CS 540 Introduction to Artificial Intelligence
Reinforcement Learning I

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Dec 2, 2021

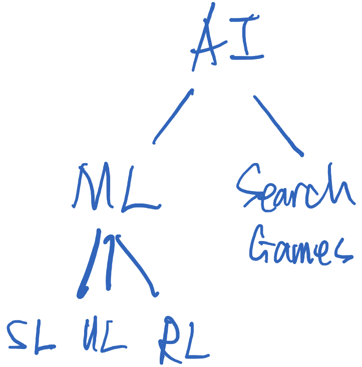
Announcements

- **Homeworks**:
  - HW9 due next Tue
  - HW10 released
- **Final**: administrative details out soon
- Class roadmap:
  - Today and next Tuesday: Reinforcement Learning

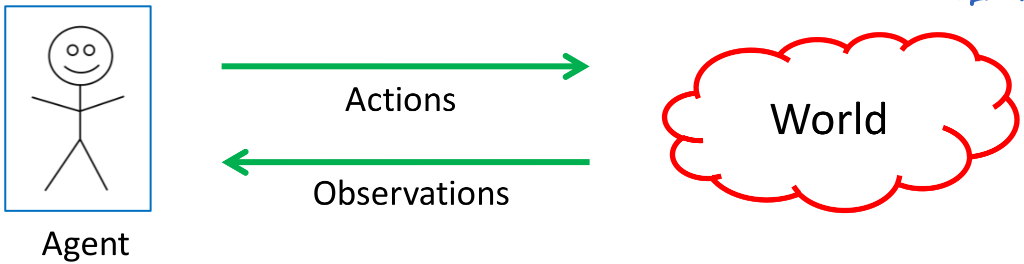
Outline

- Introduction to reinforcement learning
  - Basic concepts, mathematical formulation, MDPs, policies
- Valuing policies
  - Value functions, Bellman equation, value iteration
- Q-learning (time permitted)
  - Q function, SARSA

Back to Our General Model



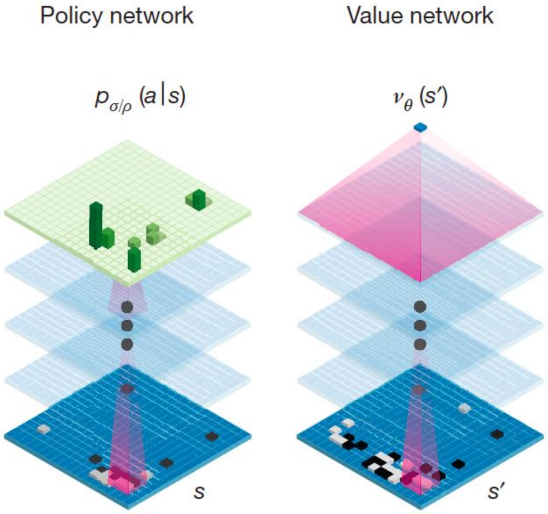
We have an **agent** **interacting** with the **world**



- Agent receives a reward based on state of the world
  - **Goal**: maximize reward / utility **($$$)**
  - Note: **data** consists of actions & observations
    - Compare to unsupervised learning and supervised learning

Examples: Gameplay Agents

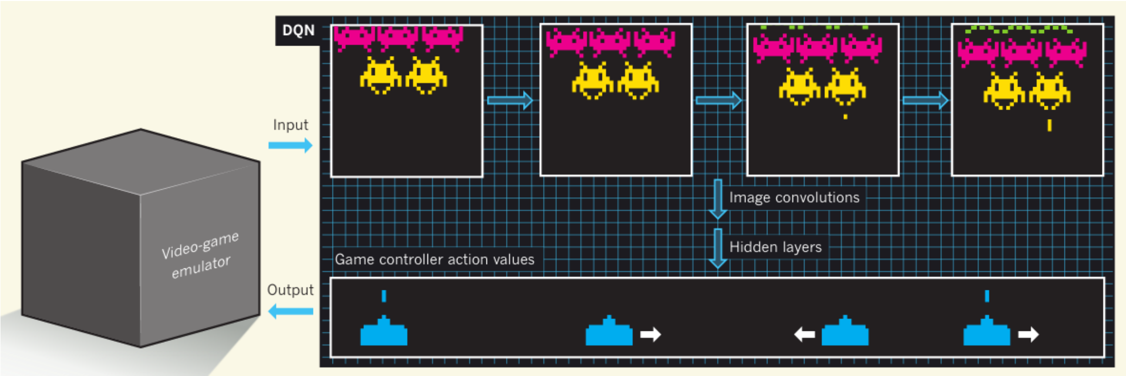
AlphaGo:

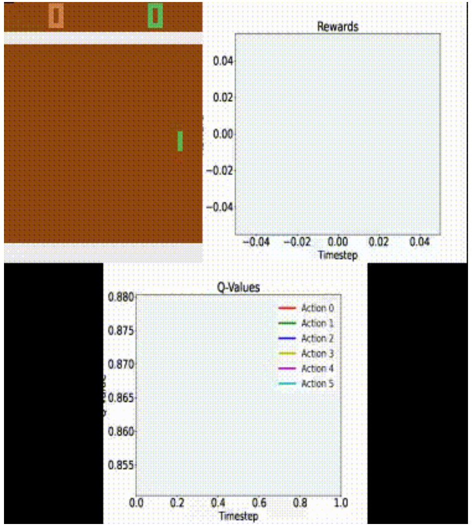


https://deepmind.com/research/alphago/

Examples: Video Game Agents

Pong, Atari



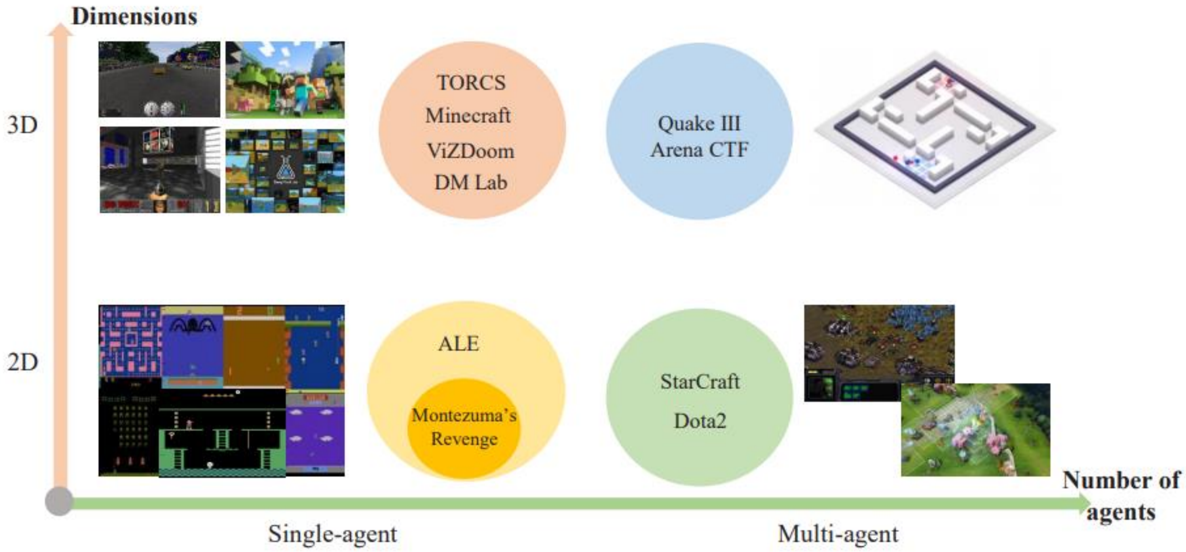


Mnih et al, "Human-level control through deep reinforcement learning"

A. Nielsen

Examples: Video Game Agents

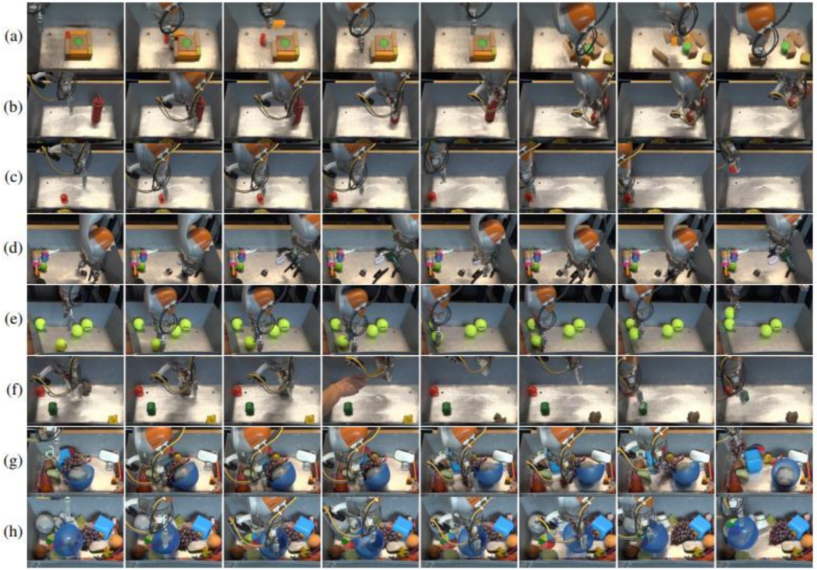
Minecraft, Quake, StarCraft, and more!



Shao et al, "A Survey of Deep Reinforcement Learning in Video Games"

Examples: Robotics

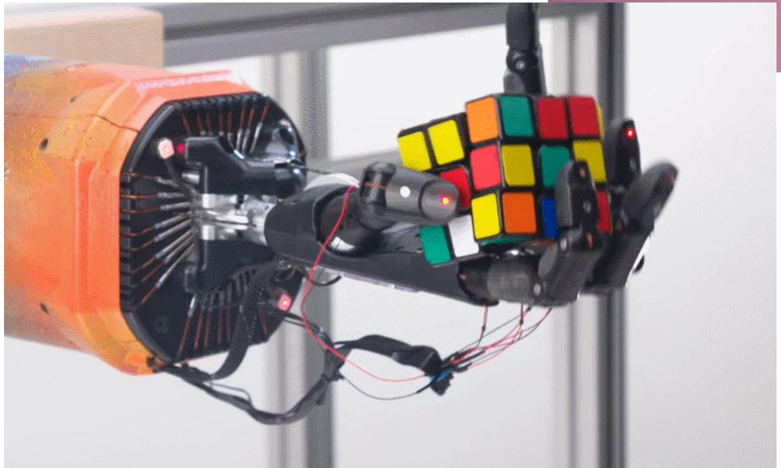
Training robots to perform tasks (e.g., grasp!)

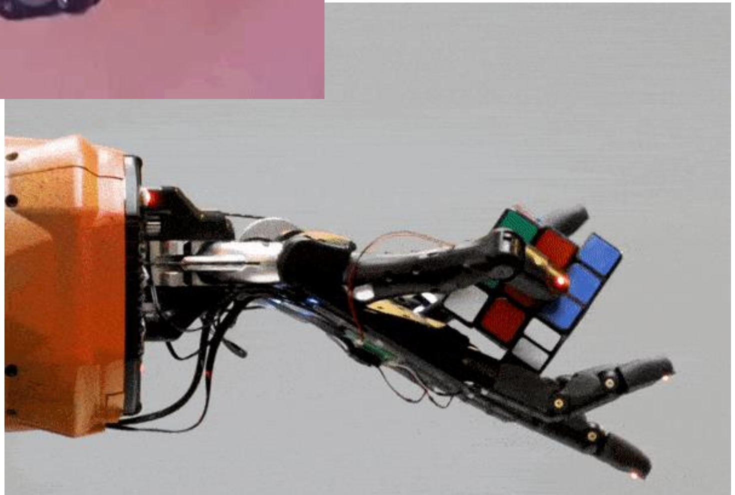


Ibarz et al, " How to Train Your Robot with Deep Reinforcement Learning – Lessons We’ve Learned "

Example: Robotics + Puzzle Solving

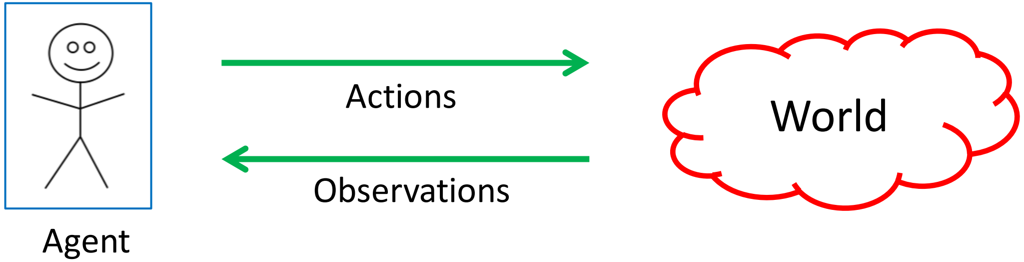
OpenAI , “Solving Rubik's Cube with a Robot Hand”, 2019





Building The Theoretical Model

Basic setup:



- Set of states, S
- Set of actions A
- Information: at time t, observe state $s_t \in$ S. Get reward r_t
- Agent makes choice $a_t \in$ A. State changes to s_{t+1}, continue

Goal: find a map from **states to actions** maximize rewards.

A "policy"

Markov Decision Process (MDP)

The formal mathematical model:

- **State set** S. Initial state s_0. **Action set** A
- **State transition model:** $P(s_{t+1} | s_t, a_t)$
  - Markov assumption: transition probability only depends on s_t and a_t, and not previous actions or states.
- **Reward function:** $r(s_t)$
- **Policy**: $\pi(s) : S \to A$ action to take at a particular state.

$= \pi(s_0)$ $= \pi(s_1)$ $= \pi(s_2)$

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \dots$$

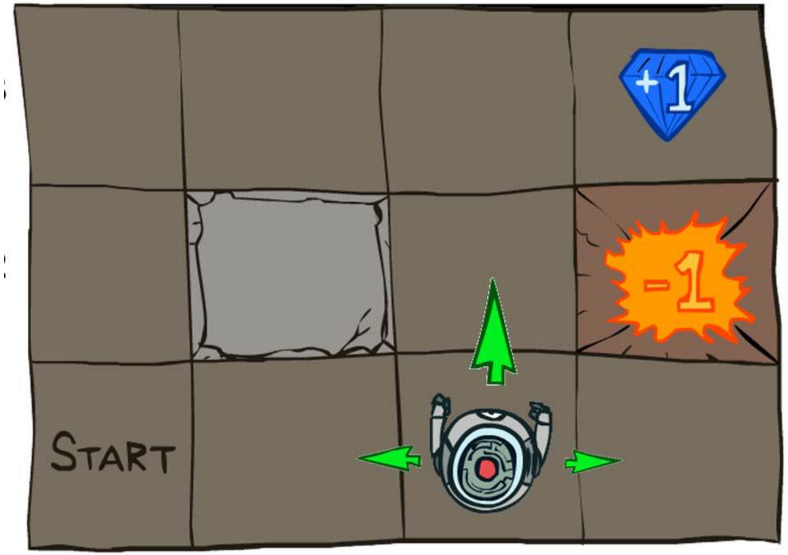
$r_0 = r(s_0)$ $r_1 = r(s_1)$ $r_2 = r(s_2)$

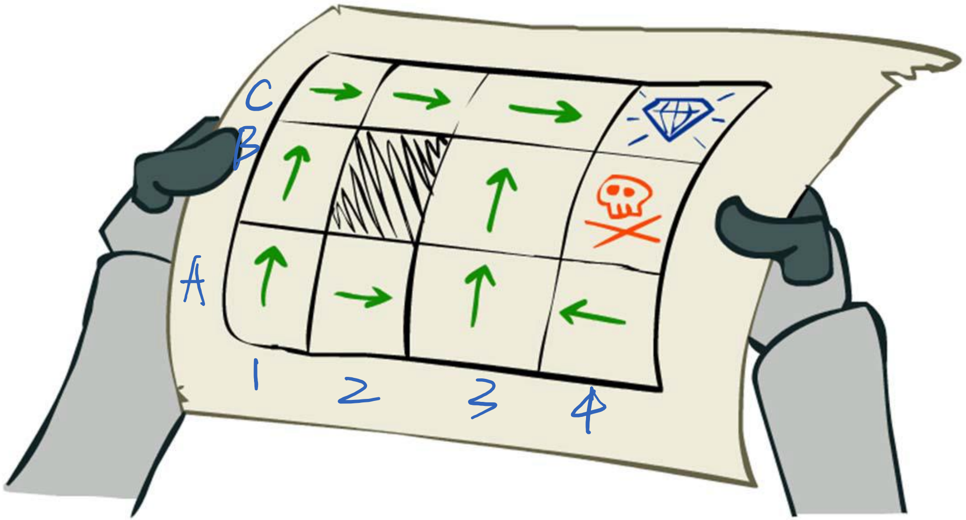
Example of MDP: Grid World

$\pi(A1) = \text{up}$, $\pi(C3) = \text{right}$, ...

Robot on a grid; goal: find the best policy

$\pi(\cdot)$

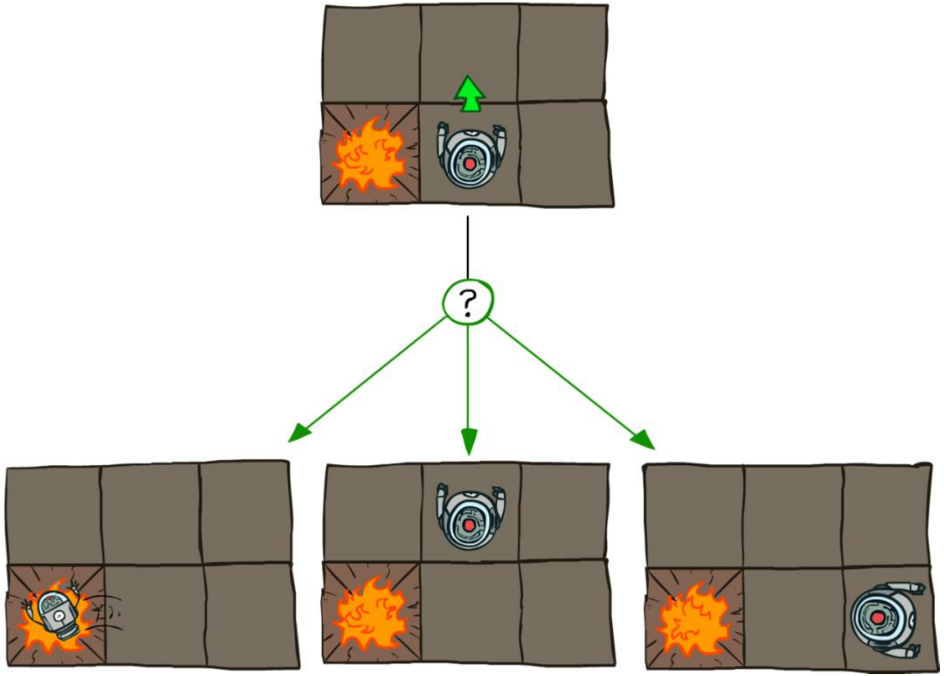


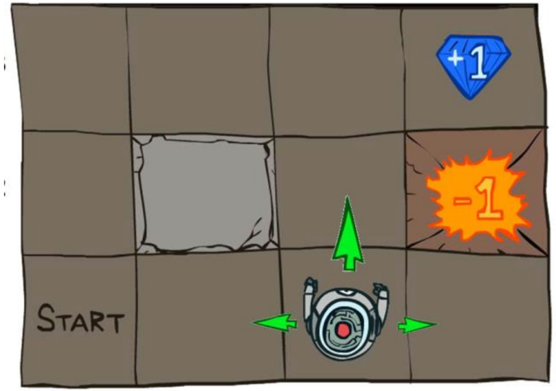


Source: P. Abbeel and D. Klein

Example of MDP: Grid World

Note: (i) Robot is unreliable (ii) Reach target fast

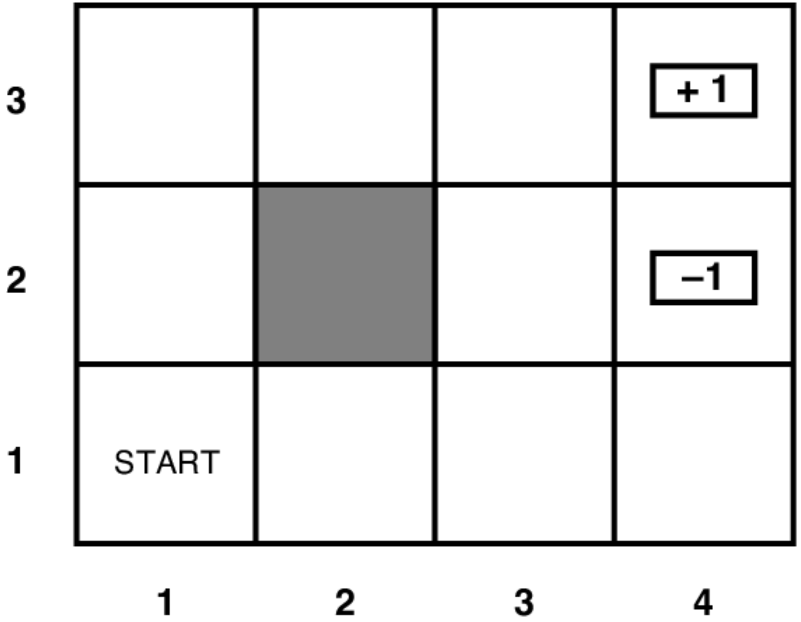


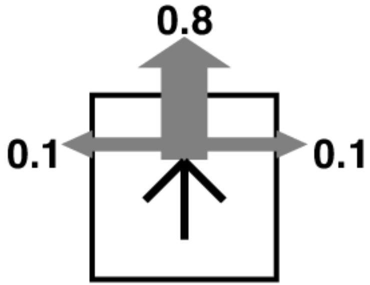


$r(s) = -0.04$ for every non-terminal state

Grid World Abstraction

Note: (i) Robot is unreliable (ii) Reach target fast

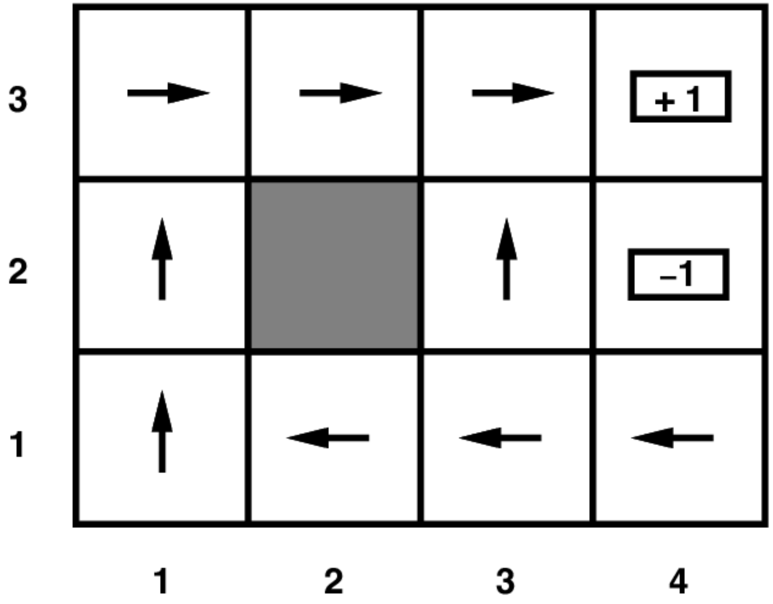


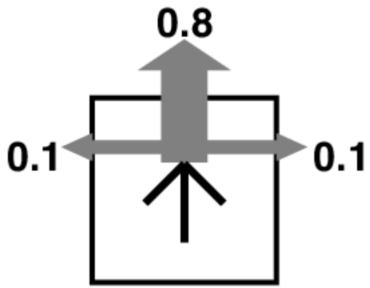


$r(s) = -0.04$ for every non-terminal state

Grid World Optimal Policy

Note: (i) Robot is unreliable (ii) Reach target fast





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Back to MDP Setup

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- **Reward function:** $r(s_t)$
- **Policy**: $\pi(s) : S \to A$ action to take at a particular state.

How do we find the best policy?

$$s_0 \xrightarrow{a_0} s_1 \xrightarrow{a_1} s_2 \xrightarrow{a_2} \ldots$$

Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value
- B. The policy maps states to actions
- C. The probability of next state can depend on current and previous states
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards

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- **C. The probability of next state can depend on current and previous states**
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards

Break & Quiz

Q 1.1 Which of the following statement about MDP is **not** true?

- A. The reward function must output a scalar value (**True: need to be able to compare**)
- B. The policy maps states to actions (**True: a policy tells you what action to take for each state**).
- **C. The probability of next state can depend on current and previous states** (**False: Markov assumption**).
- D. The solution of MDP is to find a policy that maximizes the cumulative rewards (**True: want to maximize rewards overall**).

$s_0 \xrightarrow{a_0=\pi(s_0)} s_1 \xrightarrow{a_1=\pi(s_1)} s_2 \longrightarrow s_3 \longrightarrow$ $\quad P(s_0, s_1, \ldots)$

$r_0 \quad r_1 \quad r_2 \quad r_3$

Value Function

$s_0 \xrightarrow{a_0=\pi(s_0)} s_1' \xrightarrow{a_1'=\pi(s_1')} s_2' \longrightarrow s_3' \longrightarrow$ $\quad P(s_0, s_1', s_2' \ldots)$

$r_0 \quad r_1' \quad r_2' \quad r_3$

For policy π, **expected utility** over all possible state sequences from s_0 produced by following that policy:

$$V^\pi(s_0) = \sum_{\substack{\text{sequences} \\ \text{starting from } s_0}} P(\text{sequence})\,U(\text{sequence})$$

$$U(\text{sequence}) = r_0 + \gamma r_1 + \gamma^2 r_2 + \cdots$$

$$= \sum_{s_0, s_1, \ldots} P(s_0, s_1, \ldots)\,U(s_0, s_1, \ldots)$$

Called the **value function** (for π, s_0)

Discounting Rewards

One issue: these are infinite series. **Convergence**?

- Solution utility = discounted cumulative reward

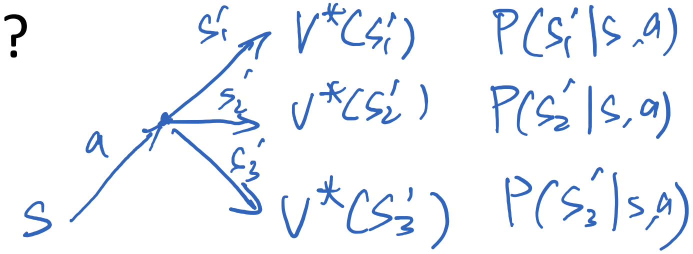
$$U(s_0, s_1 \ldots) = r(s_0) + \gamma r(s_1) + \gamma^2 r(s_2) + \ldots = \sum_{t \geq 0} \gamma^t r(s_t)$$

- Discount factor γ between 0 and 1
  - Set according to how important **present** is vs. **future**
  - Note: has to be less than 1 for convergence

From Value to Policy

Now that $V^{\pi}(s_0)$ is defined, what *a* should we take?

- First, set $V^*(s)$ to be expected utility for **optimal** policy from *s*
- What's the expected utility of an action?
  - Specifically, action *a* in state *s*?



$$\sum_{s'} P(s'|s,a)V^*(s')$$

All the states we could go to

Transition probability

Expected rewards

Obtaining the Optimal Policy

We know the expected utility of an action.

- So, to get the optimal policy, compute

$$\pi^*(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

All the states we could go to

Transition probability

Expected rewards

Slight Problem...

Now we can get the optimal policy by doing

$$\pi^*(s) = \text{argmax}_a \sum_{s'} P(s'|s,a)V^*(s')$$

- So we need to know $V^*(s)$.
  - But it was defined in terms of the optimal policy!
  - So we need some other approach to get $V^*(s)$.
  - Need some other **property** of the value function!

Bellman Equation

Let’s walk over one step for the value function:

$$V^*(s) = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a)V^*(s')$$

Current state reward

Discounted expected future **rewards**

- Bellman: inventor of dynamic programming

Value Iteration

$V_1 = \begin{bmatrix} 9 \\ .9 \\ .9 \\ .9 \\ .9 \\ .9 \\ .9 \end{bmatrix}$ $\quad V_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

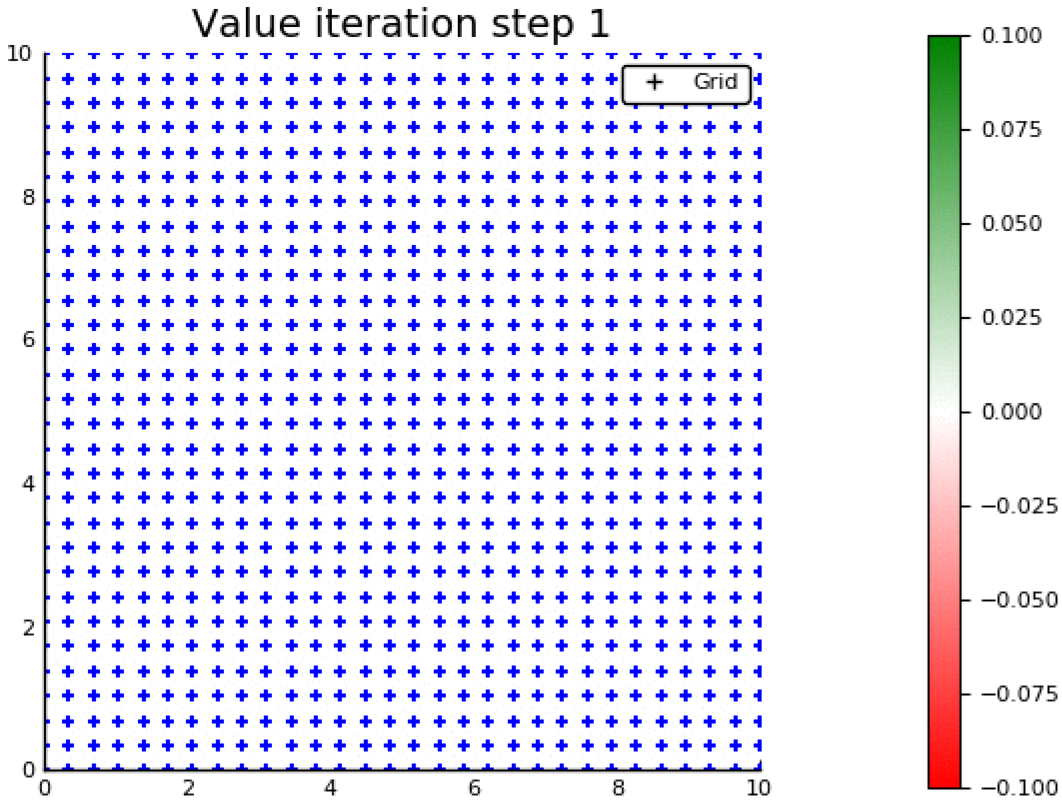
Q: how do we find $V^*(s)$?

- Why do we want it? Can use it to get the best policy
- Know: reward $r(s)$, transition probability $P(s'|s,a)$
- Also know $V^*(s)$ satisfies Bellman equation (recursion above)

A: Use the property. Start with $V_0(s)=0$. Then, update

$$\underbrace{V_{i+1}(s)}_{V_1(s)} = r(s) + \gamma \max_a \sum_{s'} P(s'|s,a)\underbrace{V_i(s')}_{V_0(s')}, \quad \forall s$$

Value Iteration: Demo

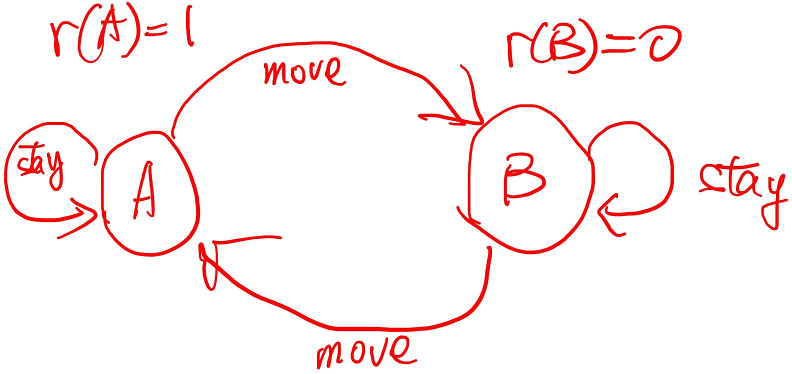


Source: POMDPBGallery Julia Package

Break & Quiz

Q 2.1 Consider an MDP with 2 states {*A, B*} and 2 actions: **"stay"** at current state and "**move**" to other state. Let **r** be the reward function such that **r**(*A*) = 1, **r**(*B*) = 0. Let γ be the discounting factor. Let π: π(*A*) = π(*B*) = **move** (i.e., an "always move" policy). What is the value function $V^{\pi}(A)$?

- A. 0
- B. $1 / (1 - \gamma)$
- C. $1 / (1 - \gamma^2)$
- D. 1



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- A. 0
- B. $1/(1-\gamma)$
- **C. $1/(1-\gamma^2)$**
- D. 1

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- A. 0
- B. 1/(1-γ)
- **C. 1/(1-γ^2)** (States: A,B,A,B,... rewards 1,0, γ^2,0, γ^4,0)
- D. 1

$V^{\pi}(A) = U(ABAB\cdots) \cdot P(ABAB\cdots)$

$= (1+0+\gamma^2 1+0+\gamma^4 1+\cdots) 1$

$V^{\pi}(B) = 0+\gamma+0+\gamma^3+\cdots+ = \gamma \cdot \frac{1}{1-\gamma^2}$

BABA ....

Q-Learning

What if we don't know transition probability $P(s'|s,a)$?

- Need a way to learn to act without it.
- **Q-learning**: get an action-value function $Q(s,a)$ that tells us the value of doing a in state s
- Note: $V^*(s) = \max_a Q(s,a)$
- Now, we can just do $\pi^*(s) = \arg\max_a Q(s,a)$
  - But need to estimate Q!

Q-Learning Iteration

How do we get Q(*s*,*a*)?

- Similar iterative procedure

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

Learning rate

- **Idea**: combine old value and new estimate of future value.

Exploration Vs. Exploitation

General question!

- **Exploration:** take an action with unknown consequences
  - Pros:
    - Get a more accurate model of the environment
    - Discover higher-reward states than the ones found so far
  - Cons:
    - When exploring, not maximizing your utility
    - Something bad might happen
- **Exploitation:** go with the best strategy found so far
  - Pros:
    - Maximize reward as reflected in the current utility estimates
    - Avoid bad stuff
  - Cons:
    - Might also prevent you from discovering the true optimal strategy

Q-Learning: Epsilon-Greedy Policy

How to **explore**?

- With some $0<\varepsilon<1$ probability, take a random action at each state, or else the action with highest Q(*s*,*a*) value.

$$a = \begin{cases} \operatorname{argmax}_{a \in A} Q(s, a) & \text{uniform}(0,1) > \epsilon \\ \text{random } a \in A & \text{otherwise} \end{cases}$$

Q-Learning: SARSA

Using the epsilon-greedy policy, an alternative:

- Just use the next action, no max over actions:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r(s_t) + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$$

Learning rate

- Called state–action–reward–state–action (**SARSA**)

Break & Quiz

Q 3.1 What is the main difficulty in applying Q-learning with a discrete Q-table to an environment with continuous numerical observations?

- A. We may not converge to the correct Q(s,a) values.
- B. It would take a potentially intractable amount of memory to represent every possible Q(s,a) value in the discrete Q-table.
- C. Q-learning with a discrete Q-table can be applied to these environments with no issues.

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- C. Q-learning with a discrete Q-table can be applied to these environments with no issues.

Summary

- Reinforcement learning setup
- Mathematica formulation: MDP
- Value functions & the Bellman equation
- Value iteration
- Q-learning

Acknowledgements: Based on slides from Fred Sala, Yin Li, Jerry Zhu, Svetlana Lazebnik, Yingyu Liang, David Page, Mark Craven, Pieter Abbeel, Dan Klein