CS368 MATLAB Programming

Lecture 8

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Based on lecture slides by Michael O'Neill and Beck Hasti

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Coordination Game

Quiz

Q1 and Q2

Q14

- For the quiz grade last week (out of 2):
- A : If $\geqslant 95$ % of you chooses A, everyone gets 3 points.
- B : If $\geqslant 75$ % of you chooses B, everyone gets 2 points.
- C : If $\geqslant 50$ % of you chooses C, everyone gets 1 point.
- Otherwise, everyone gets 0.

97%

Vectorization

Math

- Loops are used when the same task is repeated for a large number of times.
- If these tasks can be done simultaneously in parallel, vectorization is preferred in MATLAB: define the repeating task as a function, and apply the function to a vector or matrix.
- If these tasks must be done sequentially, then a for loop could be used.
- If these tasks are done for an unknown number of times until some condition is met, then a while loop could be used. More details in the next lecture.

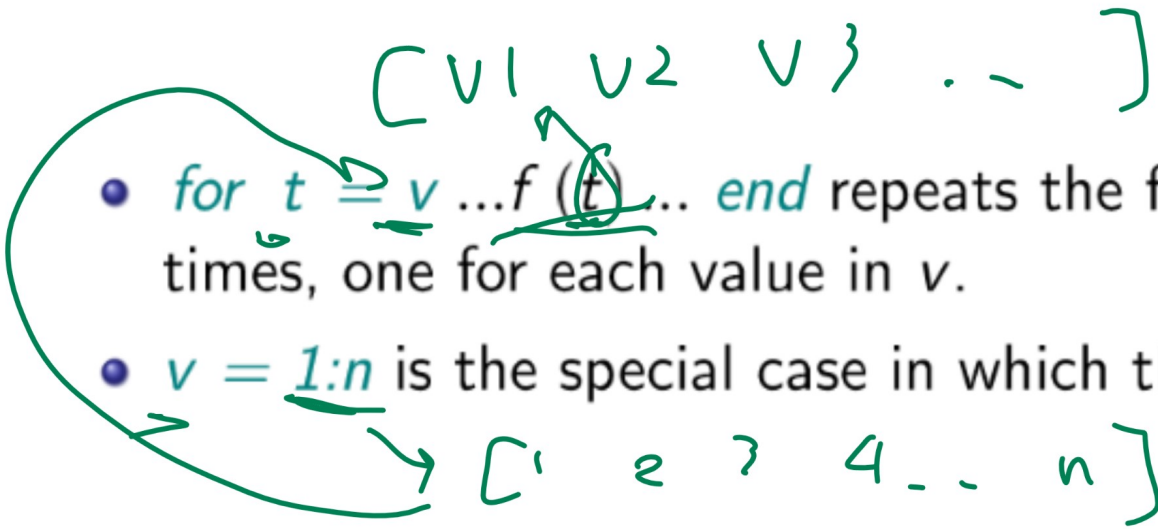
For Loop over Indices

Code

- *for* t = 1:n ... f (t) ... *end* repeats the function f for n times.
- t is the counter or index variable.
- In MATLAB, since i is the complex number $\sqrt{-1}$, using i as the index variable is not recommended.
- In MATLAB, for loop is count controlled, meaning changing the counter variable inside the loop has no impact on the number of times the loop is repeated.

For Loop over Values

Code



- *for t = v ...f (t) ... end* repeats the function s for *length(v)* times, one for each value in *v*.
- *v = 1:n* is the special case in which the set is the index set.

For Loop Example, Factorial

Code

$n(n-1)(n-2)\ldots \cdot 1$

- To compute the factorial of $n \geqslant 0$:

1. $f = 1;$ % defines the variable to store the product.
2. *for* $t = 1{:}n$ % starts the for loop for n times.
3. $f = f * t;$ % multiplies the current value to the product.
4. *end* % ends the for loop.

$n=5$

t	1	2	3	4	5
f	1	1	2	6	24	120 ...

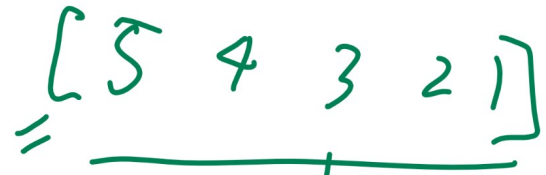
For Loop Example, Sum

Code

- To compute the sum of the values in a vector v:

1. $s = 0;$ % defines the variable to store the sum.
2. *for* $t = v$ % starts the for loop over the vector.
3. $s = s + t;$ % adds the current value to the sum.
4. *end* % ends the for loop.

print(s) → 15



t		5	4	3	2	1
s	0	5	9	12	14	15

Continue and Break

Code

- It is possible to stop a for loop without finishing all iterations.
- *continue* skips the remaining code of the current iteration.
- *break* skips the remaining code of the current iteration and all remaining iterations.
- Avoid using *continue* and *break* and use *if* and *while* instead. More details next lecture.

For Loop, Sum of Series

Quiz

Q2

- (Approximate $\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}$.)
- **1.6439**

1. $s = 0;$
2. *for* $z = 1{:}1000$
   - C : $s = s + 1 / (i \hat{} 2);$
   - D : $s = s + 1 / (z \hat{} 2);$
4. *end; s*

- Use *sum(1 ./ (1:1000).^2)* instead.

Riemann zeta function

$\sum_{i=1}^{1000} \frac{1}{i^2} + \sum_{i=1001}^{\infty} \frac{1}{i^2}$

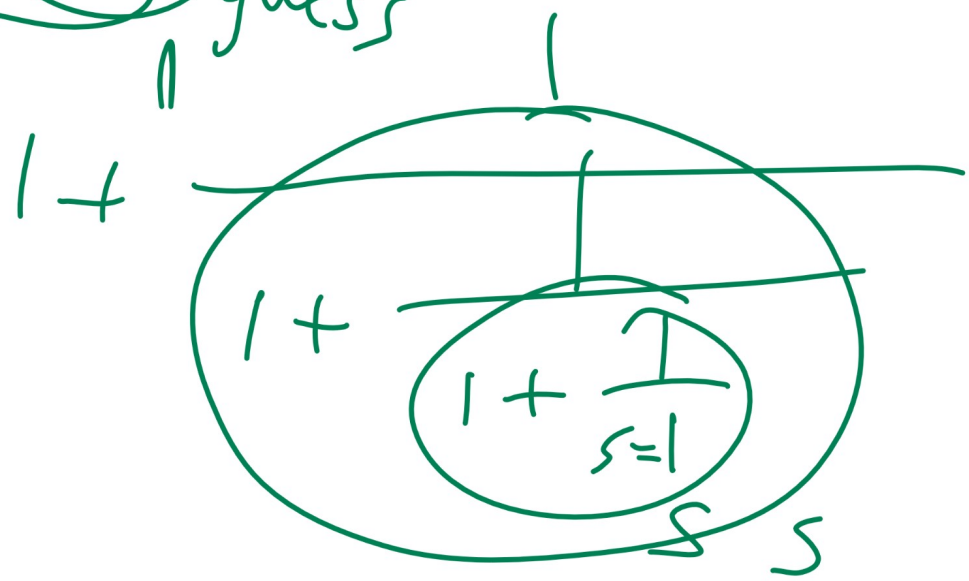
$1 + \frac{1}{4} + \frac{1}{9} + \cdots \frac{1}{1000000}$ + small $\simeq 0$

For Loop, Continued Fraction

Quiz

- (Approximate $\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}} = \dfrac{1+\sqrt{5}}{2}$.)
- **1.6180**

1. $s = 1$; % a random guess.
2. *for* $p = 1{:}1000$
   - C : $s = 1 + 1 / s$;
   - D : $s = 1 + 1 / (1 + s)$;
4. *end*; s



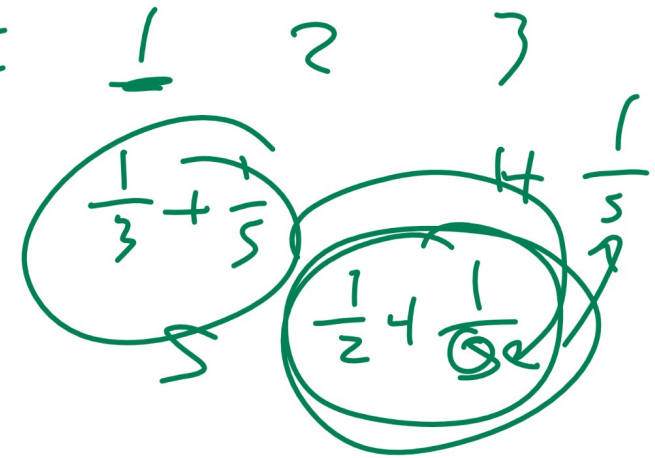
For Loop, Pie

Quiz

- (Approximate $\pi = 2 + \cfrac{2}{1 + \cfrac{1}{\frac{1}{2} + \cfrac{1}{\frac{1}{3} + \cfrac{1}{\cdots}}}}$.)

- **3.1416**

1. $s = 1;$ % a random guess.
2. *for* $t = 1{:}1000$
   - $C: s = 1 / t + 1 / s;$
   - $D: s = 1 / (1001 - t) + 1 / s;$
3. *end;* $2 + 2 / s$



For Loop, Fixed Point

Quiz

- (Approximate the solution for $\cos(x) = x$?)
- **0.7391**

1. $s = 0$; % a random guess.
2. *for* $x = 1{:}100$
   - C: $s = \cos(s)$;
   - D: $x = \cos(x)$
4. *end*; s

Q5

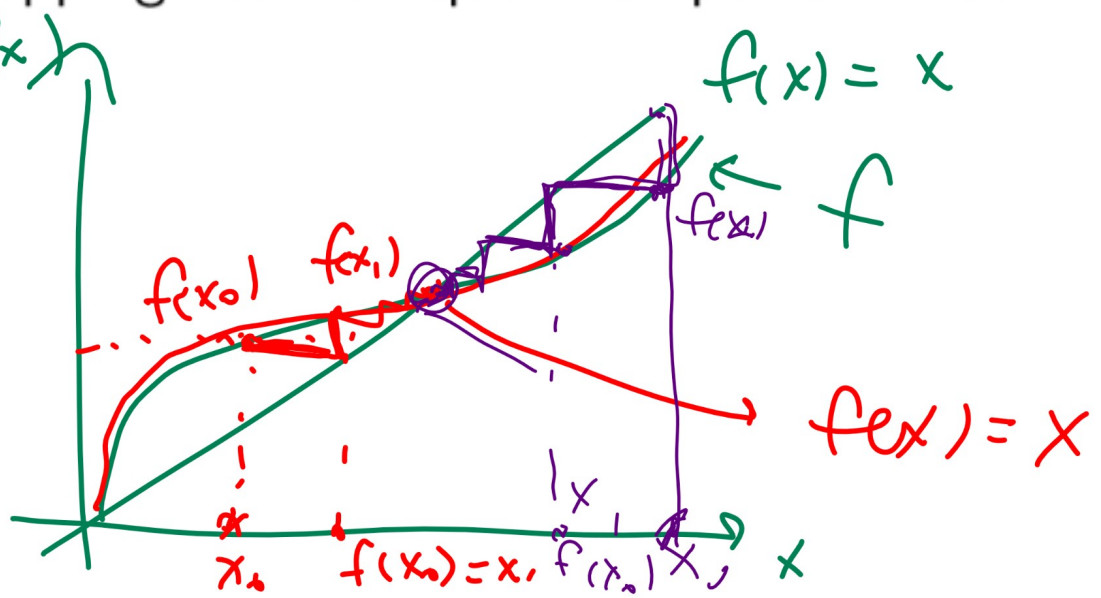
any thing

count-controlled

Contraction Mapping

Math

- A function f is a contraction map if $|f(x) - f(y)| < k|x - y|$ for some $k \in [0, 1)$, and for all x and y.
- Every contraction mapping has a unique fixed point $x^\star$ such that $f(x^\star) = x^\star$.



Fixed Point Iterations

Math

- The fixed point $x^\star$ could be found by fixed point iterations.

1. Start with any x_0.
2. Compute $x_{n+1} = f(x_n)$, for $n = 0, 1, 2, ...$
3. The sequence $x_0, x_1, x_2, ...$ converges to $x^\star$.

- Newton's method to solve non-linear system of equations is an example of a fixed point algorithm. More details in a later lecture.

Loop over a Vector

Code

- A vector can be constructed using a for loop.

1. `v = zeros(n)` % initializes an empty vector.
2. `for t = 1:n` % starts the loop.
3. `v(t) = ...` % fills in the vector.
4. `end` % ends the loop.

Loop over a Matrix

Code

- A matrix can be constructed using a nested for loop.

1. *w = zeros(n, m)* % initializes an empty matrix.
2. *for s = 1:n* % starts the outer loop.
3. *for t = 1:m* % starts the inner loop.
4. *w(s, t) = ...* % fills in the matrix.
5. *end* % ends the inner loop.
6. *end* % ends the outer loop.

rows

columns

✓ x x x x

x x x x x

Nested Loop, Cumulative Sum

Quiz

Q6

1. $s = [5\ 4\ 3\ 2\ 1];$
2. $for\ c = 2{:}5$
3. $s(c) = s(c-1) + s(c);$
4. $end;\ s$

c	2	3	4	5
s(c)	9	12	14	15

- C : **5 9 7 5 3**
- D : **5 9 12 14 15**

$s = 5\ 4\ 3\ 2\ 1$

9 12 14 15

Nested Loop, Path Count

Quiz

```
1 s = [1 1 1 1; 1 0 0 0; 1 0 0 0];
2 for r = 2:3
3   for c = 2:4
4     s(r, c) = s(r − 1, c) + s(r, c − 1);
5   end
6 end; s
```

- C:

1	1	1	1
1	2	3	4
1	2	5	9

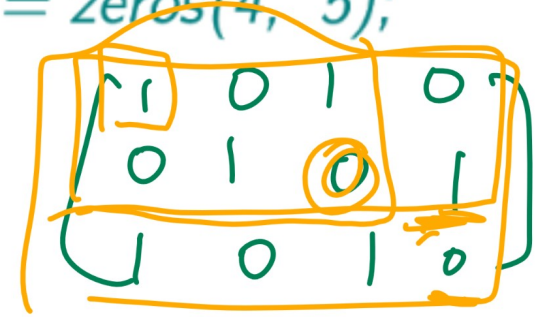
- D:

1	1	1	1
1	2	3	4
1	3	6	10

Nested Loop, Integral Image

Quiz

```
1 m = [1 0 1 0; 0 1 0 1; 1 0 1 0]; s = zeros(4, 5);
2 for r = 1:3
3     for c = 1:4
4
          s(r+1, c+1) = m(r, c) + s(r+1, c) + s(r, c+1) − s(r, c);
5     end
6 end; s(2:4, 2:5)
```



- C:

1	1	2	2
1	2	3	4
2	3	5	6

D:

1	1	2	2
1	3	5	8
2	5	11	19

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