Final Version A

CS540

August 14, 2019

1 Instruction

1. Each incorrect answer receives -0.25, each correct answer receives 1, blank answers receive 0.
2. Check to make sure your name and (numerical) student ID (if you have it) is on the (Scantron) answer sheet. Also, write your Wisc email ID at the top of the answer sheet.
3. If you think none (or more than one) of the answers are correct, choose the best (closest) one.
4. Please submit this final, the answer sheet, the formula sheet, and all your additional notes when you finish.
5. Good luck!

2 Questions

1. Given the table summarizing the Euclidean distances between clusters A to F so far. What is single-linkage Euclidean distance between the cluster formed by A, B, C and the cluster formed by D, E, F?

$-$	A	B	C	D	E	F
A	0	3	4	5	4	3
B	$-$	0	7	8	9	8
C	$-$	$-$	0	4	5	4
D	$-$	$-$	$-$	0	2	3
E	$-$	$-$	$-$	$-$	0	1
F	$-$	$-$	$-$	$-$	$-$	0

- A: 1
- B: 2
- C: 3
- D: 4
- E: 5

2. Continue from the previous question, what is the complete-linkage Euclidean distance between the cluster formed by A, B, C, and the cluster formed by D, E, F?

- A: 5
- B: 6
- C: 7
- D: 8
- E: 9

3. Continue from the previous question: start with all 6 clusters, not the A, B, C vs D, E, F clusters. Suppose hierarchical clustering with single-linkage Euclidean distance is used and the algorithm stops when there are $K = 2$ clusters. Which one of the following is one of the two clusters?

- A: $\{A\}$
- B: $\{B\}$
- C: $\{C\}$
- D: $\{D\}$
- E: $\{E\}$

4. Suppose K-Means with $K = 2$ is used to cluster the dataset $\{1, 7, 10, 12\}$ and one of the initial cluster centers $c_1 = 3$. What is a possible value of c_2 such that the two updated clusters are $\{1, 7, 10\}$ and $\{12\}$?

   - A: 16
   - B: 19
   - C: 22
   - D: 25
   - E: 28

5. Continue from the previous question and suppose the new clusters are $\{1, 7, 10\}$ and $\{12\}$. What are the updated cluster center c_1?

   - A: $c_1 = 1$
   - B: $c_1 = 3$
   - C: $c_1 = 6$
   - D: $c_1 = 7$
   - E: $c_1 = 10$

6. Suppose X is a 200×100 matrix (200 rows and 100 columns). Which one of the following expressions represents the entry of X on row 30 column 60? Notation: $e_i^{(n)}$ is the vector of length $n : (0, 0, ..., 0, 1, 0, ..., 0, 0)^T$ with a 1 at position i and 0 everywhere else, and $e^{(n)}$ is the vector of length $n : (1, 1, ..., 1, 1)^T$ with 1 at every position.

   - A: $\left(e_{30}^{(100)}\right)^T X e_{60}^{(200)}$
   - B: $\left(e_{60}^{(100)}\right)^T X e_{30}^{(200)}$
   - C: $\left(e_{30}^{(100)}\right)^T X e_{60}^{(100)}$
   - D: $\left(e_{30}^{(200)}\right)^T X e_{60}^{(100)}$
   - E: $\left(e_{60}^{(200)}\right)^T X e_{30}^{(100)}$

7. Continue from the previous question. Which one of the following expressions represents the sum of the entries of X on row 60?

   - A: $\left(e_{60}^{(100)}\right)^T X e^{(200)}$
   - B: $\left(e_{60}^{(200)}\right)^T X e^{(100)}$
   - C: $\left(e_{60}^{(100)}\right)^T X e^{(100)}$
   - D: $\left(e^{(100)}\right)^T X e_{60}^{(200)}$
   - E: $\left(e^{(200)}\right)^T X e_{60}^{(100)}$

8. What is the projection of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

- A: $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$
- B: $\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T$
- C: $\begin{bmatrix} 3 & 3 & 3 \end{bmatrix}^T$
- D: $\begin{bmatrix} 4 & 4 & 4 \end{bmatrix}^T$
- E: $\begin{bmatrix} 6 & 6 & 6 \end{bmatrix}^T$

9. What is the projection of $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$?

- A: $\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$
- B: $\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T$
- C: $\begin{bmatrix} 3 & 3 & 3 \end{bmatrix}^T$
- D: $\begin{bmatrix} 4 & 4 & 4 \end{bmatrix}^T$
- E: $\begin{bmatrix} 6 & 6 & 6 \end{bmatrix}^T$

10. What is the projected sample variance of $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$ onto $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$? Note: projected sample variance is the sample variance of the magnitude of the projection of the data points onto a principal component, which $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in this question. Use the maximum likelihood estimator of σ^2:

$$(\hat{\sigma})^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\mu})^2 , \hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- A: 0
- B: 1
- C: 2
- D: 4
- E: 16

11. Suppose a face image is represented by 3 features $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and the principal components (eigenfaces) with associated eigenvalues λ are the following. What is the reconstructed face image using the one principal component ($K = 1$)?

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ with } \lambda_1 = 1$$

$$u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \text{ with } \lambda_2 = 2$$

$$u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \text{ with } \lambda_3 = 3$$

- A: $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$
- B: $\begin{bmatrix} 0 & 0 & 3 \end{bmatrix}^T$
- C: $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}^T$
- D: $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T$
- E: $\begin{bmatrix} 0 & 2 & 3 \end{bmatrix}^T$

12. Continue from the previous question. What is the reconstructed face image using two (the first two) principal components ($K = 2$)?

- A: $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$
- B: $\begin{bmatrix} 0 & 0 & 3 \end{bmatrix}^T$
- C: $\begin{bmatrix} 1 & 0 & 3 \end{bmatrix}^T$
- D: $\begin{bmatrix} 1 & 2 & 0 \end{bmatrix}^T$
- E: $\begin{bmatrix} 0 & 2 & 3 \end{bmatrix}^T$

13. Suppose the states are integers between 0 and 9. The initial state is 1, and the goal state is 5. The successors of a state i are the first digit and the last digit of $i \cdot 7$. For example, the first digit of 14 is 1, the last digit is 4, and the first digit of 7 is 0, the last digit is 7. What is a state expansion sequence if Breadth-First Search (BFS) is used? Use the convention that a smaller integer is always enQueued before a larger integer, and a list of visited states are stored so that the same state is never enQueued twice. For example, enQueueing $\{3, 5, 7\}$ into the Queue with $\{1, 7, 9\}$ (from front to back) results in $\{1, 7, 9, 3, 5\}$ (from front to back). Use this convention for all search questions.

$$0 \cdot 7 = 00, 1 \cdot 7 = 07, 2 \cdot 7 = 14, 3 \cdot 7 = 21, 4 \cdot 7 = 28$$
$$5 \cdot 7 = 35, 6 \cdot 7 = 42, 7 \cdot 7 = 49, 8 \cdot 7 = 56, 9 \cdot 7 = 63$$

- A: $1, 0, 7, 4, 9, 8, 5$
- B: $1, 0, 7, 4, 9, 8, 3, 5$
- C: $1, 0, 7, 4, 9, 2, 8, 5$
- D: $1, 0, 7, 4, 9, 2, 8, 6, 3, 5$
- E: $1, 0, 7, 4, 9, 2, 8, 3, 6, 5$

14. Continue from the previous question. What is a state expansion sequence if Depth First Search (DFS) is used? Use the convention that a smaller integer is always pushed in a Stack after a larger integer, and a list of visited states are stored so that the same state is never pushed twice. For example, pushing $\{3, 5, 7\}$ into the Stack with $\{1, 7, 9\}$ (from top to bottom) results in $\{3, 5, 1, 7, 9\}$ (from top to bottom). Use this convention for all search questions.

- A: $1, 0, 4, 2, 8, 5$
- B: $1, 0, 4, 2, 9, 6, 8, 5$
- C: $1, 0, 4, 2, 9, 3, 6, 8, 5$
- D: $1, 0, 7, 4, 2, 8, 5$
- E: $1, 0, 7, 4, 2, 8, 3, 6, 5$

15. Suppose the states are integers between 0 and $2^{10} = 1024$. The initial state is 0, and the goal state is 1024. The successors of a state i are $2i + 1$ and $2i + 2$, if exist. How many states are expanded during a BFS search? Reminder: initial state is 0, not 1. Count both the initial state and the goal state.

   - A: 11
   - B: 12
   - C: 1024
   - D: 1025
   - E: 1026

16. Continue from the previous question. How many states are expanded during a DFS search? Count both the initial state and the goal state.

   - A: 11
   - B: 12
   - C: 1024
   - D: 1025
   - E: 1026

17. Suppose the states are integers between 0 and $2^{10} - 1 = 1023$. The initial state is 0, and the goal state is 1023. The successors of a state i are $2i+1$ and $2i+2$, if exist. How many states are expanded during a BFS search? Count both the initial state and the goal state.

   - A: 11
   - B: 12
   - C: 1024
   - D: 1025
   - E: 1026

18. Continue from the previous question. How many states are expanded during a DFS search? Count both the initial state and the goal state.

   - A: 11
   - B: 12
   - C: 1024
   - D: 1025
   - E: 1026

19. Let the states be the following five cities: Fitchburg, Madison, Middleton, Verona, Waunakee. The locations on the map are given in the following table. In the search digraph, the edge costs are the Euclidean distance between the cities corresponding to the state. The initial state is Madison, and the goal state is Verona. What is a vertex expansion sequence if Uniform Cost Search (UCS) is used? Reminder: when there are ties, the state with a smaller index has priority.

Index	City	X-coordinate	Y-coordinate	Successors
1	Fitchburg	-1	-1	Middleton, Verona
2	Madison	0	0	Fitchburg, Middleton, Waunakee
3	Middleton	-2	0	Verona
4	Verona	-2	-2	–
5	Waunakee	0	2	Middleton

- A: Madison, Middleton, Verona
- B: Madison, Fitchburg, Middleton, Verona
- C: Madison, Fitchburg, Middleton, Waunakee, Verona
- D: Madison, Fitchburg, Middleton, Waunakee, Middleton, Verona
- E: Madison, Fitchburg, Middleton, Waunakee, Middleton, Fitchburg, Verona

20. Continue from the previous question. What is a vertex expansion sequence if Best First Greedy Search is used with heuristics equal to 0 for all states? Reminder again: when there are ties, the state with a smaller index has priority.

- A: Madison, Middleton, Verona
- B: Madison, Fitchburg, Middleton, Verona
- C: Madison, Fitchburg, Middleton, Waunakee, Verona
- D: Madison, Fitchburg, Middleton, Waunakee, Middleton, Verona
- E: Madison, Fitchburg, Middleton, Waunakee, Middleton, Fitchburg, Verona

21. Continue from the previous question. How many of the following heuristic functions are admissible? The heuristic function is given as a function of the location (x, y) of the city. Hint: p-norm is decreasing in p for $p \geqslant 1$: $\left(\sum_{i=1}^{m} |x_i|^p\right)^{1/p} \geqslant \left(\sum_{i=1}^{m} |x_i|^{p'}\right)^{1/p'}$ if $1 \leqslant p \leqslant p' \leqslant \infty$.

$$
\begin{aligned}
h_1(x, y) &= |x+2| + |y+2| \\
h_2(x, y) &= \sqrt{(x+2)^2 + (y+2)^2} \\
h_3(x, y) &= \sqrt{|x+2|^3 + |y+2|^3} \\
h_4(x, y) &= \left(|x+2|^3 + |y+2|^3\right)^{1/3} \\
h_5(x, y) &= \max\left(|x+2|, |y+2|\right)
\end{aligned}
$$

- A: 1
- B: 2
- C: 3
- D: 4
- E: 5

22. Continue from the previous question. Which one of the following heuristic functions is not dominated (among the admissible ones)?

$$
\begin{aligned}
h_1(x, y) &= |x+2| + |y+2| \\
h_2(x, y) &= \sqrt{(x+2)^2 + (y+2)^2} \\
h_3(x, y) &= \sqrt{|x+2|^3 + |y+2|^3} \\
h_4(x, y) &= \left(|x+2|^3 + |y+2|^3\right)^{1/3} \\
h_5(x, y) &= \max\left(|x+2|, |y+2|\right)
\end{aligned}
$$

- A: h_1
- B: h_2
- C: h_3
- D: h_4
- E: h_5

23. Given the following tables showing the cell indices and the list of successors of a maze. The first table contains the names (indices) of the states for each cell, and the second table contains the indices of the successor states for each cells. The entrance is 1, the exit is 16. How many unique cells are visited if UCS is used? Reminder: count both initial and goal states, and the state with a smaller index has priority.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

2, 5	1, 3, 6	2, 4, 7	3, 8
1, 9	2, 10	3, 11	4, 12
5, 13	6, 14	7, 15	8, 16
9, 14	10, 13, 15	11, 14, 16	12, 15

- A: 7
- B: 10
- C: 14
- D: 15
- E: 16

24. Continue from the previous question. How many unique cells are visited if $A^\star$ is used with Manhattan distance between the cell and the goal? Reminder: count both initial and goal states, and the state with a smaller index has priority.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

- A: 7
- B: 10
- C: 14
- D: 15
- E: 16

25. Starting from the initial state $x_1 = x_2 = x_3 = x_4 = x_5 = 0$ (all variables set to False), what is the minimum number of steps required to reach the solution for the following SAT problem if first choice hill climbing is used? The score is the number of clauses satisfied. The neighbors differ by 1 variable. The following expression is 5 clauses connected by $\wedge$.

$$(x_1 \vee x_2) \wedge (x_1 \vee x_3) \wedge (x_1 \vee x_4) \wedge (x_2 \vee x_4) \wedge (x_3 \vee x_5)$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

26. Continue from the previous question. Suppose simulated annealing is used with initial (current) temporature T and the random neighbor choosen as $x_1 = 1$ and $x_2 = x_3 = x_4 = x_5 = 0$. What is the probability that the state jumps (from the initial state) to this neighbor?

- A: $\exp\left(-3/T\right)$
- B: $\exp\left(-2/T\right)$
- C: $\exp\left(-1/T\right)$
- D: $\exp\left(0/T\right)$
- E: $\exp\left(1/T\right)$

27. Continue from the previous question. Suppose genetic algorithm is used and the fitness function is the score. There are in total five initial states x_a, x_b, x_c, x_d, x_e written in the format $x = (x_1, x_2, x_3, x_4, x_5)$. What is the reproduction probability of the first state x_a? The reproduction probabilities are proportional to the fitness of the states.

$$\begin{aligned} x_a &= (1, 0, 0, 0, 0) \\ x_b &= (0, 1, 0, 0, 0) \\ x_c &= (0, 0, 1, 0, 0) \\ x_d &= (0, 0, 0, 1, 0) \\ x_e &= (0, 0, 0, 0, 1) \end{aligned}$$

- A: $\dfrac{2}{9}$
- B: $\dfrac{3}{9}$
- C: $\dfrac{2}{10}$
- D: $\dfrac{3}{10}$
- E: $\dfrac{3}{11}$

28. Consider a sequential game with Chance. Player 1, MAX, chooses an action H or T first. Then a fair coin is flipped, the outcome is either H (heads) or T (tails). Player 2, MIN, observes the outcome of the coin and chooses an action H or T. Suppose the number of outcomes or actions that are H is x and the number of outcomes or actions that are T is $3 - x$, then the value of the terminal state is $\max\{x, 3 - x\}$. In other words, the value for any path with x number of $H's$ and $(3 - x)$ number of $T's$ is $\max\{x, 3 - x\}, x = 0, 1, 2, 3$. What is the value of the whole game?

- A: 1
- B: 1.5
- C: 2
- D: 2.5
- E: 3

29. Continue from the previous question. Suppose the order in which the players are reversed: Player 2, MIN, chooses first and Player 1, MAX, chooses last. The values of the terminal states remain the same. What is the value of the whole game?

- A: 1
- B: 1.5
- C: 2
- D: 2.5
- E: 3

30. Consider a zero-sum sequential move game in which player MIN moves first, and MAX moves second. Each player has three actions labeled $1, 2, 4$. The value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 + x_2$. Alpha-Beta pruning is used. What is the number of branches (states) that can be pruned? During the search process, the actions with smaller labels are searched first. Note that a branch is pruned if $\alpha = \beta$.

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

31. Continue from the previous question. Suppose during the search process, the actions with larger labels are searched first instead (for both players). What is the number of branches (states) that can be pruned?

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

32. Continue from the previous question. Suppose during the search process, the order in which the actions are searched can be changed (the game itself cannot be changed). What is the maximum possible number of branches (states) that can be pruned?

- A: 2
- B: 3
- C: 4
- D: 5
- E: 6

33. For a simultaneous move game, both the row player and the column player are maximizing the payoff in the table. Given the game payoff table of payoff pairs (row, column), which of the following list contains all states that survive the iterated elimination of weakly dominated strategies? Eliminate weakly dominated strategies for column player first, then row player, then column player and so on. Reminder: if two actions yield identical payoffs for a player (no matter what the other player is choosing), neither actions are weakly dominated, so do not eliminate either.

–	E	F	G	H
A	(-2, 0)	(-1, 0)	(0, 1)	(0, 1)
B	(-1, 1)	(0, 2)	(0, 2)	(0, 1)
C	(0, 2)	(-1, 2)	(0, 2)	(0, 2)
D	(0, 3)	(0, 4)	(-1, 3)	(0, 3)

- A: $\{(D, F)\}$
- B: $\{(B, F), (B, G)\}$
- C: $\{(B, F), (D, F)\}$
- D: $\{(B, F), (B, G), (D, F)\}$
- E: $\{(C, E), (B, F), (D, F), (A, G), (B, G), (C, G), (A, H), (C, H)\}$

34. Continue from the previous question, but eliminate weakly dominated strategies for row player first, then column player, then row player and so on. Which of the following list contains all states that survive the iterated elimination of weakly dominated strategies? Same payoff table copied and pasted:

–	E	F	G	H
A	(-2, 0)	(-1, 0)	(0, 1)	(0, 1)
B	(-1, 1)	(0, 2)	(0, 2)	(0, 1)
C	(0, 2)	(-1, 2)	(0, 2)	(0, 2)
D	(0, 3)	(0, 4)	(-1, 3)	(0, 3)

- A: $\{(D, F)\}$
- B: $\{(B, F), (B, G)\}$
- C: $\{(B, F), (D, F)\}$
- D: $\{(B, F), (B, G), (D, F)\}$
- E: $\{(C, E), (B, F), (D, F), (A, G), (B, G), (C, G), (A, H), (C, H)\}$

35. In an endgame of Tic Tac Toe with the following configuration, how many terminal states can be reached from this configuration? Note that the same configuration reached by different paths are counted as two different terminal states: for example, the configuration XOX reached from placing the first X first and from placing the second X first are two different states.

X	O	X
O	X	$-$
$-$	$-$	$-$

In a Tic Tac Toe game, the player who places the same three symbols (X or O) on the same row, column or diagonal first is the winner. The game terminates if one player wins or if the board is filled without a winner.

- A: 14
- B: 16
- C: 18
- D: 20
- E: 24

36. Continue from the previous question, how many positions (actions) are optimal for player O given both players are using the optimal strategy in all subgames?

X	O	X
O	X	$-$
$-$	$-$	$-$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

37. Given the following simultaneous move game payoff table, what is the complete set of pure strategy Nash equilibria? Both players are MAX players. In each entry of the table, the first number is the reward to the ROW player and the second number is the reward to the COLUMN player.

$-$	L	R
U	$(1,1)$	$(0,1)$
D	$(0,0)$	$(1,1)$

- A: $\{(U,L)\}$
- B: $\{(D,R)\}$
- C: $\{(U,L),(D,R)\}$
- D: $\{(U,L),(U,R),(D,R)\}$
- E: $\{(U,L),(D,L),(D,R)\}$

38. Continue from the previous question, how many of the following are mixed strategy Nash equilibria of the game?

$$\left(\text{always U}, \left(\text{L } \frac{1}{2} \text{ of the time}, \text{R } \frac{1}{2} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{1}{2} \text{ of the time}, \text{R } \frac{1}{2} \text{ of the time}\right)\right)$$
$$\left(\left(\text{U } \frac{1}{2} \text{ of the time}, \text{D } \frac{1}{2} \text{ of the time}\right), \text{always L}\right)$$
$$\left(\left(\text{U } \frac{1}{2} \text{ of the time}, \text{D } \frac{1}{2} \text{ of the time}\right), \text{always R}\right)$$
$$\left(\left(\text{U } \frac{1}{2} \text{ of the time}, \text{D } \frac{1}{2} \text{ of the time}\right), \left(\text{L } \frac{1}{2} \text{ of the time}, \text{R } \frac{1}{2} \text{ of the time}\right)\right)$$

- A: 1
- B: 2
- C: 3
- D: 4
- E: 5

39. Continue from the previous question, how many of the following are mixed strategy Nash equilibria of the game?

$$\left(\text{always U}, \left(\text{L } \frac{1}{4} \text{ of the time}, \text{ R } \frac{3}{4} \text{ of the time}\right)\right)$$
$$\left(\text{always U}, \left(\text{L } \frac{3}{4} \text{ of the time}, \text{ R } \frac{1}{4} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{1}{4} \text{ of the time}, \text{ R } \frac{3}{4} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{3}{4} \text{ of the time}, \text{ R } \frac{1}{4} \text{ of the time}\right)\right)$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

40. Continue from the previous question, how many of the following are mixed strategy Nash equilibria of the game?

$$\left(\text{always U}, \left(\text{L } \frac{1}{8} \text{ of the time}, \text{ R } \frac{7}{8} \text{ of the time}\right)\right)$$
$$\left(\text{always U}, \left(\text{L } \frac{3}{8} \text{ of the time}, \text{ R } \frac{5}{8} \text{ of the time}\right)\right)$$
$$\left(\text{always U}, \left(\text{L } \frac{5}{8} \text{ of the time}, \text{ R } \frac{3}{8} \text{ of the time}\right)\right)$$
$$\left(\text{always U}, \left(\text{L } \frac{7}{8} \text{ of the time}, \text{ R } \frac{1}{8} \text{ of the time}\right)\right)$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

Questions 41 to 43 on next page.

41. Calculator?

    - A: Yes.
    - B: No.

42. The number of pages of additional notes? Please submit them at the end of the exam.

    - A: 0
    - B: 1
    - C: 2
    - D: 3
    - E: 4 or more.

43. Bonus mark: You received one free point for this question and you have two choices: donate 1 and keep 0 or keep 1 and donate 0. Your final grade is the points you keep plus twice the average donation (sum of the donations from everyone in your section divided by the number of people in your section).

- A: Donate the point (receive $2\cdot$ average donation).
- B: Keep the point (receive $1 + 2\cdot$ average donation).
- C, D, E (or left blank): Not participate in the game (receive 0 for this question).

$$\text{average donation} = \frac{\text{number of people in your section who chose A}}{\text{number of people in your section who chose A or B}}$$