ECO 2020 Tutorial 1

8. a) By Walras law,

$$p \cdot x = w$$

$$p \cdot x(p,w) = w \quad \text{since } x(p,w) \text{ is unique}$$

$$p \cdot \frac{\partial x(p,w)}{\partial w} = 1$$

$$\underbrace{\sum_{l=1}^{L-1} p_l \frac{\partial x_l(p,w)}{\partial w}}_{0} + p_L \frac{\partial x_L(p,w)}{\partial w} = 1$$

$$\rightarrow \frac{\partial x_L(p,w)}{\partial w} = \frac{1}{p_L}$$

b) Suppose $(y, x_L) \precsim (y', x_L') \rightarrow$

Given w s.t. (y, x_L), (y', x_L') are both feasible

$x(p, w) = (y', x_L')$. ← not true.

See solution

$p \cdot (y', x_L') = w$

w' such that $(y', x_L' + \alpha)$ and $(y, x_L + \alpha)$, $w' > w$

$$\frac{\partial x_l}{\partial w}(p,w) = 0$$

$$\frac{\partial x_L}{\partial w}(p,w) = \frac{1}{p_L}, \quad x_L' + \frac{1}{p_L}\Delta w = x_L' + \alpha$$

$$\rightarrow x(p, w') = (y', x_L' + \alpha)$$

$$(y, x_L + \alpha) \precsim (y', x_L' + \alpha)$$

$(y, x_L) \prec (y, x_L + \alpha)$ by strict monotonicity

7. $\cdot$ $(y, x_L) \precsim (y', x_L') \iff (y, x_L+\alpha) \precsim (y', x_L'+\alpha)$

$\cdot$ $\forall \alpha > 0$ $(y, x_L) \prec (y, x_L+\alpha)$

(a) "$\precsim$" rep by $u(y, x_L) = \psi(y) + x_L$

Prop 1 $A = (y, x_L)$, $B = (y', x_L')$ wlog $A \precsim B$

$\iff \psi(y) + x_L \leq \psi(y') + x_L'$

$\iff \psi(y) + x_L + \alpha \leq \psi(y') + x_L' + \alpha$

Prop 2 $u(y, x_L) = \psi(y) + x_L < \psi(y) + x_L + \alpha = u(y, x_L+\alpha)$

(b) by "$\precsim$" is continuous $\Rightarrow$ $\exists$ continuous $V(\cdot,\cdot)$ represent "$\precsim$"

(c) $Y^* \subseteq \mathbb{R}_+^{L+1}$, $\exists x$. for $\forall y \in Y^*$, $V(y, x) = 0$

By contradiction. If $\exists x_L, x_L'$ wlog $x_L > x_L' \Rightarrow x_L - x_L' = \alpha > 0$

$V(y, x_L) = V(y, x_L') = 0 \Rightarrow u(y, x_L') = u(y, x_L'+\alpha)$

contradicting Prop 2

(d) $\psi(y) = -x_L(y)$, $(y, x_L) = w$, $(y', x_L') = w'$

Aim: $(y, x_L) \precsim (y', x_L') \iff u(y, x_L) \leq u(y', x_L')$

$\forall y, y' \in \mathbb{R}_+^{L-1}$ $v(y - \psi(y)) = v(y' - \psi'(y)) = 0$

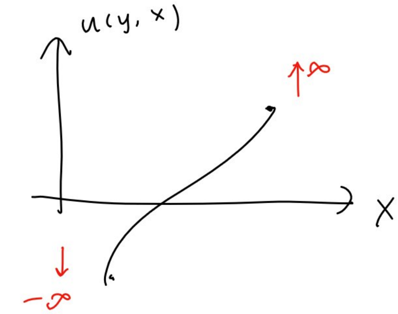
$\iff (y, -\psi(y)) \sim (y', -\psi(y'))$

$\Rightarrow (y, -\psi(y) + u(w)) \sim (y', -\psi(y') + u(w))$

$\Rightarrow (y, \underbrace{-\psi(y) + u(w)}_{x_L}) \precsim (y', \underbrace{-\psi(y) + u(w) - u(w) + u(w')}_{x_L'})$

$\iff u(w') \geq u(w)$

(e) $\forall y \in \mathbb{R}_+^{L-1}$



$\exists$ a positive monotone transformation $f(\cdot)$, continuous, s.t:

① $f(u(y,x))$ represent the same utility function

② $\lim_{x \to -\infty} f(u(y,x)) = \infty$

$\lim_{x \to \infty} f(u, y, x)) = -\infty \quad \forall y$

$\Downarrow$

apply Intermidiate Value Theorem on $f(u(y,\cdot)) = 0$

$\forall y \in \mathbb{R}_+^{L-1}$

def $m(y) = \lim_{x \to -\infty} v(y,x)$

$M(y) = \lim_{x \to \infty} v(y,x)$

By contradiction, If $\exists y$ s.t. $M(y)$ is finite

$(y, x_L), (y, x_L')$ $\quad x_L > x_L'$

$(y, x_L) \succ (y, x_L')$

$$\lim_{\alpha \to \infty} v(y, x_L + \alpha) - v(y, x_L' + \alpha) = M(y) - M(y) = 0$$

$\lim_{\alpha \to \infty} (y, x_L + \alpha) \succsim (y, x_L' + \alpha)$ $\longleftarrow$ no contradiction.

4. Suppose that $x \gg y$

Define $\varepsilon = \min\{x_1 - y_1, \ldots x_L - y_L\} > 0$

Then, for every $z \in X$, if $\|y - z\| < \varepsilon$, then $x \gg z$

$z^* \in X$ s.t $\|y - z^*\| < \varepsilon$ and $z^* > y$

By $x \gg z^*$ and weak monotonicity $x \succsim z^*$

By transitivity $x \succ y$ $\quad \succsim$ is monotone.

6. Fact monotonic $\succsim$ $\Rightarrow$ locally non-satiated

homothetic ✓

monotonic

convexity min ✓ max ✗

$X = \{(a, b) : a, b \geq 0\}$

- $\min\{a, b\}$
- $\max\{a\ b\}$

take any $x = (x_1, x_2)$, $y = (y_1, y_2) \in X$

Suppose $x \succsim y \iff (x_1, x_2) \succsim (y_1, y_2)$

$\iff (\alpha x_1, \alpha x_2) \succsim (\alpha y_1, \alpha y_2) \quad \forall \alpha > 0$

$\forall\ x, y \in X \quad y \gg x \Rightarrow y \succ x$

$(y_1, y_2) \gg (x_1, x_2)$

$y_1 > x_1$

$y_2 > x_2$

wlog $y_1 \geq y_2$

only need 2 cases:

① $x_1 \geq x_2$

② $x_1 \leq x_2$

Suppose $y_1 \geq y_2$

$x_1 \geq x_2$

$(x_1, x_2) \succsim (z_1, z_2)$

$(y_1, y_2) \succsim (z_1, z_2)$

$y_1 \leq y_2$

$x_1 \leq x_2$

$z_1 \leq z_2$

$\Rightarrow \quad x_1 \geq z_1$

$y_1 \geq z_1$

WTS: $(\alpha x_1 + (1-\alpha) y_1,\ \alpha x_2 + (1-\alpha) y_2)$

$\succsim (\alpha z_1 + (1-\alpha) y_1,\ \alpha z_2 + (1-\alpha) y_2)$

e.g. $(4, 1) \succsim (3.7, \leq 3.7)$

$(0, 4) \succsim (3.7, \leq 3.7)$

$\alpha = 0.9$

$3.6 \not\succsim 3.7$

2. Def $\forall x, y \in X$, $x \succsim y \iff u(x) \geqslant u(y)$

① $y \prec x \implies u(x) \geqslant u(y)$

proof by contrapositive

Suppose $u(y) > u(x) \implies y \succ x$

② $u(x) \geqslant u(y) \implies x \succsim y$

$$\left.\begin{array}{l} u(x) = u(y) \implies x \sim y \\ u(x) > u(y) \implies x \succ y \end{array}\right\} \quad u(x) \geqslant u(y) \implies x \succsim y$$

3 (1) $B = \{\{x, y, z\}, \{w, x, y\}\}$

$C\{x, y, z\} = \{x, y\}$

$C\{w, x, y\} = \{x, y\}$

(2) Show $C(B_1 \cup B_2) = C(C(B_1) \cup C(B_2))$

$\implies$ $x \succsim y \quad \forall y \in (B_1 \cup B_2)$

$C(B_1) \subseteq B_1$

$\implies C(B_1) \cup C(B_2) \subseteq B_1 \cup B_2$

$x \succsim z \quad \forall z \in (C(B_1) \cup C(B_2))$

$\implies x \in C(C(B_1) \cup C(B_2))$

$\Longleftarrow$ Suppose $x_1 \in C(B_1)$, $x_2 \in C(B_2)$

$x_1, x_2 \in C(B_1) \cup C(B_2)$

let $x_1 \in C(C(B_1) \cup C(B_2))$

$\implies x_1 \succsim x_2$

$\implies x_1 \succsim y_1 \quad \forall y_1 \in B_1$

$x_2 \succsim y_2 \quad \forall y_2 \in B_2$

$x_1 \succsim y \quad \forall y \in B_1 \cup B_2$

$\Rightarrow x_1 \in C(B_1 \cup B_2)$

5. Lexicographic

complete

transitive

strongly monotone

strictly convex

use: indifference curves are singletons.

$\succsim$ is convex $\quad x \succsim y, \; z \succsim y, \; x \neq z$

$\Rightarrow \alpha \in [0, 1]$

$\alpha x + (1-\alpha) z \succ y$

$x_1 > y_1$ OR $x_1 > y_1$

$z_1 > y_1$ $\quad z_1 = y_1 \Rightarrow x_2 \geq y_2$

need all cases.

1. $\succsim$ is rational

(iii) $x \succ y \succsim z \Rightarrow x \succ z$

$z \succsim x \Rightarrow y \succsim x$

(i) $\succ$ is irreflexive

$x \succ x \Rightarrow x \succsim x$ but not $x \not\succsim x$

$x \succ y \Rightarrow x \succsim y$

$y \succ z \Rightarrow x \succsim z$

$z \succsim x \Rightarrow z \succsim y$

(ii) $\sim$ $\quad x \succsim x$ and $x \succsim x$

$x \sim y \sim z$ $\quad x \succ z$ or $z \succ x$

$x \succsim y, \; y \succsim z$

$\Downarrow \qquad \Downarrow$

$x \succ y \qquad z \succ y$

$x \sim y \Rightarrow x \succsim y$ and $y \succsim x \Rightarrow y \succsim x$ and $x \succsim y \Rightarrow y \sim x$