CS368 MATLAB Programming

Lecture 8

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Based on lecture slides by Michael O'Neill and Beck Hasti

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Coordination Game

Quiz

- For the quiz grade last week (out of 2):
- A : If $\geqslant 95$ % of you chooses A, everyone gets 3 points.
- B : If $\geqslant 75$ % of you chooses B, everyone gets 2 points.
- C : If $\geqslant 50$ % of you chooses C, everyone gets 1 point.
- Otherwise, everyone gets 0.

Vectorization

Math

- Loops are used when the same task is repeated for a large number of times.
- If these tasks can be done simultaneously in parallel, vectorization is preferred in MATLAB: define the repeating task as a function, and apply the function to a vector or matrix.
- If these tasks must be done sequentially, then a for loop could be used.
- If these tasks are done for an unknown number of times until some condition is met, then a while loop could be used. More details in the next lecture.

For Loop over Indices

Code

- *for t = 1:n* ...$f(t)$... *end* repeats the function f for n times.
- t is the counter or index variable.
- In MATLAB, since i is the complex number $\sqrt{-1}$, using i as the index variable is not recommended.
- In MATLAB, for loop is count controlled, meaning changing the counter variable inside the loop has no impact on the number of times the loop is repeated.

For Loop over Values

Code

- *for* $t = v$ ...$f(t)$ ... *end* repeats the function s for *length*(v) times, one for each value in v.
- $v =$ *1:n* is the special case in which the set is the index set.

For Loop Example, Factorial

Code

- To compute the factorial of $n \geqslant 0$:

1. $f = 1;$ % defines the variable to store the product.
2. $for \;\; t = 1{:}n$ % starts the for loop for n times.
3. $\quad f = f * t;$ % multiplies the current value to the product.
4. end % ends the for loop.

For Loop Example, Sum

Code

- To compute the sum of the values in a vector v:

1. $s = 0;$ % defines the variable to store the sum.
2. *for* $t = v$ % starts the for loop over the vector.
3. $s = s + t;$ % adds the current value to the sum.
4. *end* % ends the for loop.

Continue and Break

Code

- It is possible to stop a for loop without finishing all iterations.
- *continue* skips the remaining code of the current iteration.
- *break* skips the remaining code of the current iteration and all remaining iterations.
- Avoid using *continue* and *break* and use *if* and *while* instead. More details next lecture.

For Loop, Sum of Series

Quiz

- (Approximate $\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}$.)
- **1.6439**

1. *s = 0;*
2. *for z = 1:1000*
   - *C : s = s + 1 / (i ^ 2);*
   - *D : s = s + 1 / (z ^ 2);*
4. *end; s*

- Use *sum(1 ./ (1:1000).^2)* instead.

For Loop, Continued Fraction

Quiz

- (Approximate $\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}} = \cfrac{1+\sqrt{5}}{2}$.)
- **1.6180**

1. *s = 1;* % a random guess.
2. *for p = 1:1000*
   - *C* : *s = 1 + 1 / s;*
   - *D* : *s = 1 + 1 / (1 + s);*
4. *end; s*

For Loop, Pie

Quiz

- (Approximate $\pi = 2 + \cfrac{2}{1 + \cfrac{1}{\cfrac{1}{2} + \cfrac{1}{\cfrac{1}{3} + \cfrac{1}{\ldots}}}}$.)
- **3.1416**

1. *s = 1;* % a random guess.
2. *for t = 1:1000*
   - *C* : *s = 1 / t + 1 / s;*
   - *D* : *s = 1 / (1001 − t) + 1 / s;*
4. *end; 2 + 2 / s*

For Loop, Fixed Point

Quiz

- (Approximate the solution for $\cos(x) = x$?)
- **0.7391**

1. *s = 0;* % a random guess.
2. *for x = 1:100*
   - *C* : *s = cos(s);*
   - *D* : *x = cos(x)*
4. *end; s*

Contraction Mapping

Math

- A function f is a contraction map if $|f(x) - f(y)| < k|x - y|$ for some $k \in [0, 1)$, and for all x and y.
- Every contraction mapping has a unique fixed point $x^\star$ such that $f(x^\star) = x^\star$.

Fixed Point Iterations

Math

- The fixed point $x^{\star}$ could be found by fixed point iterations.

1. Start with any x_0.
2. Compute $x_{n+1} = f(x_n)$, for $n = 0, 1, 2, ...$
3. The sequence $x_0, x_1, x_2, ...$ converges to $x^{\star}$.

- Newton's method to solve non-linear system of equations is an example of a fixed point algorithm. More details in a later lecture.

Loop over a Vector

Code

- A vector can be constructed using a for loop.

1. *v = zeros(n)* % initializes an empty vector.
2. *for t = 1:n* % starts the loop.
3. *v(t) = ...* % fills in the vector.
4. *end* % ends the loop.

Loop over a Matrix

Code

- A matrix can be constructed using a nested for loop.

1. *w = zeros(n, m)* % initializes an empty matrix.
2. *for s = 1:n* % starts the outer loop.
3. *for t = 1:m* % starts the inner loop.
4. *w(s, t) = ...* % fills in the matrix.
5. *end* % ends the inner loop.
6. *end* % ends the outer loop.

Nested Loop, Cumulative Sum

Quiz

1. $s = [5\ 4\ 3\ 2\ 1];$
2. $for\ \ c = 2{:}5$
3. $\quad s(c) = s(c-1) + s(c);$
4. $end;\ s$

- C : **5 9 7 5 3**
- D : **5 9 12 14 15**

Nested Loop, Path Count

Quiz

1. $s = [1\ 1\ 1\ 1;\ 1\ 0\ 0\ 0;\ 1\ 0\ 0\ 0];$
2. $for\ \ r = 2{:}3$
3. $\quad for\ \ c = 2{:}4$
4. $\quad\quad s(r,\ c) = s(r-1,\ c) + s(r,\ c-1);$
5. $\quad end$
6. $end;\ s$

- C :

1	**1**	**1**	**1**
1	**2**	**3**	**4**
1	**2**	**5**	**9**

D :

1	**1**	**1**	**1**
1	**2**	**3**	**4**
1	**3**	**6**	**10**

Nested Loop, Integral Image

Quiz

1. $m = [1\ 0\ 1\ 0;\ 0\ 1\ 0\ 1;\ 1\ 0\ 1\ 0];\ s = zeros(4,\ 5);$
2. $for\ \ r = 1{:}3$
3. $\quad for\ \ c = 1{:}4$
4. $$s(r{+}1,\ c{+}1) = m(r,\ c) + s(r{+}1,\ c) + s(r,\ c{+}1) - s(r,\ c);$$
5. $\quad end$
6. $end;\ s\,(2{:}4,\ 2{:}5)$

- C :

1	1	2	2
1	2	3	4
2	3	5	6

D :

1	1	2	2
1	3	5	8
2	5	11	19

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