Final Formula Sheet

CS540

August 19, 2019

1 Formulas

- Euclidean Distance: $\rho_2(x_i, x_{i'}) = \sqrt{\sum_{j=1}^{m} (x_{ij} - x_{i'j})^2}$
- Manhattan Distance: $\rho_1(x_i, x_{i'}) = \sum_{j=1}^{m} |x_{ij} - x_{i'j}|$
- Chebyshev Distance: $\rho_\infty(x_i, x_{i'}) = \max_{j=1,2,\ldots,m} \{|x_{ij} - x_{i'j}|\}$
- Single Linkage Distance: $\rho(C_k, C_{k'}) = \min\{\rho(x_i, x_{i'}) : x_i \in C_k, x_{i'} \in C_{k'}\}$
- Complete Linkage Distance: $\rho(C_k, C_{k'}) = \max\{\rho(x_i, x_{i'}) : x_i \in C_k, x_{i'} \in C_{k'}\}$
- Length of a vector (in Euclidean space): length of $x = (x_1, x_2, ..., x_m)$ is $\|x\|_2 = \sqrt{x^T x} = \sqrt{x_1^2 + x_2^2 + ... + x_m^2}$
- Projection onto any vector v_k : $\text{proj}_{v_k} x_i = \left(\frac{v_k^T x_i}{v_k^T v_k}\right) v_k$, and its length is $\|\text{proj}_{v_k} x_i\|_2 = \frac{v_k^T x_i}{\sqrt{v_k^T v_k}}$
- Projection onto a unit vector: first $u_k = \frac{v_k}{\sqrt{v_k^T v_k}}$ to convert v_k into a unit vector u_k and $\text{proj}_{u_k} x_i = u_k^T x_i u_k$, and its length is $\|\text{proj}_{u_k} x_i\|_2 = u_k^T x_i$
- Sample Variance (MLE): $\hat{\Sigma} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\mu})(x_i - \hat{\mu})^T$, with $\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$
- PCA Max Variance Formulation: $\max_{u_k} u_k^T \hat{\Sigma} u_k$ such that $u_k^T u_k = 1$
- PCA-reconstruction: $x_i = \sum_{k=1}^{m} (u_k^T x_i) u_k \approx \sum_{k=1}^{K} (u_k^T x_i) u_k$
- Discounted Expected Reward (Cost): $\mathbb{E}[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ...], \gamma \in [0,1], \mathbb{E}[r_t] = \sum_{r_t \in R} r_t \mathbb{P}\{r_t | s_{t-1}, a_{t-1}\}$
- Logic Notations: x_i or $x_j : x_i \vee x_j, x_i$ and $x_j : x_i \wedge x_j$, not $x_i : \neg x_i$
- Simulated Annealing Update Probability: if $f(s')$ is worse than $f(s_i)$ then $p_i = \exp\left(-\frac{|f(s') - f(s_i)|}{\text{Temp}}\right)$.

e.g. if the f is the cost (minimization problem), then $s_{t+1} = \begin{cases} s' & \text{if } f(s') < f(s_t)\,,\ \text{else} \\ s' & \text{with probability } \exp\left(-\dfrac{|f(s') - f(s_t)|}{\text{Temp }(t)}\right) \\ s_t & \text{otherwise} \end{cases}$

e.g. if the f is the reward (score) (maximization problem), then $s_{t+1} = \begin{cases} s' & \text{if } f(s') > f(s_t)\,,\ \text{else} \\ s' & \text{with probability } \exp\left(-\dfrac{|f(s') - f(s_t)|}{\text{Temp }(t)}\right) \\ s_t & \text{otherwise} \end{cases}$

- Genetic Algorithm Reproduction Probability: if F is fitness then $p_i = \dfrac{F(s_i)}{\sum_{j=1}^{N} F(s_j)}$
- Alpha Beta Pruning: prune if $\alpha \geqslant \beta$ where $\alpha(s) = \max_{s' \in s'(s)} \beta(s')\,, \beta(s) = \min_{s' \in s'(s)} \alpha(s')$
- Geometric Sum: $1 + x + x^2 + ... + x^n = \dfrac{1 - x^n}{1 - x}, x \in (0, 1)$ and $1 + x + x^2 + ... = \dfrac{1}{1 - x}$
- Arithmetic Sum: $1 + 2 + 3 + ... + n = \dfrac{(1 + n) \cdot n}{2}$