- Two documents A and B. Suppose A contains 1 "Groot" and 9 other words, and B contains 8 "Groot" and 2 other words. One document is taken out at random (with equal probability), and one word is picked out at random (all words with equal probability). The word is "Groot". What is the probability that the document is A?
- A: $\frac{1}{2}$, B: $\frac{1}{3}$, C: $\frac{1}{4}$, D: $\frac{1}{8}$, E: $\frac{1}{9}$

$$\frac{\Pr\{Groot, A\}}{\Pr\{A\}} = \frac{1/20}{1/2}$$

$\Pr\{Groot \mid A\} + \Pr\{other \mid A\} = 1$ ($\frac{1}{10}$, $\frac{9}{10}$)

$\Pr\{Groot \mid B\} + \Pr\{other \mid B\} = 1$ ($\frac{8}{10}$, $\frac{2}{10}$)

marginal distribution of X

conditional distribution

X \ Y	Groot	other	Sum
A	1/20	9/20	1/2
B	8/20	2/20	1/2
Sum	9/20	11/20	

Sum marginal of Y

joint distribution

$\Pr\{A \mid Groot\}$

$\Pr\{B \mid Groot\}$

$$\frac{\Pr\{A, Groot\}}{\Pr\{Groot\}} = \frac{1/20}{9/20} = \frac{1}{9}$$

$$\frac{\Pr\{G \mid A\}\cdot\Pr\{A\}}{\Pr\{G \mid A\}\Pr\{A\} + \Pr\{G \mid B\}\Pr\{B\}}$$

$$\frac{1/10 \cdot 1/2}{1/10 \cdot 1/2 + 8/10 \cdot 1/2} = \frac{1}{9}$$

$$\Pr\{Coat \mid A\} = \frac{\Pr\{Coat, A\}}{\Pr\{A\}}$$

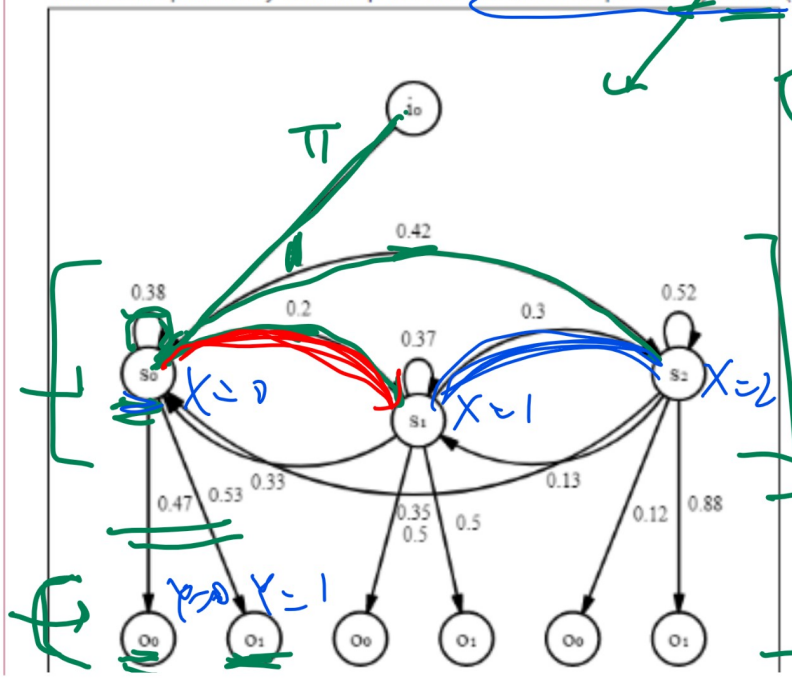
$$\Pr\{Coat, A\} = \Pr\{Coat \mid A\} \cdot \Pr\{A\} = \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{20}$$

Question 14

• [5 points] Andy is a three-month old baby. He can be happy (state 0), hungry (state 1), or having a wet diaper (state 2). Initially when he wakes up from his nap at 1pm, he is happy. If he is happy, there is a 0.38 chance that he will remain happy one hour later, a 0.2 chance to be hungry by then, and a 0.42 chance to have a wet diaper. Similarly, if he is hungry, one hour later he will be happy with 0.33 chance, hungry with 0.37 chance, and wet diaper with 0.3 chance. If he has a wet diaper, one hour later he will be happy with 0.35 chance, hungry with 0.13 chance, and wet diaper with 0.52 chance. He can smile (observation 0) or cry (observation 1). When he is happy, he smiles 0.47 of the time and cries 0.53 of the time; when he is hungry, he smiles 0.5 of the time and cries 0.5 of the time; when he has a wet diaper, he smiles 0.12 of the time and cries 0.88 of the time.
What is the probability that the particular observed sequence cry, smile (or 1, 0 in the diagram) happens?

M14 Q14

from slides.



Transition A

Observation B

$$\Pr\{X_4 = 1, X_5 = 2 \mid X_3 = 0\}$$

$$= \Pr\{X_4 = 1 \mid X_3 = 0\} \cdot \Pr\{X_5 = 2 \mid X_4 = 1\}$$

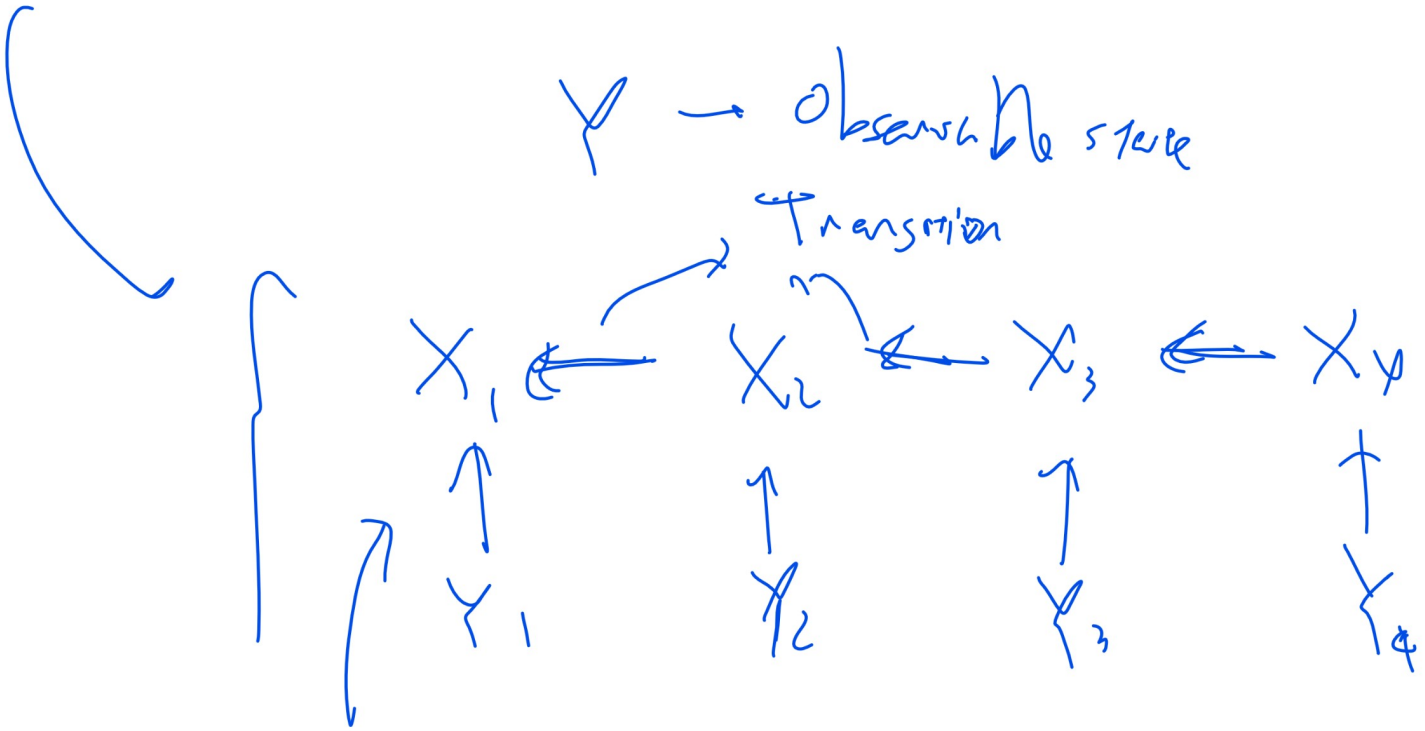
$$= 0.2 \cdot 0.3$$

$$= 0.06$$

HMM

X – hidden state

Y – observable state



HMM

$t = 1 - 4$

CPT Observation

$Pr\{X_4, X_5 / X_3\}$

$= \dfrac{Pr\{X_4, X_5, X_3\}}{Pr\{X_3\}}$

Markov property $=$

$Pr\{X_t | X_{t-1}, X_{t-2}, X_{t-3}\}$

$Pr\{X_t | X_{t-1}\}$

X_5 indep of X_3 conditioning on X_4

$$= \frac{Pr\{X_4 \mid X_3\} \cdot Pr\{X_3 \mid X_4\} \cdot Pr\{X_3\}}{Pr\{X_3\}}$$

or $\vee$

and $\wedge$

$$= Pr\{X_4 \mid X_3\} \cdot Pr\{X_3 \mid X_4\}$$

② $Pr\{Y_1 = 0,\ Y_2 = 1\}$ → same as above question

$X_1 = 0$ → $Pr\{X_1 = 0\} = 1$

Markov

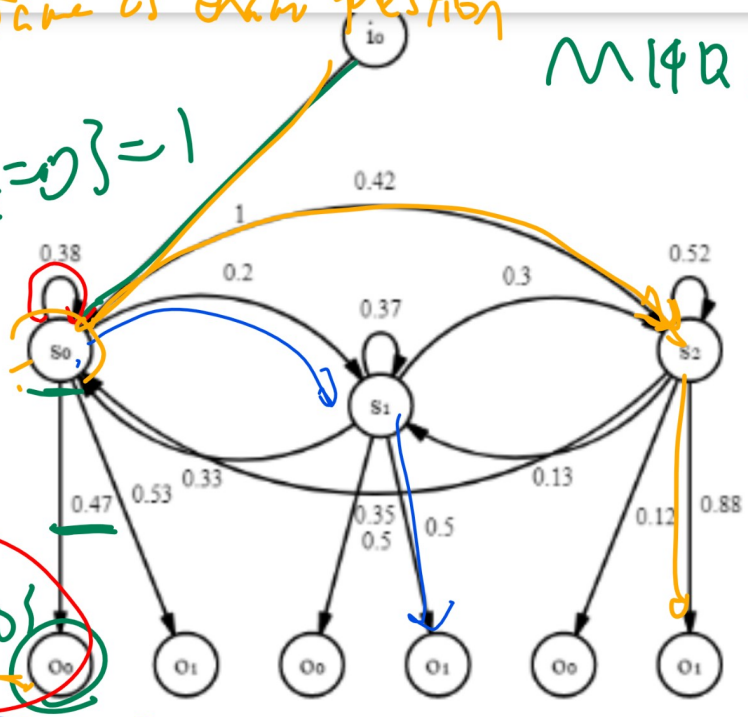
$= Pr\{Y_1 = 0 \mid X_1 = 0\}$

0.47

$Pr\{Y_2 = 1 \mid X_2 = 0\}\ Pr\{X_2 = 0 \mid X_1 = 0\}$ (0.53, 0.38)

$+ Pr\{Y_2 = 1 \mid X_2 = 1\} \cdot Pr\{X_2 = 1 \mid X_1 = 0\}$ (0.5, 0.2)

$+ Pr\{Y_2 = 1 \mid X_2 = 2\} \cdot Pr\{X_2 = 2 \mid X_1 = 0\}$ (0.88, 0.42)



③ $Pr\{X_1=0, X_2=2 \mid Y_1=0, Y_2=1\}$

$$= \frac{Pr\{X_1=0, X_2=2, Y_1=0, Y_2=1\}}{Pr\{Y_1=0, Y_2=1\}}$$ ← from ②

$$= Pr\{X_2=2 \mid X_1=0\} \cdot Pr\{Y_1=0 \mid X_1=0\} \cdot Pr\{Y_2=1 \mid X_2=2\}$$

$$= 0.42 \cdot 0.47 \cdot 0.88$$

part ②