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M15 Practice Exam Problems

• Enter your ID (the wisc email ID without @wisc.edu) here: yw and click Confirm (or hit enter key)

• The same ID should generate the same set of questions. Your answers are not saved when you close the browser. You could print the page: Print, solve the problems, then enter all your answers at the end.
• Please do not refresh the page: your answers will not be saved.

ID: yw

Question 1

• [4 points] Given the following game payoff table, suppose the the row player uses a pure strategy, and column player uses a mixed strategy playing L with probability q. What is the smallest and largest value of q in a mixed strategy Nash equilibrium?

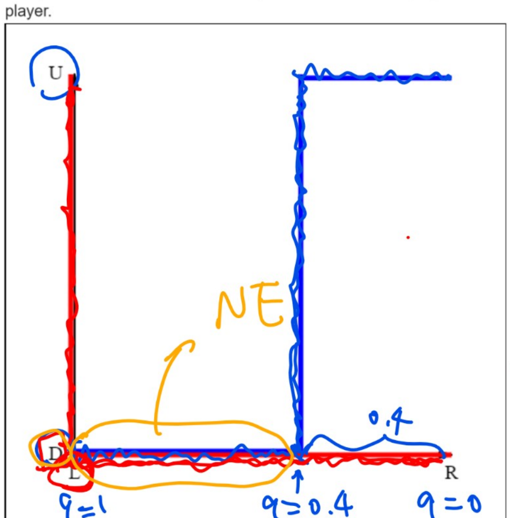
Row \ Col	L (q)	R ($1-q$)
U (p)	0,7	4,0
D ($1-p$)	6,7	0,7

$U > D$

$0q + 4(1-q) > 6q + 0(1-q)$

$q < 0.4$

Note: the following is a diagram of the best responses (make sure you understand what they are and how to draw them). The red curve is the best response for the column player and the blue curve is the best response for the row player.



$br_{ROW}(q) = \begin{cases} U \; 1 & q < 0.4 \\ [0,1] & q = 0.4 \\ D \; 0 & q > 0.4 \end{cases}$ (p^*)

$L > R$

$7 > 7(1-p) + 0p$

$br_{COL}(p) = \begin{cases} [0,1] & p = 0 \; D \\ L \; 1 & p > 0 \end{cases}$ (q^*)

• Answer (comma separated vector): 0.4,1 Calculate

$(D, L^{(q)} R^{(1-q)}; q \in [0.4, 1])$

Question 2

• [3 points] Consider a variant of the II-nim game. There are two piles, each pile has n = 5 sticks. A player can take one stick from a single pile; or take two sticks, one from each pile (when available). The player who takes the last stick wins. Let the game value be 1 if the first player wins (and -1 if the second player wins). What is the game theoretical value of this game?

• Answer: 1 Calculate

$n=1 \to 1,1 \to$ 1st wins
$n=2 \to 2,2 \to$ 2nd wins
$n=3 \to 3,3 \to$ 1st win
$n=4 \to 4,4 \to$ 2nd wins
4,3 ; 4,2 ; 3,3 → 2,2 2nd win

$n=5$, 5,5 → 4,4 1st

2, 2nd wins

Question 3

• [2 points] Alice, Bob and Cindy go to the same school and live on a straight street lined with evenly spaced telephone poles. Alice's house is at the pole 7, Bob's is at the pole 5, Cindy's is at the pole 3. Where should the school set up a school bus stop so that the sum of distances (from house to bus stop) walked by the three students is minimized?

• Answer: 5 Calculate

$\min |x-7| + |x-5| + |x-3|$

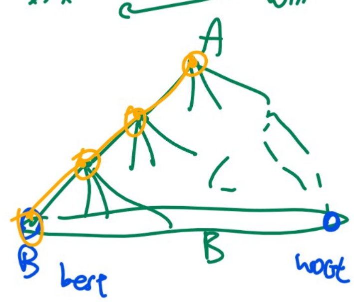
3, 4, 5, 6, 7

$x = \text{median}(7,5,3) = 5$

Question 4

• [4 points] Imagine a world where each person has 4 friends. Alice and Bob are d = 3 "friendship links" away (i.e. if d = 1, Alice and Bob are friends; if d = 2, there is a third person X such that Alice and X are friends, and Bob and X are friends; and so on). Imagine a depth first search (DFS) algorithm that has access to the friendship links. The algorithm starts at Alice and the goal is to find Bob. In the best (luckiest) case, how many people the algorithm needs to visit (including Alice and Bob)?

• Answer: 4 Calculate



Question 5

• [4 points] Imagine a population of N = 160 individuals. Each of them simultaneously chooses between taking the vaccine and not. All individuals have the same payoffs. Suppose there are n people who choose not to take the vaccine, then the payoff from not taking the vaccine is $-\alpha \cdot \dfrac{n}{N}$, and the payoff from taking the vaccine is

for n player unvaxed:

Not $\geq$ Vax

$-\alpha \cdot \frac{n}{N} \geq -c - \beta \frac{n-1}{N}$ → me, unvax → vax for N-n players vax : Vax $\geq$ Not

$-c-\beta\frac{n}{N} \geq -\alpha\frac{n+1}{N}$ → me, vax → unvax

$-c - \beta \cdot \dfrac{n}{N}$, α = 17 is the herd immunity coefficient, β = 3 measures the ineffectiveness of the vaccine, and c = 4 is the cost of getting the vaccine. In a Nash equilibrium, what is the largest number of individuals who choose NOT to take the vaccine.

• Note: n is the number of people NOT taking the vaccine, and the question is asking for the largest number of individuals who choose NOT to take the vaccine.

• Answer: 45 . Calculate

$T \leq n \leq T$ ↑

Question 6

• [4 points] Given the following BoS game, what is the column (Juliet) player's (expected) value (i.e. payoff) in the mixed strategy Nash equilibrium?

q = 1−q =

Romeo \ Juliet	Bach	Stravinsky
Bach p	4, 3	0, 0
Stravinsky 1−p	0, 0	3, 4

• Answer: 12/7 . Calculate

$q\frac{12}{7} + (1-q)\frac{12}{7}$ → $\frac{12}{7}$

B = S ← for col

→ $3p + 0(1-p) = 0p + 4(1-p)$

$p = \frac{4}{7}$

$\frac{12}{7} = \frac{12}{7}$

Question 7

• [4 points] There are 10 lights in a row. The initial state is $[1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0]$, 0 is "off", 1 is "on". A valid move finds two adjacent lights where one is on and the other is off, and switches them while keeping all other lights the same. That is, locally, you may do 01 to 10 or 10 to 01. What is the smallest number of moves to reach the goal state $[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$

• Answer: 2+3+3+4+5 . Calculate

2+3+3+4+5

Question 8

• [4 points] Suppose the state space has 163 states that form a tree. What is the shape of the tree that makes iterative deepening realize that a goal does not exist as quickly as possible (i.e. one that minimizes the number of expanded nodes)? Enter the number of nodes that will be searched? Include the root and repeated nodes.

• Answer: 163+1 . Calculate

162

Question 9

• [4 points] What is the projected variance of $\begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -9 \\ 7 \end{bmatrix}$ onto the principal component $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$? Use the MLE formula for the variance: $\sigma^2 = \dfrac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$ with $\mu = \dfrac{1}{n} \sum_{i=1}^{n} x_i$.

unit

$x^T u$

• Answer: 0 . Calculate

$x_1 = 3$, $x_2 = 3$

$\mu = \frac{1}{2}(x_1 + x_2) = \frac{1}{2}(3+3) = 3$

$\sigma^2 = \frac{1}{2}((x_1-3)^2 + (x_2-3)^2) = 0$

Question 10

• [4 points] In GoogSoft, software engineers A and B form a two-person team. Their year-end bonus depends on their relative performance. The bonus outcomes are summarized in the following table. The value of slacking to each person is s = 5. The total payoff to each person is the sum of the bonus and the value from slacking. What is the smallest value of x such that both players will work hard in a Nash equilibrium?

-	B works hard	B slacks
A works hard	x, x	16, 2 +5
A slacks	+5 2, 16	+5 +5 9, 9

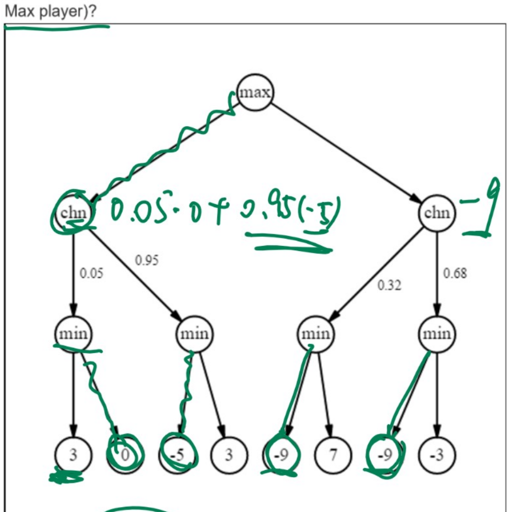
• Answer: 5+2 . Calculate

$x \geq 2+5 = 7$

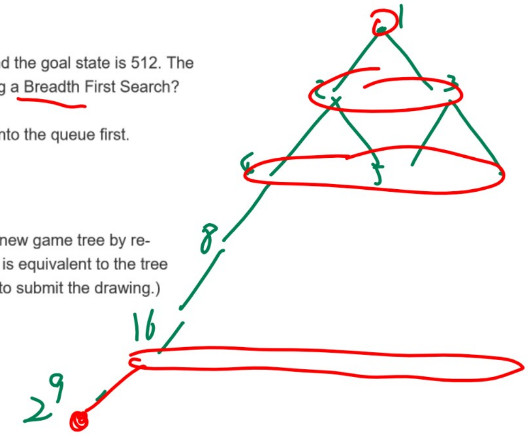
$x \geq 2+5$

Question 11

• [4 points] Consider a zero-sum sequential move game with Chance. Max player moves first, then Chance, then Min. The values of the terminal states are shown in the diagram. What is the (expected) value of the game (for the Max player)?



• Answer: 0.05*0+0.95*(-5) . Calculate



Question 12

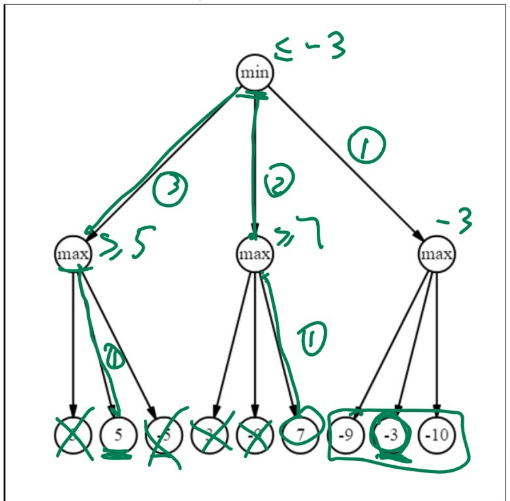
• [4 points] Suppose the states are integers between 1 and 512. The initial state is 1, and the goal state is 512. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded using a Breadth First Search? Include both the initial and goal states.

• Note: use the convention used in the lectures, enqueue the states with smaller index into the queue first.

• Answer: 512 . Calculate

Question 13

• [4 points] Consider the following zero-sum game tree. MIN player moves first. Draw a new game tree by re-ordering the children of each internal node (including the root), such that the new game is equivalent to the tree above, but alpha-beta pruning will prune as many nodes as possible. (You do not have to submit the drawing.) Enter the number of nodes pruned.



4

at most 4

• Answer: 4 .

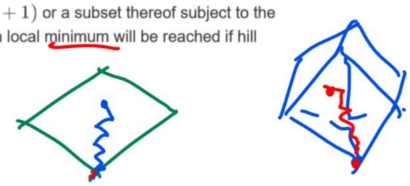
Question 14

2D

• [4 points] Let the states be 3D integer points with integer coordinates (i, j, k) with boundary constrains $-5 \le i \le 5$ and $-10 \le j \le 10$ and $-5 \le k \le 5$. Each state (i, j, k) has six successors $(i - 1, j, k), (i + 1, j, k), (i, j - 1, k), (i, j + 1, k), (i, j, k - 1), (i, j, k + 1)$ or a subset thereof subject to the boundary constraints. The score of state (i, j, k) is $3\,i - 5\,j + 3\,k$. Which local minimum will be reached if hill climbing is used starting from $\begin{bmatrix} -2 \\ -10 \\ -2 \end{bmatrix}$? Enter the state, not the score.

• Answer (comma separated vector): -5,10,-5

-5 10 -5



Question 15

• [1 points] Please enter any comments including possible mistakes and bugs with the questions or your answers. If you have no comments, please enter "None": do not leave it blank.

• Answer: None.

Grade

Grade

***** ***** ***** ***** *****

ID: yw

Question 1 is correct. (4/4)
Question 2 is correct. (3/3)
Question 3 is correct. (2/2)
Question 4 is correct. (4/4)
Question 5 is correct. (4/4)
Question 6 is correct. (4/4)
Question 7 is correct. (4/4)
Question 8 is correct. (4/4)
Question 9 is correct. (4/4)
Question 10 is correct. (4/4)
Question 11 is correct. (4/4)
Question 12 is correct. (4/4)
Question 13 is correct. (4/4)
Question 14 is correct. (4/4)
Question 15 is correct. (1/1)
Grade: 54 out of 54.
Code: tz79"9&>">&?"?&9"9&9"9&9"9&9"9&9"9&9"9&9"9&9"9&9"9&9"9&9"9&<"<&089"89

***** ***** ***** ***** *****

```
##m: 15
##id: yw
##1: 0.4,1
##2: 1
```

• You could save the text in the above text box to a file using the button Download or copy and paste it into a file yourself Copy.

• You could load your answers from the text (or txt file) in the text box below using the button Load. The first two lines should be "##m: 15" and "##id: your id", and the format of the remaining lines should be "##1: your answer to question 1" newline "##2: your answer to question 2", etc. Please make sure that your answers are loaded correctly before submitting them.

Choose File M15.txt

```
##m: 15
##id: yw
```

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