CS368 MATLAB Programming

Lecture 3

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Based on lecture slides by Michael O’Neill and Beck Hasti

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Matrix Operations, Multiplication Again

Quiz

(What is the row 2, column 3 entry of \([1 \ 2 \ 3] \\
4 \ 5 \ 6 \\
7 \ 8 \ 9\)?)

Q1

$e_i' \cdot m \cdot e_j$

$m = [1 \ 2 \ 3; \ 4 \ 5 \ 6; \ 7 \ 8 \ 9]; \ e_i = [0; \ 1; \ 0]; \ e_j = [0; \ 0; \ 1];$

1. reshape($[9; 1; 3, 3]$)

$A: e_i' \cdot m \cdot e_j$

$B: e_j' \cdot m \cdot e_i$

$C: e_i' \cdot e_j \cdot m$

$D: m \cdot e_i \cdot e_j'$

\[
\begin{pmatrix}
0 & 1 & 0 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix} \cdot 
\begin{pmatrix}
0 & 0 \\
0 & 1
\end{pmatrix} = 
\begin{pmatrix}
6
\end{pmatrix}
\]
Matrix Operations, Division

Quiz

\[ \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \]

Later lecture

\[ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

\[ B = \begin{bmatrix} 2 & 4; 6 & 8 \end{bmatrix} \]

\[ \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} \]

\[ = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

Element-wise division

\[ \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \]

\[ \begin{bmatrix} 2 \\ 2 \end{bmatrix} \]

\[ \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \]

\[ \begin{bmatrix} 2 \\ 2 \end{bmatrix} \]
A curve can be the graph of a function described by $y = f(x)$, or the trace of a moving point, in which the movement of the point is described by its position $(f_x(t), f_y(t))$ at time $t$.

A curve is plotted using a large number of line segments.
Function Curves
Math

To plot $y = f(x)$ from $x = x_1$ to $x = x_n$, find $x_1 < x_2 < x_3 < ... < x_n$ and use lines to connect the following points,

$$(x_1, f(x_1)), (x_2, f(x_2)), (x_3, f(x_3)), ..., (x_n, f(x_n)).$$
Parametric Curves
Math

- To plot \((f_x(t), f_y(t))\) from \(t = t_1\) to \(t_n\), find
  \(t_1 < t_2 < t_3 < \ldots < t_n\) and use lines to connect the following points,
  \((f_x(t_1), f_y(t_1)), (f_x(t_2), f_y(t_2)), (f_x(t_3), f_y(t_3)), \ldots, (f_x(t_n), f_y(t_n))\).
Curve Discretization

Math

- $t_1, t_2, t_3, ..., t_n$ is a partition of the domain $t \in [t_1, t_n]$.

1. The partition is usually uniform, meaning $t_i = t_{i-1} + \delta$ with $\delta = \frac{t_n - t_1}{n}$ and some large $n$.

2. $t_i$ can also be sampled randomly. More details in a later lecture.

3. $t_i$ can also be chosen according to how fast the function is changing.

4. $t_i$ can also be chosen so that the lengths of the line segments are the same.

arc length parameterization
Curve Plotting

Code

- Suppose \( x, y \) are vectors of length \( n \).

- \( \text{plot}(x, y) \) plots line segments connecting \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\).

1. For example, define \( x = 0:0.01:1 \) and use \( \text{plot}(x, f(x)) \) to plot \( f(x) \) between 0 and 1 with a partition of size 100.

2. Another example, define \( t = 0:0.01:1 \) and use \( \text{plot}(fx(t), fy(t)) \) to plot \((f_x(t), f_y(t))\) between 0 and 1 with a partition of size 100.
Line Specs
Code

- `plot(x, y, s)` s specifies the style, marker, and color of the lines.

1. Line style: `'-'` solid, `'-.-'` dashed, `':.'` dotted, `'-.'` dash-dotted.
2. Marker: `'o'` circle, `'.` dot, `'x'` cross, `'s'` square, `'d'` diamond ...
3. Color: `'r'` red, `'g'` green, `'b'` blue, `'k'` black, `'w'` white ...

- `plot(x1, y1, s1, x2, y2, s2, ...)` plots multiple lines in the same figure.
Curve Plotting, Square

Quiz

(Plot a unit square.)

B: \texttt{plot ([0, 1], [0, 1], 'r')}

C: \texttt{plot ([0, 0, 1, 1, 0], [0, 1, 1, 0], 'r')}

D: \texttt{plot ([0, 0, 1, 1, 0], [0, 1, 1, 0, 0], 'r')}
Curve Plotting, Circle

Quiz

- (Plot a full circle.)
- \( B : \text{plot}(\text{sind}(0:360), \text{sind}(0:360), \ '.') \)
- \( C : \text{plot}(\text{sind}(0:360), \text{cosd}(0:360), \ '.') \)
- \( D : \text{plot}(-1:0.01:1, \sqrt{1 - (-1:0.01:1)^2}, \ '.') \)
Curve Plotting, Aliasing

Quiz

- (Plot the horizontal dashed line at $y = 0$.)
- $B$ : plot (0:10:1800, sind (0:10:1800), '— —')
- $C$ : plot (0:90:1800, sind (0:90:1800), '— —')
- $D$ : plot (0:180:1800, sind (0:180:1800), '— —')
Plotting Features

Code

- Texts can be added to the plot. More details about text manipulation in the next lecture.
  - `title(t)` adds title t.
  - `xlabel(t)` adds x-axis label t.
  - `ylabel(t)` adds y-axis label t.
  - `legend(c1, c2, ...)` adds legend (names of the curves $c_1, c_2, ...$).
  - `text(x, y, t)` adds text t at position $(x, y)$.
  - `axis([x0, x1, y0, y1])` changes the range of the axes to $x \in [x_0, x_1]$ and $y \in [y_0, y_1]$. 
3D Curve Plotting

Code

- Suppose $x, y, z$ are vectors of length $n$.
- $\text{plot3}(x, y, z, s)$ plots the lines in 3D connecting $(x_1, y_1, z_1), (x_2, y_2, z_2), ..., (x_n, y_n, z_n)$, with specs $s$. 
A surface can be a graph of a function described by \( z = f(x, y) \), or the trace of a moving point, in which the movement of the point is described by its position \( (f_x(s, t), f_y(s, t), f_z(s, t)) \).

A surface is plotted using a large number of faces, usually triangles, but in MATLAB, four sided polygons.
Surface Plotting

Code

- Suppose $x, y, z$ are matrices representing points on the surface.
- $\text{contour}(x, y, z, n)$ plots $n$ contours of the surface, and $\text{contour3}(x, y, z, n)$ plots them in 3D.
- $\text{mesh}(x, y, z)$ plots the surface mesh.
- $\text{surf}(x, y, z)$ plots the surface.

If $x$ and $y$ are omitted, the $x$ and $y$ coordinates are assumed to be the column and row indices of the elements in $z$. 
Surface Plotting, Pyramid

Quiz

Q6

\[ x = [1, 2, 3] \]
\[ y = [1, 2, 3] \]

(Plot a unit height pyramid centered at (2, 2).)

- \( B \) : surf([0 0 0; 0 1 0; 0 0 0])
- \( C \) : surf([0 1 0; 0 1 0; 0 1 0])
- \( D \) : surf([0 0 0; 1 1 1; 0 0 0])

\[ z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]
Surface Plotting, Plane

Quiz

(Plot a flat square surface at $z = 1$ with side lengths 1.)

- $B: \text{surf}([0 \ 1; \ 0 \ 1], \ [0 \ 0; \ 1 \ 1], \ [1 \ 1; \ 1 \ 1])$
- $C: \text{surf}([0 \ 1; \ 0 \ 1], \ [0 \ 1; \ 1 \ 1], \ [1 \ 1; \ 1 \ 1])$
- $D: \text{surf}([0 \ 1; \ 0 \ 1], \ [1 \ 0; \ 1 \ 1], \ [1 \ 1; \ 1 \ 1])$
Surface Plotting, Grid

Quiz

(Plot $z = x + 2y$ for $x = y = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$.)

$B:$
$x = \text{repmat}([1 2 3], [3 1]);$  $y = \text{repmat}([1 2 3]', [3 1]);$

$C:$
$x = \text{repmat}([1 2 3]', [1 3]);$  $y = \text{repmat}([1 2 3], [3 1]);$

$\text{surf}(x + 2*y)$
Surface Plotting, Bowl

Quiz

Plot \( z = x^2 + y^2 \) for \( x = y = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \end{bmatrix} \).

1. \( x = \text{repmat}(-2:2, [5 1]); \ y = x'; \)

B: \( \text{surf}(x, y, x.\hat{}^2 + y.\hat{}^2) \)

C: \( \text{surf}(x, y, x^2 + y^2) \)

D: \( \text{surf}(x, y, x' \times x + y' \times y) \)
Mesh Grid Shortcut

Code

- \([x, y] = \text{meshgrid}(u, v)\) creates \(x = \text{repmat}(u, [\text{length}(v), 1])\) and \(y = \text{repmat}(v', [1, \text{length}(u)])\). The matrices \(x, y\) then can be used to plot the surface \(z = f(x, y)\) using \(\text{surf}(x, y, f(x, y))\).

- \([x, y, z] = \text{sphere}()\) and \([x, y, z] = \text{cylinder}()\) create meshes of a unit sphere and a unit cylinder. The surface then can be plotted using \(\text{surf}(x, y, z)\).
Other Plots

Code

- Under "PLOTS" tab, many other plots can be created based on a matrix.
- *m* files are MATLAB scripts and can be used to store a list of commands or the definition of a function. More details in the next lecture.

- The script and its output can be published as a PDF file or an HTML web page.
Blank Slide