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Since we need find a distribution $\mathcal{P}$ to satisfy

$$\sum_{k=1}^{n-j+1} p_k \cdot \frac{j \cdot \binom{n-j}{k-1}}{\binom{n}{k}} = O(\frac{1}{logn})$$

and the right $\frac{1}{logn}$ is quite hard to eliminated by summation. So I guess

$$p_k = \frac{1}{logn} \cdot q_k$$

where $\sum_{k=1}^{n} q_k = logn$. Recall that

$$\sum_{i=1}^{\infty} \frac{1}{i} \longrightarrow logn$$

So Let $q_k = \frac{1}{k}$,what remained I should do is to prove

$$\sum_{k=1}^{n-j+1} \frac{1}{k} \cdot \frac{j \cdot \binom{n-j}{k-1}}{\binom{n}{k}} = O(1)$$

that is equivalent to prove

$$\sum_{k=1}^{n-j+1} \frac{1}{k} \cdot \frac{\binom{n-j}{k-1}}{\binom{n}{k}} = O(\frac{1}{j})$$

And let

$$\Phi(k) = \frac{1}{k} \cdot \frac{\binom{n-j}{k-1}}{\binom{n}{k}} \;\; obviously \;\; \Phi(k) > 0$$

Easily

$$\frac{\Phi(k+1)}{\Phi(k)} = \frac{n-j-k+1}{n-k} \leq \frac{n-j+1}{n} \leq 1$$

for $j \geq 1$ and$k \geq 1$.

So, we can get

$$\Phi(k) \leq \Phi(1) \cdot \hbar^{k-1}$$

where $\hbar = \frac{n-j+1}{n} \leq 1$.And

$$\sum_{k=1}^{n-j+1} \Phi(k) \leq \sum_{k=1}^{n-j+1} \Phi(1) \cdot \hbar^{k-1} \leq \sum_{k=1}^{\infty} \Phi(1) \cdot \hbar^{k-1}$$

where $\hbar = \frac{n-j+1}{n} \leq 1$. Because $\Phi(1) = \frac{1}{n}$

$$\sum_{k=1}^{\infty} \Phi(1) \cdot \hbar^{k-1} = \frac{\Phi(1)}{1-\hbar} = \frac{1}{j-1}$$

and

$$\frac{1}{j-1} \leq 3 \cdot \frac{1}{i} \in O(\frac{1}{j})$$

*Q.E.D.*