# CS368 MATLAB Programming

## Lecture 9

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Based on lecture slides by Michael O'Neill and Beck Hasti

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# Pseudo Randomness

## Math

- Truly random numbers are difficult or impossible to generate.
- A sequence of pseudo-random numbers is a deterministic sequence with complicated pattern that looks random to users who do not know the pattern therefore cannot predict the next number in the sequence.

# Random Number Generator

## Math

- A simple pseudo-random number generator is the Linear Congruential Generator (LCG).

1. Start with a seed $x_0$.
2. Compute $x_{n+1} = (ax_n + c) \mod m$ for some $m, a, c$ unknown to the user.

- The resulting sequence $\frac{x_0}{m}, \frac{x_1}{m}, \ldots$ is approximately uniformly distributed between 0 and 1, including 0, not including 1.
- For example, Java uses $m = 2^{48}$, $a = 25214903917$, and $c = 11$.
- MATLAB uses another more complicated algorithm called Mersenne Twister.

# Random Number Generation

## Code

- *rand()* generates a uniform random number between 0 and 1, including 0, not including 1.
- *rand() * u* generates a uniform random number between 0 and $u$, including 0, not including $u$.
- *rand() * (u − l) + l* generates a uniform random number between $l$ and $u$, including $l$, not including $u$.
- *rand(n, m)* creates an $n \times m$ matrix of random numbers between 0 and 1.

# Integer Random Numbers

## Code

- *randi(n)* generates a uniform random integer between 1 and $n$, including 1 and $n$.
- *randi([l, u])* generates a uniform random integer between $l$ and $u$, including $l$ and $u$.
- *randperm(n)* generates a random permutation of *1:n*.
- *randperm(n, k)* with $k \leqslant n$ generates a random sample from *1:n* of size $k$, sampled without replacement.

# Random Variable, Discrete

## Math

- A discrete random variable is a random variable that takes on a finite (or countable) number of values with positive probabilities.
- The probability that the random variable $X$ takes on value $x$ in $\{1, 2, 3, ...\}$ is denoted by $f(x) = \mathbb{P}\{X = x\}$. The function $f$ is called the probability mass function.
- A random number generated based on the probabilities specified by $f$ is called a realization of the $X$.

# Cumulative Distribution Functions, Discrete

## Math

- The cumulative probability that the random variable $X$ takes on value less than $x$ is denoted by $F(x) = \mathbb{P}\{X \leqslant x\} = \sum_{i=1}^{x} \mathbb{P}\{X = i\}$. The function $F$ is called the cumulative distribution function (CDF).
- The CDF can be efficiently computed using a for loop.

1. $F(1) = \mathbb{P}\{X = 1\}$.
2. $F(x) = F(x-1) + \mathbb{P}\{X = x\}, x > 1$.

# Inverse Transform Sampling

## Math

- The inverse transform sampling (also called CDF inversion method) can be use to generate a realization of the random variable $X$.

1. Generate $u \sim \text{Uniform}(0, 1)$.
2. Compute CDF of $X$, call it $F(x)$.
3. Find the largest $x$ such that $F(x) < u$.

# For Loop, Review

## Code

- Use a for loop to compute the CDF based on the probabilities stored in a vector $p$, where $p_i = \mathbb{P}\{X = i\}, i = 1, 2, ..., n$.

1. $f = zeros(n);$
2. $f(1) = p(1);$
3. $for \ \ t = 1{:}n$
4. $\quad f(t) = f(t-1) + p(t);$
5. $end$

- $cumsum(x)$ finds the same CDF.

# While Loop

## Code

- A while loop is used when the loop stops after an unknown number of iterations until some condition is met.
- Use a while loop to compute the inverse of the CDF stored in a vector $f$, where $f_i = \mathbb{P}\{X \leqslant i\}$.

1. $t = 1;$
2. $while \;\; f(t) <= x$
3. $\quad t = t + 1;$
4. $end$

- $sum(f <= x) + 1$ or $find(x < f, \; 1)$ finds the same inverse CDF.

# Random Variable Quiz Questions

## Quiz

# Random Variable, Continuous

## Math

- A continuous random variable is a random variable that takes on uncountably infinite number of values.
- The (theoretical) probability that the random variable is equal to any number is 0.
- Inverse transform sampling can also be used to generate a random value of the variable.

# Cumulative Distribution Functions, Continuous

## Math

- The CDF of a distribution $X$ taking values $(-\infty, \infty)$ is given by $F(x) = \mathbb{P}\{X \leqslant x\}$.
- The derivative of this function is called the probability density function $f(x) = F'(x)$, meaning $F(x) = \int_{-\infty}^{x} f(\hat{x})\, d\hat{x}$.
- More details in the next next lecture.

# Simulation

## Math

- Direct computation of the probability of an event is sometimes difficult, and simulation can be used to approximate this probability by repeating the same random process a large number of times and find the fraction of times the event occurs.

# Simulation Quiz Questions

## Quiz

# Reproducibility

## Code

- The same code can produce a different output every time it is excuted.
- In order to make a simulation reproducible, the best practice is to always set a seed at the beginning of the simulation using *rng(seed)*.

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