Final Version B

CS540

August 19, 2019

1 Instruction

1. Each incorrect answer receives -0.25, each correct answer receives 1, blank answers receive 0.
2. Check to make sure your name and (numerical) student ID (if you have it) is on the (Scantron) answer sheet. Also, write your Wisc email ID at the top of the answer sheet.
3. If you think none (or more than one) of the answers are correct, choose the best (closest) one.
4. Please submit this final, the answer sheet, the formula sheet, and all your additional notes when you finish.
5. Good luck!

2 Questions

1. There are six data points $\{1, 2, 4, 8, 16, 32\}$ and suppose hierarchical clustering with single-linkage Manhanttan distance is used. Suppose the algorithm stops when there are $K = 2$ clusters. What is the cluster containing 1?

   - A: $\{1\}$
   - B: $\{1, 2\}$
   - C: $\{1, 2, 4\}$
   - D: $\{1, 2, 4, 8\}$
   - E: $\{1, 2, 4, 8, 16\}$

2. Continue from the previous question, what is the cluster containing 1 if hierarchical clustering with complete-linkage Manhanttan distance is used?

   - A: $\{1\}$
   - B: $\{1, 2\}$
   - C: $\{1, 2, 4\}$
   - D: $\{1, 2, 4, 8\}$
   - E: $\{1, 2, 4, 8, 16\}$

3. Suppose K-Means clustering with $K = 2$ is used on the dataset $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$ and the initial cluster centers are $c_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $c_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Which one of the following is one of the new cluster centers after the first iteration? Use Euclidean distance.

- A: $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- B: $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$
- C: $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$
- D: $\begin{bmatrix} 1.5 \\ 1.5 \end{bmatrix}$
- E: $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$

4. Continue from the previous question and suppose the initial cluster centers are changed to $c_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $c_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Which one of the following is one of the new cluster centers after the first iteration?

- A: $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$
- B: $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$
- C: $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$
- D: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- E: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

5. What is the projection of $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

- A: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$
- B: $\begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \end{bmatrix}^T$
- C: $\begin{bmatrix} 2 & 2 & 2 \end{bmatrix}^T$
- D: $\begin{bmatrix} \frac{2}{\sqrt{14}} & \frac{4}{\sqrt{14}} & \frac{6}{\sqrt{14}} \end{bmatrix}^T$
- E: $\begin{bmatrix} 2 & 4 & 6 \end{bmatrix}^T$

6. What is the projection of $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$?

- A: $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$
- B: $\begin{bmatrix} \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \end{bmatrix}^T$
- C: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T$
- D: $\begin{bmatrix} \frac{-2}{\sqrt{14}} & \frac{1}{\sqrt{14}} & \frac{0}{\sqrt{14}} \end{bmatrix}^T$
- E: $\begin{bmatrix} -2 & 1 & 0 \end{bmatrix}^T$

7. What is the projected sample variance of $\left\{ \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$ onto $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$? Note: projected sample variance is the sample variance of the magnitude of the projection of the data points onto a principal component, which $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in this question. Use the maximum likelihood estimator of $\sigma^{2:}$

$$(\hat{\sigma})^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\mu})^2, \hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

- A: 0
- B: 2
- C: 5
- D: 7
- E: 14

8. Suppose X is a 100×100 matrix, not necessarily symmetric. Which one of the following expressions represents the sum of entries of X on column 60? Notation: e_i is the length 100 vector $(0, 0, ..., 0, 1, 0, ..., 0, 0)^T$ with a 1 at position i and 0 everywhere else, and e is the length 100 vector $(1, 1, ..., 1, 1)^T$ with 1 at every position.

   - A: $e_{60}^T X^T e$
   - B: $e_{60} X^T e^T$
   - C: $e X^T e_{60}^T$
   - D: $e^T X^T e_{60}$
   - E: $e_{60} X^T e$

9. Continue from the previous question. Which one of the following expressions represents the sum of the entries of X on row 60?

   - A: $e_{60}^T X^T e$
   - B: $e_{60} X^T e^T$
   - C: $e X^T e_{60}^T$
   - D: $e^T X^T e_{60}$
   - E: $e_{60} X^T e$

10. Continue from the previous question. Which one of the following expressions do NOT represent the sum of all entries of X ?

   - A: $e^T X e$
   - B: $e^T X^T e$
   - C: $\sum_{i=1}^{100} e_i^T X e$
   - D: $\sum_{i=1}^{100} e^T X^T e_i$
   - E: $\sum_{i=1}^{100} \sum_{j=1}^{100} e_i X^T e_j^T$

11. Suppose a face image is represented by 3 features $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and the principal components (eigenfaces) are the following. What is u_1 ?

$$u_1 = ??? \text{ with } \lambda_1 = 3$$

$$u_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ with } \lambda_2 = 2$$

$$u_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \text{ with } \lambda_3 = 1$$

- A: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$
- B: $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
- C: $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
- D: $\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$
- E: $\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

12. Continue from the previous question. What is the reconstructed face image using the one principal component ($K = 1$)?

- A: $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$
- B: $\begin{bmatrix} 0 & 2 & 0 \end{bmatrix}^T$
- C: $\begin{bmatrix} 0 & 0 & 3 \end{bmatrix}^T$
- D: $\begin{bmatrix} 2 & 0 & 2 \end{bmatrix}^T$
- E: $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}^T$

13. Suppose the states are integers between 0 and 9. The initial state is 6, and the goal state is 0. The successors of a state i are the first and second decimal place of $\dfrac{i}{7.0}$. For example, the first decimal place of 0.149 is 1 and the second decimal place of 0.149 is 4; the first decimal place of 1.2 is 2 and the second decimal place of 1.2 is 0. What is a state expansion sequence if Breadth-First Search (BFS) is used? Use the convention that a smaller integer is always enQueued before a larger integer, and a list of visited states are stored so that the same state is never enQueued twice. For example, enQueueing $\{3, 5, 7\}$ into the Queue with $\{1, 7, 9\}$ (from front to back) results in $\{1, 7, 9, 3, 5\}$ (from front to back). Use this convention for all uniformed search questions.

$$\frac{0}{7} = 0.000000, \frac{1}{7} = 0.142857, \frac{2}{7} = 0.285714, \frac{3}{7} = 0.428571, \frac{4}{7} = 0.571428$$
$$\frac{5}{7} = 0.714285, \frac{6}{7} = 0.857142, \frac{7}{7} = 1.00000, \frac{8}{7} = 1.142857, \frac{9}{7} = 1.285714$$

- A: $6, 8, 5, 1, 4, 7, 0$
- B: $6, 8, 1, 4, 5, 7, 0$
- C: $6, 5, 8, 7, 1, 0$
- D: $6, 5, 8, 1, 7, 4, 0$
- E: $6, 5, 1, 4, 7, 0$

14. Continue from the previous question. What is a state expansion sequence if Depth First Search (DFS) is used? Use the convention that a smaller integer is always pushed in a Stack after a larger integer, and a list of visited states are stored so that the same state is never pushed twice. For example, pushing $\{3, 5, 7\}$ into the Stack with $\{1, 7, 9\}$ (from top to bottom) results in $\{3, 5, 1, 7, 9\}$ (from top to bottom). Use this convention for all search questions.

- A: $6, 8, 5, 1, 4, 7, 0$
- B: $6, 8, 1, 4, 5, 7, 0$
- C: $6, 5, 8, 7, 1, 0$
- D: $6, 5, 8, 1, 7, 4, 0$
- E: $6, 5, 1, 4, 7, 0$

15. Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i+1, 3i+2$, if exist. How many states are expanded during a BFS search? Include both the initial and goal states.

    - A: 4
    - B: 5
    - C: 14
    - D: 15
    - E: 28

16. Continue from the previous question. How many states are expanded during a DFS search?

    - A: 4
    - B: 5
    - C: 14
    - D: 15
    - E: 28

17. Continue from the previous question. How many unique states are expanded during an IDS search? Suppose the same state expanded multiple times at different depth are counted as only once.

    - A: 4
    - B: 5
    - C: 14
    - D: 15
    - E: 28

18. Continue from the previous question. How many states in total are expanded during an Iterative Deepening Search (IDS)? Suppose the same state expanded twice at different depth are counted twice for this question. Start from depth 0. At depth 0, only state 1 is expanded. Let the answer to the previous question be n.

    - A: $n+8$
    - B: $n+9$
    - C: $n+13$
    - D: $n+17$
    - E: $n+18$

19. Let the states be the following five cities: Fitchburg, Madison, Middleton, Verona, Waunakee. The locations on the map are given in the following table. In the search digraph, the edge costs are the Euclidean distance between the cities corresponding to the state. The initial state is Madison, and the goal state is Verona. What is a state expansion sequence if $A^{\star}$ Search is used with heuristics equal to the Euclidean distance between the state and the goal state, for all states? Reminder: when there are ties, the state with a smaller index has priority.

$$h\left(s=(x,y)\right)=\sqrt{(x+2)^{2}+(y+2)^{2}}\leqslant h^{\star}\left(s\right)$$

Index	City	X-coordinate	Y-coordinate	Successors
1	Fitchburg	-1	-1	Middleton, Verona
2	Madison	0	0	Fitchburg, Middleton, Waunakee
3	Middleton	-2	0	Verona
4	Verona	-2	-2	–
5	Waunakee	0	2	Middleton

- A: Madison, Fitchburg, Verona
- B: Madison, Fitchburg, Middleton, Verona
- C: Madison, Fitchburg, Middleton, Waunakee, Verona
- D: Madison, Fitchburg, Middleton, Waunakee, Middleton, Verona
- E: Madison, Fitchburg, Middleton, Waunakee, Middleton, Fitchburg, Verona

20. Continue from the previous question. What is a state expansion sequence if Best First Greedy Search is used with the same ? Reminder: when there are ties, the state with a smaller index has priority.

$$h\left(s=(x,y)\right)=\sqrt{(x+2)^{2}+(y+2)^{2}}\leqslant h^{\star}\left(s\right)$$

- A: Madison, Fitchburg, Verona
- B: Madison, Fitchburg, Middleton, Verona
- C: Madison, Fitchburg, Middleton, Waunakee, Verona
- D: Madison, Fitchburg, Middleton, Waunakee, Middleton, Verona
- E: Madison, Fitchburg, Middleton, Waunakee, Middleton, Fitchburg, Verona

21. Continue from the previous question, how many of the following heuristic functions are admissible?

$$h_1(s) = 1 - \mathbb{1}_{\{h^\star(s)<0\}}$$
$$h_2(s) = 1 - \mathbb{1}_{\{h^\star(s)<1\}}$$
$$h_3(s) = \mathbb{1}_{\{h^\star(s)<0\}}$$
$$h_4(s) = \mathbb{1}_{\{h^\star(s)>1\}}$$
$$h_5(s) = \mathbb{1}_{\{h^\star(s)\geqslant 0\}}$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

22. Continue from the previous question, which one of these heuristic functions is dominated (among the admissible ones)?

$$h_1(s) = 1 - \mathbb{1}_{\{h^\star(s)<0\}}$$
$$h_2(s) = 1 - \mathbb{1}_{\{h^\star(s)<1\}}$$
$$h_3(s) = \mathbb{1}_{\{h^\star(s)<0\}}$$
$$h_4(s) = \mathbb{1}_{\{h^\star(s)>1\}}$$
$$h_5(s) = \mathbb{1}_{\{h^\star(s)\geqslant 0\}}$$

- A: h_1
- B: h_2
- C: h_3
- D: h_4
- E: h_5

23. Given the following tables showing the cell indices and the list of successors of a maze. The first table contains the names (indices) of the states for each cell, and the second table contains the indices of the successor states for each cells. The entrance is 1, the exit is 16. How many unique cells are visited if Uniform Cost Search is used? Count both initial and goal states.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

$2, 5$	$1, 3$	$2, 4$	$3, 8$
$1, 6, 9$	$5, 7$	$6, 8$	$4, 7, 12$
$5, 10, 13$	$9, 11$	$10, 12$	$8, 11, 16$
$9, 14$	$13, 15$	$14, 16$	$12, 15$

- A: 7
- B: 10
- C: 14
- D: 15
- E: 16

24. Continue from the previous question. How many unique cells are visited if $A^{\star}$ is used with Manhattan distance between the cell and the goal?

- A: 7
- B: 10
- C: 14
- D: 15
- E: 16

25. Starting from $x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = 0$ (all variables set to False), what is the minimum number of steps required to reach the solution for the following SAT problem if first choice hill climbing is used? The score is the number of clauses satisfied. Neighbors differ in one variable. The following express is 5 clauses connected by $\wedge$.

$$(\neg x_1 \vee x_2) \wedge (\neg x_1 \vee x_3) \wedge (\neg x_2 \vee x_4) \wedge (\neg x_2 \vee x_5) \wedge (x_1 \vee x_6) \wedge (x_2 \vee x_7)$$

- A: 2
- B: 3
- C: 4
- D: 5
- E: 6

26. Suppose the states are given by five integers from $\{0, 1\}$. The fitness function is the position of the last 1 in the sequence. $F(x_1, x_2, x_3, x_4, x_5) = \max\{t \in \{0, 1, 2, 3, 4, 5\} : x_t = 1\}$, with $x_0 = 1$.
There are in total five states. Using the genetic algorithm, the reproduction probability of first state a is $\frac{1}{5}$. What is the reproduction probability of the last state e? The reproduction probabilities are proportional to the fitness of the states.

$$\begin{aligned} x_a &= (0, 0, 1, 0, 0) \\ x_b &= (0, 1, 0, 0, 1) \\ x_c &= (1, 0, 1, 1, 0) \\ x_d &= (0, 0, 0, 0, 0) \\ x_e &= ??? \end{aligned}$$

- A: $\frac{1}{3}$
- B: $\frac{1}{4}$
- C: $\frac{1}{5}$
- D: $\frac{2}{9}$
- E: $\frac{3}{10}$

27. Consider a zero-sum sequential move game in which player MAX moves first, and MIN moves second. Each player has three actions labeled $1, 2, 4$. The value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. Alpha-Beta pruning is used. What is the number of branches (states) that can be pruned? During the search process, the actions with smaller labels are searched first. Note that a branch is pruned if $\alpha = \beta$.

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

28. Continue from the previous question. Suppose during the search process, the actions with larger labels are searched first instead (for both players). What is the number of branches (states) that can be pruned?

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

29. Continue from the previous question. Suppose during the search process, the order in which the actions are searched can be changed (the game itself cannot be changed). What is the maximum possible number of branches (states) that can be pruned?

- A: 2
- B: 3
- C: 4
- D: 5
- E: 6

30. In a two player simultaneous move game, each player picks an integer between 2 and N. Let the integers be x_1 and x_2. If $x_1 < x_2$, player 1 gets $x_1 + 2$ and player 2 gets $x_1 - 2$. If $x_1 > x_2$, player 1 gets $x_2 - 2$ and player 2 gets $x_2 + 2$. If $x_1 = x_2$, both players get x_1. Both players are MAX players. For $N = 3$, how many states (not actions, a state is a pair of actions) survive the iterated elimination of strictly dominated strategies?

   - A: 1
   - B: 2
   - C: 4
   - D: 6
   - E: 9

31. Continue from the previous question. For $N = 4$, how many states survive the iterated elimination of strictly dominated strategies?

   - A: 1
   - B: 2
   - C: 4
   - D: 6
   - E: 9

32. Continue from the previous question. For $N = 99$, how many states survive the iterated elimination of strictly dominated strategies?

   - A: $2 \cdot 2$
   - B: $97 \cdot 97$
   - C: $98 \cdot 98$
   - D: $99 \cdot 99$
   - E: $1 \cdot 1$

33. Continue from the previous question. For $N = 100$, how many states survive the iterated elimination of strictly dominated strategies?

   - A: $1 \cdot 1$
   - B: $2 \cdot 2$
   - C: $98 \cdot 98$
   - D: $99 \cdot 99$
   - E: $100 \cdot 100$

34. Consider a zero-sum sequential move game with Chance. Player MAX first chooses between actions $x_1 \in \{-1, +1\}$, then Chance chooses $x_2 \in \{-1, +1\}$, $\begin{cases} -1 & \text{with probability } \dfrac{1}{2} + \dfrac{x_1}{4} \\ +1 & \text{with probability } \dfrac{1}{2} - \dfrac{x_1}{4} \end{cases}$, and at the end, player MIN chooses between actions $x_3 \in \{-1, +1\}$. The value of the terminal states corresponding to the actions (x_1, x_2, x_3) is $(x_1 + 2 + x_3) \cdot x_2$. Not a typo: $\left(x_1 + \boxed{2} (\text{ not } x_2) + x_3\right) \cdot x_2$. What is the value of the whole game?

- A: -2.5
- B: -1.5
- C: -0.5
- D: 0.5
- E: 1.5

35. For a zero-sum simultaneous move game, the row player is the MAX player and the column player is the MIN player. Given the following game payoff table, which one of the following is a complete list of states that survive the iterated elimination of weakly dominated strategies? For this question, eliminate the weakly dominated strategies for column player first. Reminder: if two actions yield identical payoffs for a player (no matter what the other player is choosing), neither actions are weakly dominated, so do not eliminate either.

–	L	M	R
U	0	-1	1
M	1	0	1
D	1	-1	0

- A: $\{(M, M)\}$
- B: $\{(U, M), (D, M)\}$
- C: $\{(M, L), (M, R)\}$
- D: $\{(U, L), (M, M), (D, R)\}$
- E: All 9 states.

36. Continue from the previous question, but eliminate the weakly dominated strategies for row player first, which one of the following is a complete list of states that survive the iterated elimination of weakly dominated strategies?

- A: $\{(M, M)\}$
- B: $\{(U, M), (D, M)\}$
- C: $\{(M, L), (M, R)\}$
- D: $\{(U, L), (M, M), (D, R)\}$
- E: All 9 states.

37. Given the following game payoff table, what is the complete set of pure strategy Nash equilibria? Both players are MAX players. In each entry of the table, the first number is the reward to the row player and the second number is the reward to the column player.

−	L	R
U	$(3, 1)$	$(0, 0)$
D	$(0, 1)$	$(1, 1)$

- A: $\{(U, L)\}$
- B: $\{(D, R)\}$
- C: $\{(U, L), (D, R)\}$
- D: $\{(U, L), (U, R), (D, R)\}$
- E: $\{(U, L), (D, L), (D, R)\}$

38. Continue from the previous question, how many of the following are mixed strategy Nash equilibria of the game?

$$\left(\text{always U}, \left(\text{L } \frac{1}{2} \text{ of the time}, \text{ R } \frac{1}{2} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{1}{2} \text{ of the time}, \text{ R } \frac{1}{2} \text{ of the time}\right)\right)$$
$$\left(\left(\text{U } \frac{1}{2} \text{ of the time}, \text{ D } \frac{1}{2} \text{ of the time}\right), \text{always L}\right)$$
$$\left(\left(\text{U } \frac{1}{2} \text{ of the time}, \text{ D } \frac{1}{2} \text{ of the time}\right), \text{always R}\right)$$
$$\left(\left(\text{U } \frac{1}{2} \text{ of the time}, \text{ D } \frac{1}{2} \text{ of the time}\right), \left(\text{L } \frac{1}{2} \text{ of the time}, \text{ R } \frac{1}{2} \text{ of the time}\right)\right)$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

–	L	R
U	$(3,1)$	$(0,0)$
D	$(0,1)$	$(1,1)$

39. Continue from the previous question, how many of the following are mixed strategy Nash equilibria of the game?

$$\left(\text{always U}, \left(\text{L } \frac{1}{4} \text{ of the time}, \text{ R } \frac{3}{4} \text{ of the time}\right)\right)$$
$$\left(\text{always U}, \left(\text{L } \frac{3}{4} \text{ of the time}, \text{ R } \frac{1}{4} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{1}{4} \text{ of the time}, \text{ R } \frac{3}{4} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{3}{4} \text{ of the time}, \text{ R } \frac{1}{4} \text{ of the time}\right)\right)$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

40. Continue from the previous question, how many of the following are mixed strategy Nash equilibria of the game?

$$\left(\text{always D}, \left(\text{L } \frac{1}{8} \text{ of the time}, \text{ R } \frac{7}{8} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{3}{8} \text{ of the time}, \text{ R } \frac{5}{8} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{5}{8} \text{ of the time}, \text{ R } \frac{3}{8} \text{ of the time}\right)\right)$$
$$\left(\text{always D}, \left(\text{L } \frac{7}{8} \text{ of the time}, \text{ R } \frac{1}{8} \text{ of the time}\right)\right)$$

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

Questions 41 to 43 on next page.

41. Calculator?

    - A: Yes.
    - B: No.

42. The number of pages of additional notes? Please submit them at the end of the exam.

    - A: 0
    - B: 1
    - C: 2
    - D: 3
    - E: 4 or more.

43. Bonus mark: You received one free point for this question and you have two choices: donate 1 and keep 0 or keep 1 and donate 0. Your final grade is the points you keep plus twice the average donation (sum of the donations from everyone in your section divided by the number of people in your section).

- A: Donate the point (receive $2\cdot$ average donation).
- B: Keep the point (receive $1 + 2\cdot$ average donation).
- C, D, E (or left blank): Not participate in the game (receive 0 for this question).

$$\text{average donation} = \frac{\text{number of people in your section who chose A}}{\text{number of people in your section who chose A or B}}$$