

# CS368 MATLAB Programming

## Lecture 2

Young Wu

Based on lecture slides by Michael O'Neill and Beck Hasti

February 2, 2022

# Quiz

## Admin

- Are you in room 168 Noland or on Zoom?
- *A* : I don't know
- *B* : 168 Noland
- *C* : Zoom
- *D* : Both

# Scalar Operations, Binary

## Code

- $x + y$  is  $x + y$ .
- $x - y$  is  $x - y$ .
- $x * y$  is  $xy$ .
- $x / y$  and  $y \setminus x$  are  $\frac{x}{y}$ .
- $x \wedge y$  is  $x^y$ .

# Scalar Operations, Unary

## Code

- $\text{sqrt}(x)$  is  $\sqrt{x}$  .
- $\text{exp}(x)$  is  $e^x$  .
- $\text{log}(x)$  is natural log or  $\ln(x)$ ,  $\text{log10}(x)$  is  $\log_{10}(x)$ .
- $\text{sin}(x)$  with  $x$  in radians,  $\text{sind}(x)$  with  $x$  in degrees,  $\text{asin}(y)$  is  $\arcsin(y)$  in radians,  $\text{asind}(x)$  is  $\arcsin(y)$  in degrees.
- $\text{cos}(x)$ ,  $\text{cosd}(x)$ ,  $\text{acos}(y)$ ,  $\text{acosd}(y)$ .
- $\text{tan}(x)$ ,  $\text{tand}(x)$ ,  $\text{atan}(y)$ ,  $\text{atand}(y)$ .

# Scalar Operations, Unary Integer

## Code

- *round*( $x$ ) is rounding  $x$  to nearest integer.
- *floor*( $x$ ) is  $\lfloor x \rfloor$ , largest integer  $\leq x$ .
- *ceil*( $x$ ) is  $\lceil x \rceil$ , smallest integer  $\geq x$ .
- *mod*( $x, y$ ) is  $x \bmod y$ , or  $x$  modulo  $y$ , the remainder when  $x$  is divided by  $y$ , integer division,  $x \% y$  does not work in MATLAB.

# Scalar Operations, Rounding

## Quiz

- $1.00001 + 1$
- $B : 2$
- $C : 2.00001$
- $D : 2.0000$

# Scalar Math Operations, Infinity

## Quiz

- $10^{309}$
- $B : 1e+309$
- $C : 1.0000e+309$
- $D : \text{Inf}$

# Scalar Operations, Precision

## Quiz

- $10^{-309}$
- $B : 1e-309$
- $C : 1.0000e-309$
- $D : 0$



# Scalar Operations, Numerical Instability

## Quiz

- $(1 + 10^{-16} - 1) * 10^{16}$
- $B : 0$
- $C : 0.0000$
- $D : 1$
- $E : 1.0000$

# Scalar Math Operations, Numerical Instability Again

## Quiz

- $(1 - 1 + 10^{-16}) * 10^{16}$
- $B : 0$
- $C : 0.0000$
- $D : 1$
- $E : 1.0000$

# Numerical Instability

## Code

- The number of decimal places that is displayed can be changed.
- The number of decimal places that can be stored is fixed.
- ①  $\pi$ ,  $e$ ,  $\sqrt{2}$  etc are approximate values (accurate up to 16 decimal places).
- ② Underflow may occur: numbers that are too close to 0 are stored as **0**.
- ③ Overflow may occur: numbers that are too large are stored as **Inf**.

# Vector Multiplication

## Math

- $\begin{bmatrix} a \\ b \end{bmatrix} \odot \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ bd \end{bmatrix}$  is the element-wise product.
- $\begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} c \\ d \end{bmatrix} = [a \ b] \begin{bmatrix} c \\ d \end{bmatrix} = ac + bd$  is the inner product, also called the dot product for matrices.
- $\begin{bmatrix} a \\ b \end{bmatrix} \otimes \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} [c \ d] = \begin{bmatrix} ac & ad \\ bc & bd \end{bmatrix}$  is the outer product.

# Vector Multiplication

## Code

- Suppose  $M$  and  $W$  are two row vectors having the same size.
- $M .* W$  is the element-wise product  $M \odot W$ .
- $M * W'$  and  $\text{dot}(M, W)$  are the inner product  $M \cdot W = MW^T$ .
- $M' * W$  is the outer product  $M \otimes W = M^T W$ .

# General Vector Operations

## Code

- Suppose  $M$  and  $W$  are two row vectors having the same size, and  $c$  is a scalar.
- $M + W$  and  $M - W$  are element-wise and also vector addition and subtraction.
- $M.^W$  and  $M.^c$  are element-wise exponentiation.

# General Vector Operations, Unary

## Code

- Most of the built-in unary operations are element-wise when applied to vectors.
- For example, the square root function can be applied element-wise to vectors directly.
- ① `sqrt([a b c])` is  $[\sqrt{a} \quad \sqrt{b} \quad \sqrt{c}]$ .
- ② `[a b c] .^ 0.5` is also  $[\sqrt{a} \quad \sqrt{b} \quad \sqrt{c}]$ .
- ③ `[a b c] ^ 0.5` results in an error.

# Vector Operations, Multiplication

## Quiz

- **2 6 12 20 30 42 56 72**
- $A: (1:8)' * (2:9)$
- $B: (1:8) * (2:9)'$
- $C: (1:8) * (2:9)$
- $D: (1:8) .* (2:9)$



# Vector Operations, Exponentiation

## Quiz

- **1 2 4 8 16 32 64 128**
- $A: (0:7) \wedge 2$
- $B: (0:7) \cdot \wedge 2$
- $C: 2 \wedge (0:7)$
- $D: 2 \cdot \wedge (0:7)$

# Vector Operations, Trig Operations

## Quiz

- **1 0 -1 0 1 0 -1 0**
- $A : \text{round}(\text{sind}((0:7) * 180))$
- $B : \text{round}(\text{cosd}((0:7) * 180))$
- $C : \text{round}(\text{sind}((0:7) * 90))$
- $D : \text{round}(\text{cosd}((0:7) * 90))$

# Matrix Multiplication

## Math

- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \odot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae & bf \\ cg & dh \end{bmatrix}$  is the element-wise product.
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} ae + bf \\ ce + df \end{bmatrix}$  and  
 $\begin{bmatrix} e & f \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea + fc & eb + fd \end{bmatrix}$  are matrix products.
- $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$  is also the matrix product.

# Matrix Multiplication

## Code

- Suppose  $M$  and  $W$  are two matrices.
- $M .* W$ , when  $M$  and  $W$  have the same size, is the element-wise product  $M \odot W$ .
- $M * W$ , when number of columns of  $M$  is the same as the number of rows of  $W$ , is the matrix product  $MW$ .

# General Matrix Operations, Binary Code

- Suppose  $M$  and  $W$  are two matrices, and  $c$  is a scalar.
- $M + W$  and  $M - W$  are element-wise and also matrix addition and subtraction.
- $M .* W$  is element-wise, and  $M * W$  is matrix multiplication.
- $M ./ W$  and  $W \setminus M$  are element-wise, and  $M / W$  and  $W \setminus M$  find the matrix  $X$  such that  $MX = W$ , the solution of systems of linear equations. More details in a later lecture.
- $M.^c$  and  $M.^c$  are element-wise, and  $M^c$  is matrix exponentiation.

# General Matrix Operations, Unary

## Code

- Most of the built-in unary operations are element-wise when applied to matrices.
- For example, the square root function can be applied element-wise to matrices directly.

①  $\text{sqrt}([a \ b; \ c \ d])$  is  $\begin{bmatrix} \sqrt{a} & \sqrt{b} \\ \sqrt{c} & \sqrt{d} \end{bmatrix}$ .

②  $[a \ b; \ c \ d] .^ 0.5$  is also  $\begin{bmatrix} \sqrt{a} & \sqrt{b} \\ \sqrt{c} & \sqrt{d} \end{bmatrix}$ .

③  $[a \ b; \ c \ d] ^ 0.5$  is the actual square root of the matrix, it finds a matrix  $\begin{bmatrix} e & f \\ g & h \end{bmatrix}$  such that  $\begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ .

# Matrix Operations, Multiplication

## Quiz

- (What is the second column of  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ?)

2

- 5
- 8

①  $m = [1 \ 2 \ 3; 4 \ 5 \ 6; 7 \ 8 \ 9];$

- $A : m * [0 \ 1 \ 0]$
- $B : m * [0; 1; 0]$
- $C : [0 \ 1 \ 0] * m$
- $D : [0; 1; 0] * m$

# Matrix Operations, Multiplication Again

## Quiz

- (What is the row 2, column 3 entry of  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ ?)
- **6**
- ①  $m = [1 \ 2 \ 3; 4 \ 5 \ 6; 7 \ 8 \ 9]; \ e_i = [0; 1; 0]; \ e_j = [0; 0; 1];$ 
  - $A: e_i' * m * e_j$
  - $B: e_j' * m * e_i$
  - $C: e_i' * e_j * m$
  - $D: m * e_i * e_j'$



# Matrix Operations, Division

## Quiz

- $[1 \ 2; 3 \ 4] \ .\ \ [2 \ 4; 6 \ 8]$

- $A: \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$$B: \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

- $C: \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$$D: \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

# Blank Slide