# A Nontrivial Inequality

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$$Claim: \quad q(s)\sum_{\ell=1}^{logn} 2^{\ell}(1-q(s))^{2^{\ell}-1} \leq 2\cdot q(s)\sum_{j=1}^{n-1}(1-q(s))^{j}$$

PROOF:
Suppose $j = 2^{\ell}-1$,

$$q(s)\sum_{\ell=1}^{logn} 2^{\ell}(1-q(s))^{2^{\ell}-1}$$

$$= q(s)[2(1-q(s))^{1} + 4(1-q(s))^{3} + 8(1-q(s))^{7} + \cdots + n(1-q(s))^{n-1}]$$

$$= q(s)[2(1-q(s))^{1} + \underbrace{2(1-q(s))^{3} + 2(1-q(s))^{3}}_{2} + \underbrace{2(1-q(s))^{7} + \cdots + 2(1-q(s))^{7}}_{4} + \cdots$$
$$\cdots + \underbrace{2(1-q(s))^{n-1} + 2(1-q(s))^{n-1} + \cdots + 2(1-q(s))^{n-1}}_{2^{logn-1}}]$$

$$\leq q(s)[2(1-q(s))^{1} + \underbrace{2(1-q(s))^{2} + 2(1-q(s))^{3}}_{2} + \underbrace{2(1-q(s))^{4} + \cdots + 2(1-q(s))^{7}}_{4} + \cdots$$
$$\cdots + \underbrace{2(1-q(s))^{2^{log(n-1)}} + 2(1-q(s))^{2^{log(n-1)}+1} + \cdots + 2(1-q(s))^{n-1}}_{2^{logn-1}}]$$

$$= q(s)\sum_{j=1}^{n-1}(1-q(s))^{j}$$

**Q.E.D**