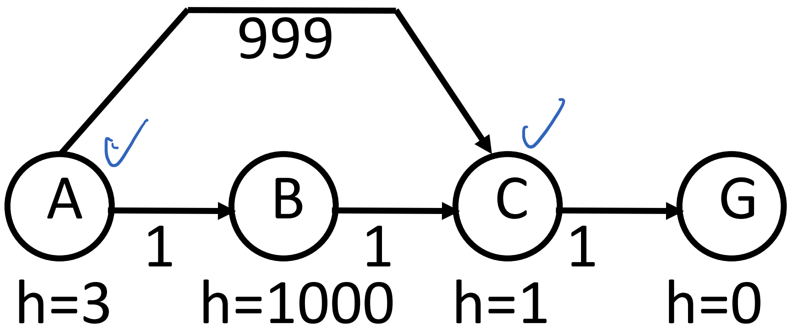


- Not optimal! **Get** A → C → G. **Want**: A →B → C → G

Attempt 2: **A Search**

Next approach: use both ***g*(*s*)** + ***h*(*s*)** alone

- Specifically, expand node with smallest ***g*(*s*)** + ***h*(*s*)**
- Again, use a priority queue
- Called **"A" search**



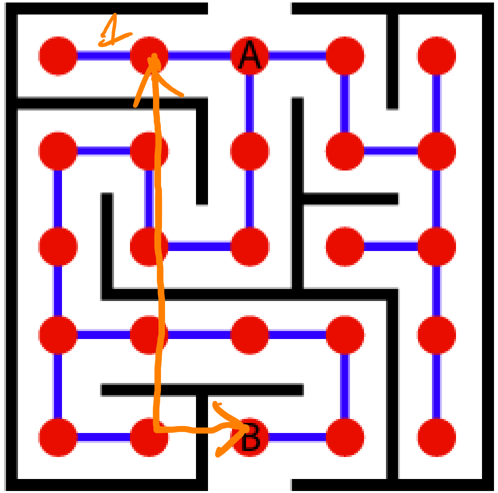
- **Still not optimal!** (Does work for former example).

Attempt 3: **A* Search**

h(s)=5

Same idea, use $g(s) + h(s)$, with one **requirement**

- Demand that $h(s) \leq h^*(s)$
- If heuristic has this property, "admissible"
  - Optimistic! Never over-estimates
- Still need $h(s) \geq 0$
  - Negative heuristics can lead to strange behavior
- This is **A* search**



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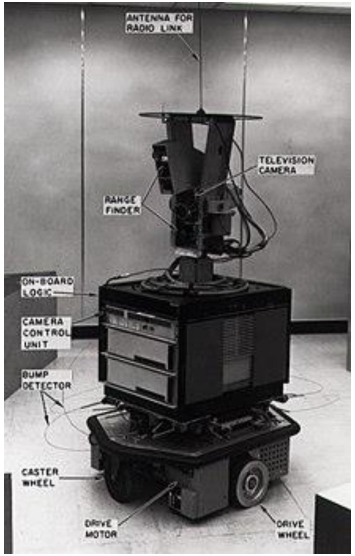
Attempt 3: **A* Search**

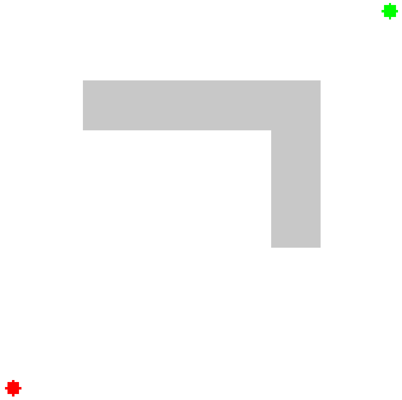
O : fringe
color : h(s).

Origins: robots and planning



Shakey the Robot, 1960's

Credit: Wiki



Animation: finding a path around obstacle

Credit: Wiki

Admissible Heuristic Functions

Have to be careful to ensure admissibility (**optimism!)**

- Example: **8 Game**

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- One useful approach: **relax constraints**
  - *h*(*s*) = number of tiles in wrong position
    - allows tiles to fly to destination in a single step

$h(s) = 5.$

Break & Quiz

Q 1.1: Consider finding the fastest driving route from one US city to another. Measure cost as the number of hours driven when driving at the speed limit. Let ***h*(*s*)** be the number of hours needed to ride a bike from city s to your destination. ***h*(*s*)** is

- A. An admissible heuristic
- B. Not an admissible heuristic

Break & Quiz

Q 1.1: Consider finding the fastest driving route from one US city to another. Measure cost as the number of hours driven when driving at the speed limit. Let ***h*(*s*)** be the number of hours needed to ride a bike from city s to your destination. ***h*(*s*)** is

- A. An admissible heuristic
- **B. Not an admissible heuristic**

Break & Quiz

Q 1.1: Consider finding the fastest driving route from one US city to another. Measure cost as the number of hours driven when driving at the speed limit. Let ***h*(*s*)** be the number of hours needed to ride a bike from city s to your destination. ***h*(*s*)** is

- A. An admissible heuristic No: riding your bike take longer.
- **B. Not an admissible heuristic**

Break & Quiz

Q 1.2: Which of the following are admissible heuristics?

✓ (i) $h(s) = h^*(s)$

✗ (ii) $h(s) = \max(2, h^*(s))$

✓ (iii) $h(s) = \min(2, h^*(s))$ $\leq h^*(s)$

✗ (iv) $h(s) = h^*(s) - 2$ may be negative

(v) $h(s) = \mathrm{sqrt}(h^*(s))$

- A. All of the above
- B. (i), (iii), (iv)
- C. (i), (iii)
- D. (i), (iii), (v)

Break & Quiz

Q 1.2: Which of the following are admissible heuristics?

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- A. All of the above
- B. (i), (iii), (iv)
- **C. (i), (iii)**
- D. (i), (iii), (v)

Break & Quiz

Q 1.2: Which of the following are admissible heuristics?

(i) $h(s) = h^*(s)$

(ii) $h(s) = \max(2, h^*(s))$ No: $h(s)$ might be too big

(iii) $h(s) = \min(2, h^*(s))$

(iv) $h(s) = h^*(s)-2$ No: $h(s)$ might be negative

(v) $h(s) = \text{sqrt}(h^*(s))$ No: if $h^*(s) < 1$ then $h(s)$ *is bigger*

- A. All of the above
- B. (i), (iii), (iv)
- **C. (i), (iii)**
- D. (i), (iii), (v)

Heuristic Function Tradeoffs

Dominance: h_2 dominates h_1 if for all states *s*,

$$h_1(s) \leq h_2(s) \leq h^*(s)$$

- **Idea**: we want to be as close to h^* as possible
  - But not over!
- **Tradeoff**: being very close might require a very complex heuristic, expensive computation
  - Might be better off with cheaper heuristic & expand more nodes.

Heuristic Function Tradeoffs

h_2 dominates h_1.

- Example: **8 Game**

Example State

1		5
2	6	3
7	4	8

Goal State

1	2	3
4	5	6
7	8	

- Previous heuristic: $\boldsymbol{h_1(s)}$ = number of tiles in wrong position
- Better heuristic?

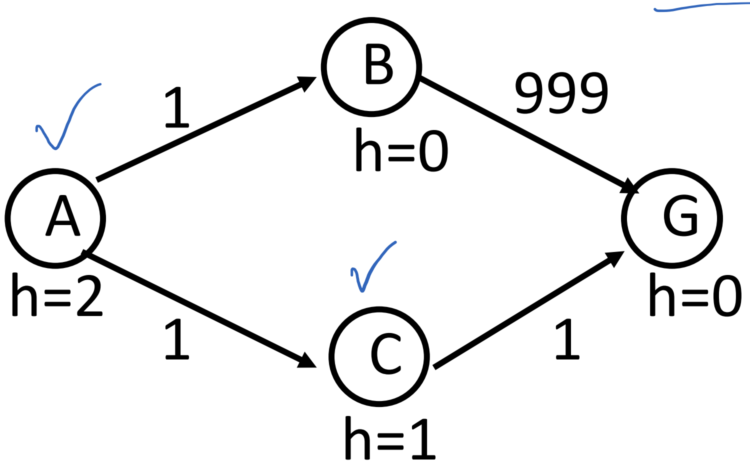
$$h_2(s) = \sum_{tile = wrong} \text{Manhattan distance}(tile, destination)$$

A* Termination

g(s) + h(s)
B 1 + 0
C 1 + 1
G 1000 + 0

When should A* **stop**?

- One idea: as soon as we reach goal state?

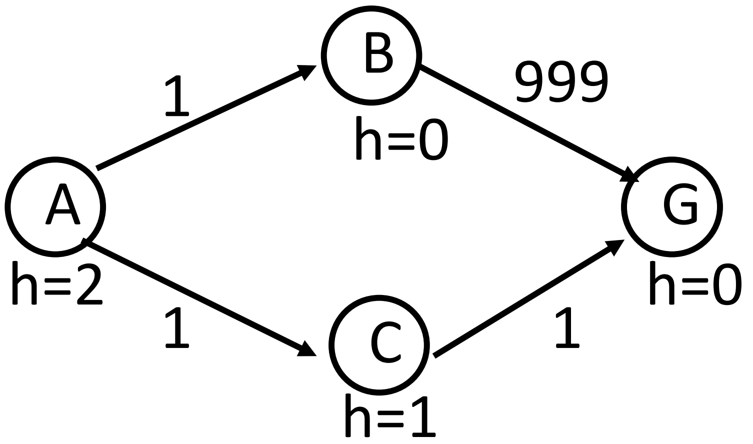


- ***h*** admissible, but note that we get A →B → G (**cost 1000**)!

A* Termination

When should A* stop?

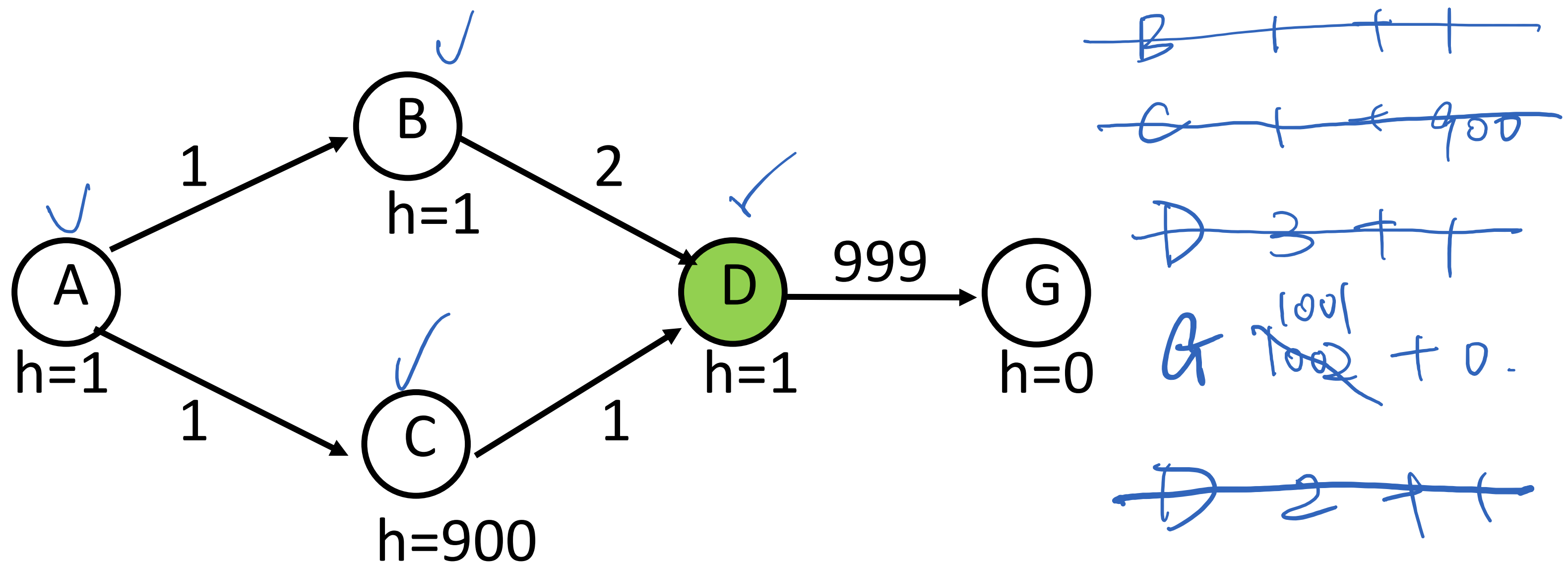
- **Rule**: terminate **when a goal is popped** from queue.



- Note: taking ***h*** =0 reduces to uniform cost search rule.

A* Revisiting Expanded States

Possible to revisit an expanded state, get a shorter path:



- Put D back into priority queue, smaller **g**+**h**

A* Full Algorithm

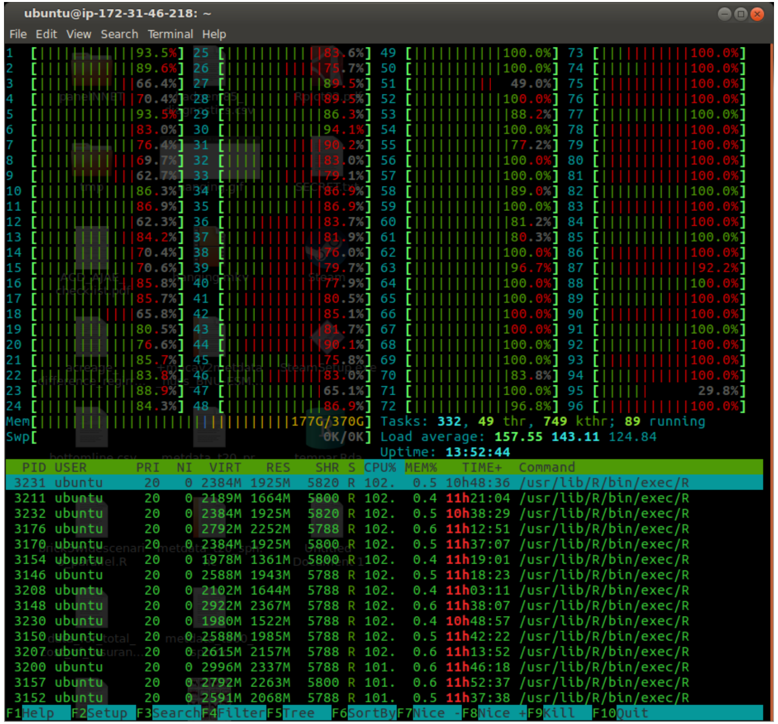
1. Put the start node S on the priority queue, called OPEN
2. If OPEN is empty, exit with failure
3. Remove from OPEN and place on CLOSED a node n for which f(n) is minimum (note that f(n)=g(n)+h(n))
4. If n is a goal node, exit (trace back pointers from n to S)
5. Expand n, generating all successors and attach to pointers back to n. For each successor n' of n
    1. If n' is not already on OPEN or CLOSED estimate h(n'), g(n')=g(n)+ c(n,n'), f(n')=g(n')+h(n’), and place it on OPEN.
    2. If n' is already on OPEN or CLOSED, then check if g(n') is lower for the new version of n'. If so, then:
        1. Redirect pointers backward from n' along path yielding lower g(n').
        2. Put n' on OPEN.
    3. If g(n') is not lower for the new version, do nothing.
6. Goto 2.

A* Analysis

Some properties:

- Terminates!
- A* can use **lots of memory**: O(# states).
- Will run out on large problems.

- Next, we will consider some alternatives to deal with this.



Break & Quiz

Q 2.1: Consider two heuristics for the 8 puzzle problem. h_1 is the number of tiles in wrong position. h_2 is the l_1/Manhattan distance between the tiles and the goal location. How do h_1 and h_2 relate?

- A. h_2 dominates h_1
- B. h_1 dominates h_2
- C. Neither dominates the other

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- **A. h_2 dominates h_1**
- B. $\boldsymbol{h_1}$ dominates $\boldsymbol{h_2}$ (No: $\boldsymbol{h_1}$ is a distance where each entry is at most 1, $\boldsymbol{h_2}$ can be greater)
- C. Neither dominates the other

Break & Quiz

Q 2.2: Consider the state space graph below. Goal states have bold borders. ***h*(*s*)** is show next to each node. What node will be expanded by A* after the initial state I?

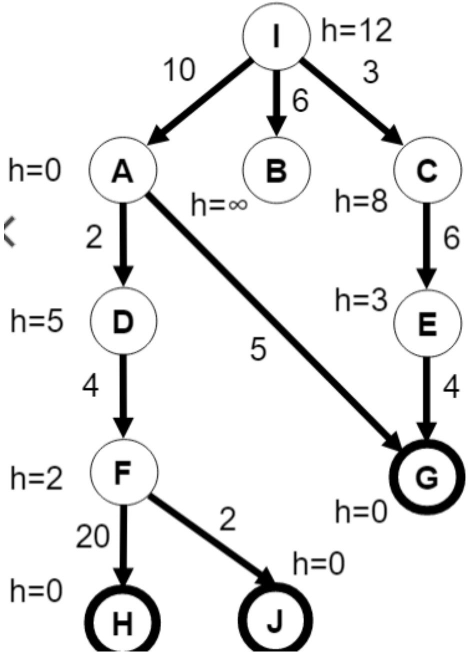
- A. A
- B. B
- C. C

g(s) + h(s)

A 10 + 0

B 6 + ∞

C 3 + 8



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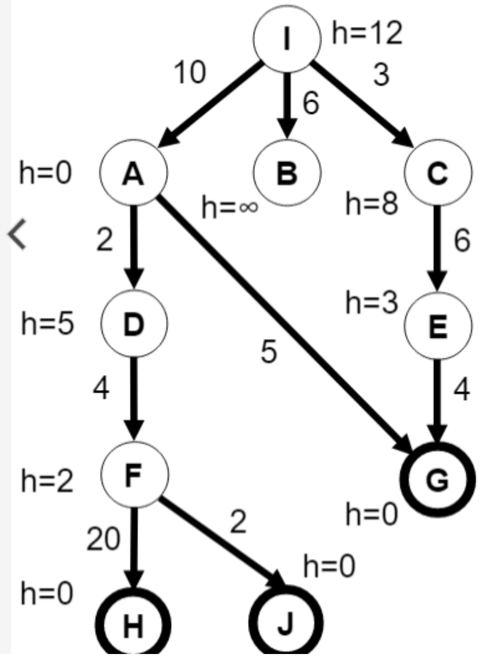
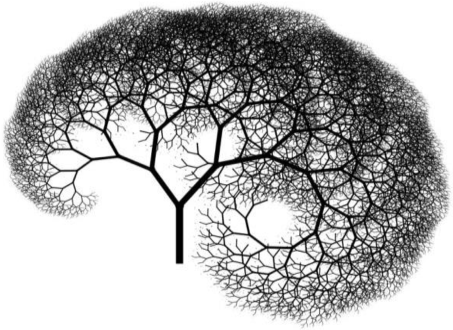


IDA*: Iterative Deepening A*

Similar idea to our earlier iterative deepening.

- Bound the memory in search.
- At each phase, don't expand any node with $g(s) + h(s) > k$,
  - Assuming integer costs, do this for k=0, then k=1, then k=2, and so on
- Complete + optimal, might be costly time-wise
  - Revisit many nodes
- Lower memory use than A*



Fractalsaco

IDA*: Properties

How many restarts do we expect?

- With integer costs, optimal solution *C**, at most *C**

What about non-integer costs?

- Initial threshold *k*. Use the same rule for non-expansion
- Set new *k* to be the min ***g*(*s*)** + ***h*(*s*)** for non-expanded nodes
- Worst case: restarted for each state

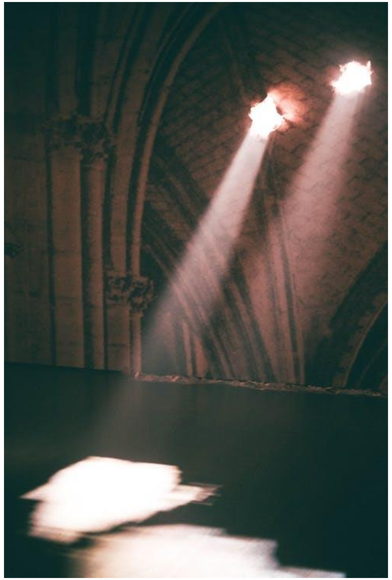
Beam Search

General approach (beyond A* too)

- Priority queue with fixed size *k*; beyond *k* nodes, **discard**!
- **Upside**: good memory efficiency
- **Downside**: not complete or optimal

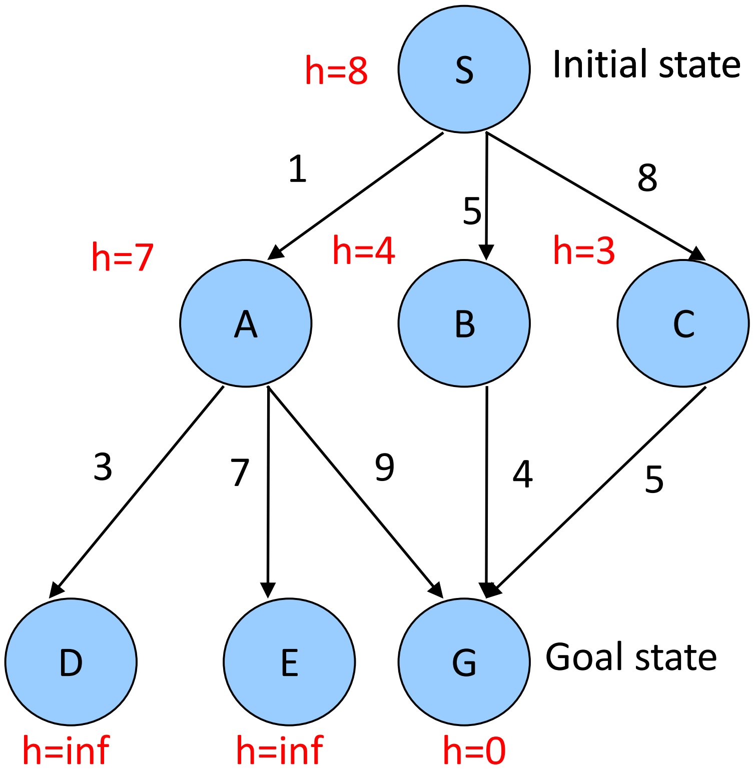
Variation:

- Priority queue with nodes that **are at most ε worse** than best node.



Recap and Examples

Example for A*:

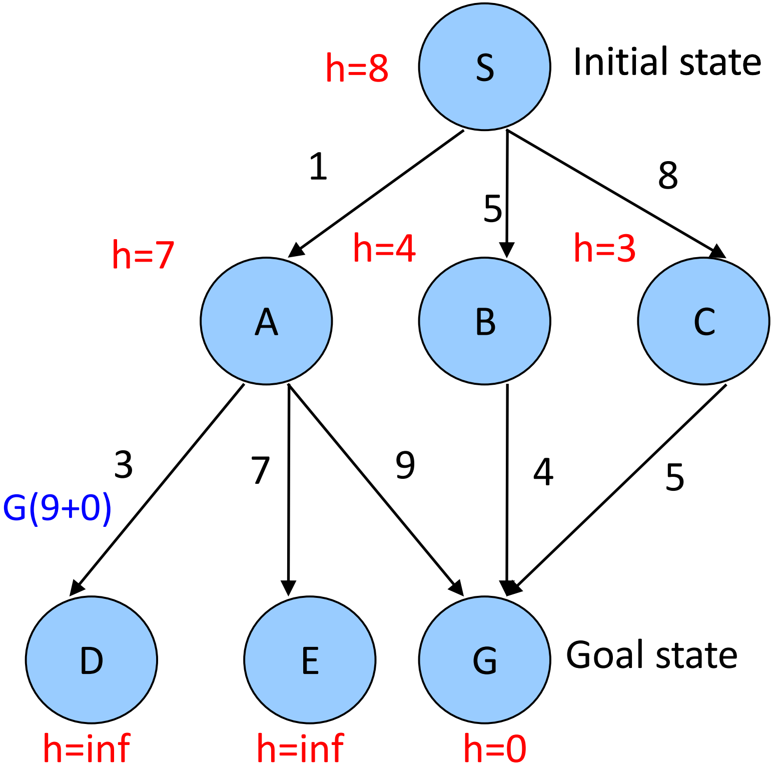


Recap and Examples

Example for A*:

OPEN	CLOSED
S(0+8)	-
A(1+7) B(5+4) C(8+3)	S(0+8)
B(5+4) C(8+3) D(4+inf) E(8+inf) G(10+0)	S(0+8) A(1+7)
C(8+3) D(4+inf) E(8+inf) G(9+0)	S(0+8) A(1+7) B(5+4)
C(8+3) D(4+inf) E(8+inf)	S(0+8) A(1+7) B(5+4) G(9+0)

G → B → S

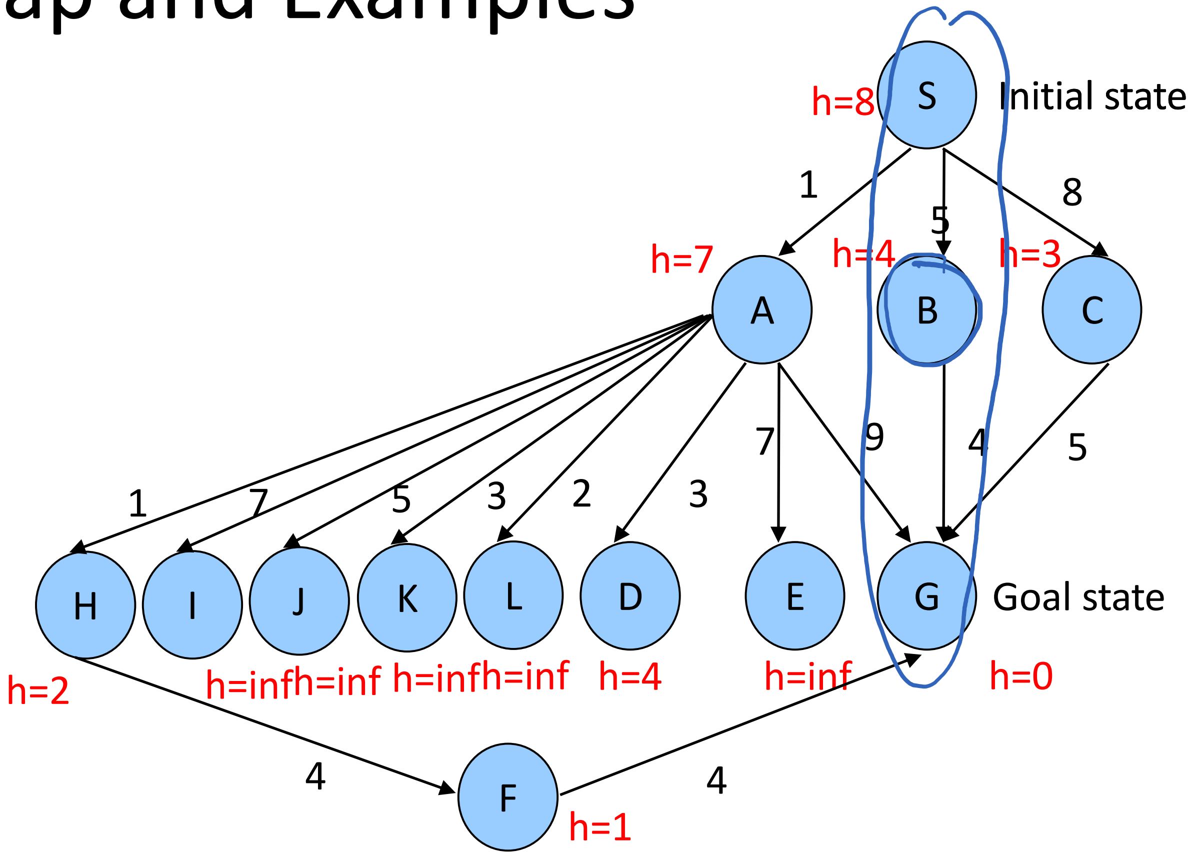


Recap and Examples

Example for IDA*:

Threshold = 8

PREFIX	OPEN
-	S(0+8)
S	A(1+7)
S A	H(2+2) D(4+4)
S A H	D(4+4) F(6+1)
S A H F	D(4+4)
S A D	

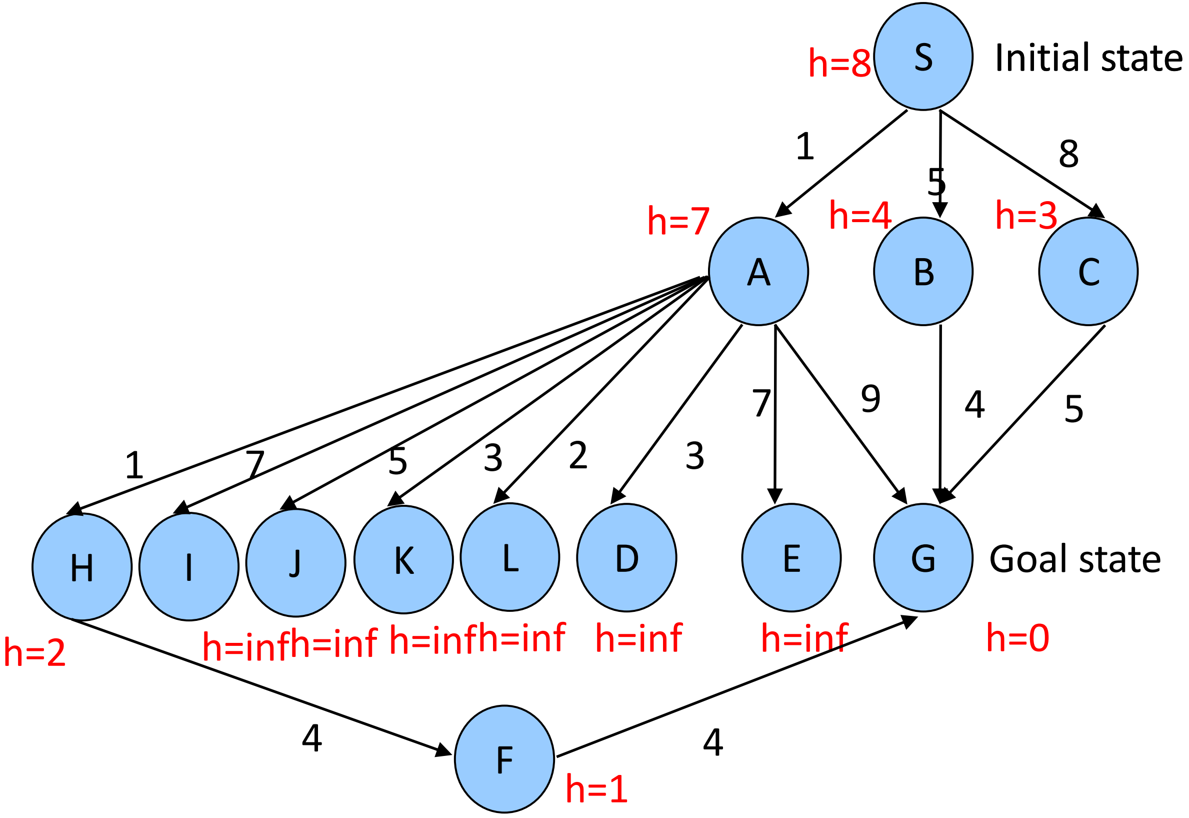


Recap and Examples

Example for IDA*:

Threshold = 9

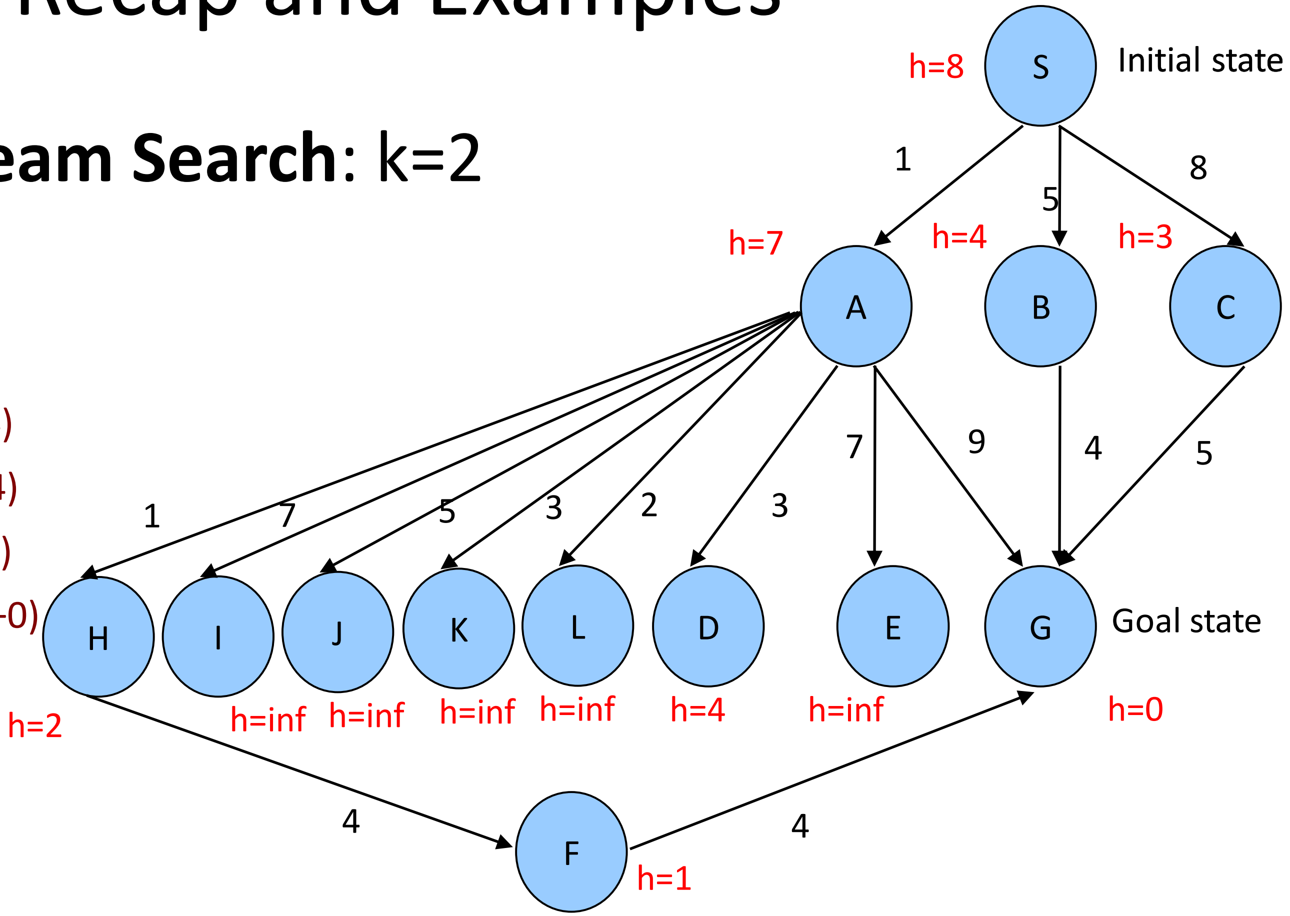
PREFIX	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
S A	B(5+4) H(2+2) D(4+4)
S A H	B(5+4) D(4+4) F(6+1)
S A H F	B(5+4) D(4+4)
S A D	B(5+4)
S B	G(9+0)
S B G	



Recap and Examples

Example for **Beam Search**: k=2

CURRENT	OPEN
-	S(0+8)
S	A(1+7) B(5+4)
A	H(2+2) D(4+4)
H	D(4+4) F(6+1)
F	D(4+4) G(10+0)
D	G(10+0)
G	



Summary

- Informed search: introduce heuristics
  - Not all approaches work: best-first greedy is bad
- A* algorithm
  - Properties of A*, idea of admissible heuristics
- Beyond A*
  - IDA*, beam search. Ways to deal with space requirements.

Acknowledgements: Adapted from materials by Jerry Zhu and Fred Sala (University of Wisconsin-Madison).