ECO 2020 Tutorial 5

(4) By Lemma 6

$$F_z(z) = \Pr\{Y \le z\} = \Pr\{\gamma X \le z\}$$
$$= \Pr\{X \le \frac{z}{\gamma}\}$$
$$= F_x(\frac{z}{\gamma})$$

$$\begin{cases} \text{if } z \geq 0 \implies \frac{z}{\gamma} \le z \implies F_Y(z) \le F_X(z) \\ \text{if } z \le 0 \implies \frac{z}{\gamma} \geq z \implies F_Y(z) \geq F_X(z) \end{cases}$$

$$\left.\begin{array}{l} E[X] = 0 \\ E[Y] = E[\gamma X] = \gamma E[X] = 0 \end{array}\right\} \implies X \trianglerighteq^2 Y$$

(8) Strongly more risk averse

$u^*(\ )$ and $u(\ )$

$\exists\ k$ and $v(\cdot)$ $[v'(\cdot) \le 0$ and $v'' \le 0]$ s.t

$$u^*(x) = k\, u(x) + v(x)$$

a) If u^* is strongly more risk averse than $u(\cdot)$

$\implies r_{u^*}(x) \geq r_u(x) \quad \forall x \in [0,1]$

$$r_{u^*}(x) = \frac{-u^{*''}(x)}{u^{*'}(x)} \implies \begin{array}{l} u^{*'}(x) = k\, u'(x) + v'(x) \\ u^{*''}(x) = k\, u''(x) + v''(x) \end{array}$$

$$= \frac{k\, u''(x) + v''(x)}{k\, u'(x) + v'(x)}$$

$$r_{u^*}(x) - r_u(x) = -\frac{k u''(x) + v''(x)}{k u'(x) + v'(x)} + \frac{u''(x)}{u'(x)}$$

$$= \frac{-u'(x) v''(x) + u''(x) v'(x)}{u'(x) u^{*\prime}(x)} \geq 0$$

$$u''(x) v'(x) \geq 0$$

$$-u'(x) v''(x) \geq 0$$

b) If $u(\cdot)$ is bounded $\Rightarrow$ $u^*(x) + k u(\cdot) + c$

$u(x+1) - u(x)$ is decreasing

$v(x+1) - v(x)$ is non-increasing in x

$$u^*(x+1) - u^*(x) = k u(x+1) + v(x+1) - k u(x) - v(x)$$

$$= k(u(x+1) - u(x)) + v(x+1) - v(x)$$

if $v(\cdot)$ is decreasing

$v(x+1) - v(x)$ is decreasing in x

$$\lim_{x \to +\infty} u^*(x+1) - u^*(x) = k\underbrace{(u(x+1) - u(x))}_{\to 0} + \underbrace{v(x+1) - v(x)}_{\to -y} = -y$$

c) If SMRA $\Rightarrow$ AP holds

AP $\not\Rightarrow$ SMRA

$u^*(x) = -\exp(-\beta x)$

$u(x) = -\exp(-\alpha x)$

for any $v(\cdot)$ decreasing $\Rightarrow$ SMRA does not hold.

(6) a) Let $x, y \in \mathbb{R}^L_+$, $\alpha \in [0,1]$

Exp utility $\alpha u(x) + (1-\alpha) u(y)$ —— (1)

Utility of expected value $u(\alpha x + (1-\alpha) y)$ —— (2)

b) Fix $p \gg 0$

w, w': wealth

$\lambda \in [0,1]$ $\quad x = (p, w) \quad x' = (p, w')$

$$p(\lambda x + (1-\lambda) x') \leq (\lambda w + (1-\lambda) w')$$

$$u(\lambda x + (1-\lambda) x') \leq \tilde{u}(\lambda w + (1-\lambda) w')$$

$$\geq \lambda u(x) + (1-\lambda) u(x')$$

$$= \lambda \tilde{u}(w) + (1-\lambda) \tilde{u}(w')$$

c) $u(x) = \sqrt{\max(x_1, x_2)} = \sqrt{w} = \tilde{u}(w)$

Fix $p_1 > 1 \Rightarrow x(p, w) = (0, w)$

(11) a) $1 \longrightarrow 1$

$1 \longrightarrow a, \pi$

$\quad\quad b, 1-\pi$

$\min(a, b) < 1$

b) $a\pi + b(1-\pi) > 1$

c) $\max \; \pi u(x_1 + a x_2) + (1-\pi)(x_1 + b x_2)$

s.t. $x_1 + x_2 = 1$

$$\mathcal{L} = \pi u(x_1 + a x_2) + (1-\pi) u(x_1 + b x_2) + \lambda_1 (1 - x_1 - x_2)$$

FOC: $\underbrace{\pi u'(x_1 + a x_2)(1-a) + (1-\pi) u'(x_1 + b x_2)(1-b)}_{f} = 0$

d) $a < 1$

$b > 1 > a$

$$\frac{\partial x_1}{\partial a} = -\frac{\partial f / \partial a}{\partial f / \partial x_1}$$

$$\frac{\partial f}{\partial a} = -\pi u'(x_1 + a(1 - x_1)) + \pi (1-a)(1-x_1) u''(x_1 + a(1-x_1))$$

$$< 0$$

$$\frac{\partial f}{\partial x_1} = \pi (1-a)^2 u''(x_1 + a(1-x_1))$$
$$+ (1-\pi)(1-b)^2 u''(x_1 + b(1-x_1))$$
$$< 0$$

$$\Rightarrow \frac{\partial x_1}{\partial a} < 0$$

e) f) WTS $\frac{\partial x_1}{\partial \pi} > 0$

$$\frac{\partial x_1}{\partial \pi} = -\frac{\partial f / \partial \pi}{\underbrace{\partial f / \partial x_1}_{< 0}} > 0$$

$$\frac{\partial f}{\partial \pi} = (1-a) u'(x_1 + a(1-x_1)) - (1-b) u'(x_1 + b(1-x_1))$$

$$> 0$$

(12) $R(x, x') = \sum_s \pi_s h(\max\{0, x_s' - x_s\})$

$x \geq_R x'$ iff $R(x, x') \leq R(x', x)$

$S = 3, \quad \pi_s = \frac{1}{3}, \quad h(x) = \sqrt{x}$

$$\begin{cases} x = (0, -2, 1) \\ x' = (0, 2, -2) \\ x'' = (2, -3, 1) \end{cases}$$

$$\Rightarrow \quad R(x, x') = \frac{2}{3} \qquad R(x', x) = \frac{\sqrt{3}}{3}$$

$$R(x', x'') = \frac{\sqrt{2}+1}{3} \qquad R(x'', x') = \frac{\sqrt{6}}{3}$$

$$R(x'', x) = \frac{\sqrt{2}+1}{3} \qquad R(x, x'') = \frac{\sqrt{2}}{3}$$

$$x' >_R x >_R x'' >_R x'$$

⑦ $E[p \cdot y] = E[p] \cdot y \leq E[p]\, y^*(E[p]) = \pi(E[p])$

⑩ a) $E_{F_\delta}[u(x)] = \frac{1}{2} u(x_0 - \delta) + \frac{1}{2} u(x_0 + \delta)$

$$= \frac{1}{2}\left(u(x_0) - \delta u'(x_0) + \frac{1}{2}\delta^2 u''(x_0) + o(\delta^2)\right)$$

$$+ \frac{1}{2}\left(u(x_0) + \delta u'(x_0) + \frac{1}{2}\delta^2 u''(x_0) + o(\delta^2)\right)$$

$$= u(x_0) + \frac{1}{2}\delta^2 u''(x_0) + o(\delta^2)$$

$$u(c(F_\delta, u)) = u(x_0 - c_\delta)$$

$$= u(x_0) - c_\delta u'(x_0) + o(c_\delta)$$

b) As $E_{F_\delta}[u(x)] = u(c(F_\delta, u))$

$$\Rightarrow c_\delta = -\frac{1}{2}\frac{u''(x_0)}{u'(x_0)}\delta^2 + o(\delta^2) - o(c_\delta)$$

$$c(F_\delta, u) = x_0 - c_\delta = -\frac{1}{2}\gamma_{A,u}(x_0)\delta^2 + o(\delta^2)$$

c) $c(u, F) \geq c(v, F)$

$$\Longleftrightarrow x_0 - \frac{1}{2}\gamma_{A,u}(x_0)\delta^2 \geq x_0 - \frac{1}{2}\gamma_{A,v}(x_0)\delta^2$$

$$\Longleftrightarrow \gamma_{A,u}(x_0) \leq \gamma_{A,v}(x_0)$$

(4) $u(x) = x^{\frac{1}{2}}$

(a) $r_A(x) = \frac{1}{2x} \implies x = 5, \quad r_A(5) = \frac{1}{10}$

$r_R(x) = \frac{1}{2}$

(b) $u(c(F, u)) = \frac{1}{2}\sqrt{16} + \frac{1}{2}\sqrt{4} = 3$

$\implies c(F, u) = 9$

$u(10) = (\frac{1}{2} + \pi)\sqrt{16} + (\frac{1}{2} - \pi)\sqrt{4}$

$\implies \pi = \frac{1}{2}\sqrt{10} - 3$

(c) The same. $c(F, u) = 25, \quad \pi = \frac{1}{2}(\sqrt{26} - 5)$

(5)
$$\frac{u'(w - \alpha^* q)}{u'(w - D + \alpha^*(1-q))} = \frac{\pi}{1-\pi}\frac{(1-q)}{q} < 1$$

$q > \pi = 0 \qquad \frac{\pi}{q} < 1$

$\implies \frac{1-q}{1-\pi} < 1$

$u'(w - \alpha q) < u'(w - D + \alpha(1-q))$

$w - \alpha q > w - D + \alpha(1-q)$

$0 > -D + \alpha$

$D > \alpha$

③ $p, p' \in \Delta S$

$u, u' : Z \to \mathbb{R}$

$\forall f, g \in X \quad f \lesssim g \longleftrightarrow \sum p_s \sum u(z) f(z|s)$

$\leq \sum_s p_s \sum u(z) g(z|s)$

a) $u(z^*) > u(z_*)$

b) $q_\alpha \in \Delta Z$

$z^* \quad p_v = \alpha$

$z_* \quad p_r = 1-\alpha$

$\alpha^s \in (0,1)$

$f_s \sim f_{q\alpha^s}$

$f_s = (z_* \dots\dots z^* \dots\dots z_*)$

$f^* = (z^* \dots\dots z^*)$

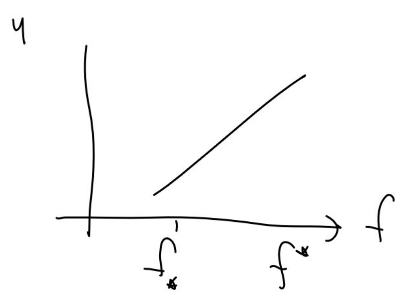
$f_* = (z_* \dots\dots z_*)$

$f^* \succ f_*$

$f^* \succ h \succ f_*$

$u(f^*) > u(h)\ u(f_*)$

$f = \alpha f^* + (1-\alpha) f_*$



c) $f_s(z_* \dots z^* \dots z_*) \sim f_{q\alpha^s}$

$f_s : p_s u(z^*) + (1-p_s) u(z_*)$

$f_{q_{\alpha_s}} : \alpha_s u(z^*) + (1-\alpha_s) u(z_*)$

$p_s = \alpha_s$

$p'_s = \alpha_s$

$p_s = p'_s$