

CS540 Introduction to Artificial Intelligence

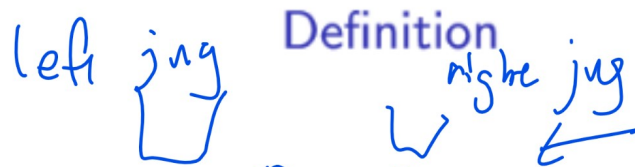
Lecture 15

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

July 8, 2020

Water Jugs Example

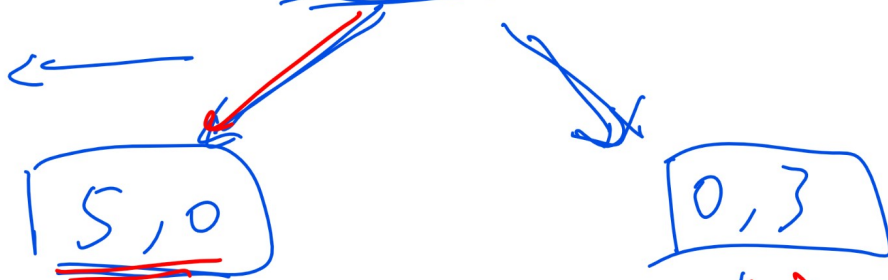


0, 0

initial state

State

Successor function based on rule of jugs

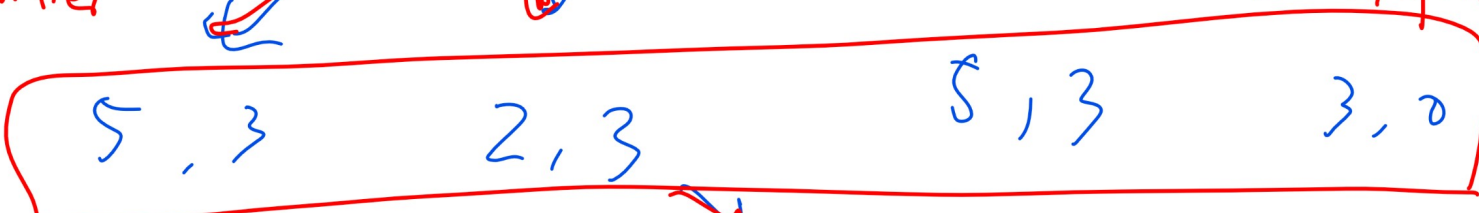


Search tree

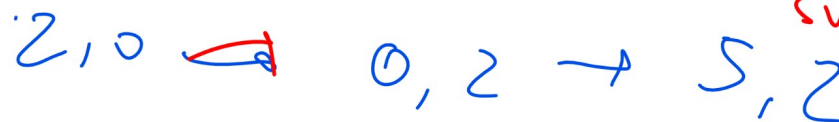
frontier

expand

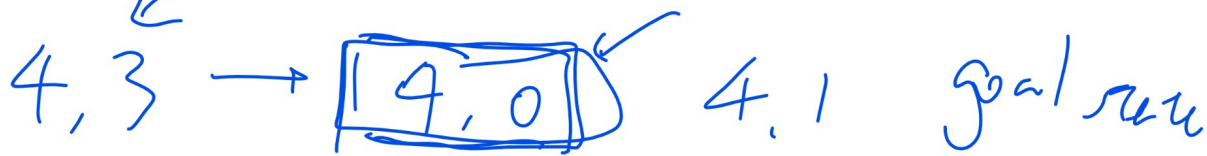
find all successors.



BFS



DFS



Performance

Definition

- A search strategy is complete if it finds at least one solution.
- A search strategy is optimal if it finds the optimal solution.
- For uninformed search, the costs are assumed to be 1 for all edges $c = 1$.

Complexity

Definition

- The time complexity of a search strategy is the worst case maximum number of vertices expanded.
- The space complexity of a search strategy is the worst case maximum number of states stored in the frontier at a single time.
- Notation: the goals are d edges away from the initial state. This means assuming a constant cost of 1, the optimal solution has cost d . The maximum depth of the graph is D .
- Notation: the branching factor is b , the maximum number of actions associated with a state.

$$b = \max_{s \in V} |s'(s)|$$

Search Tree Diagram

Definition

max # branches = b .

initial

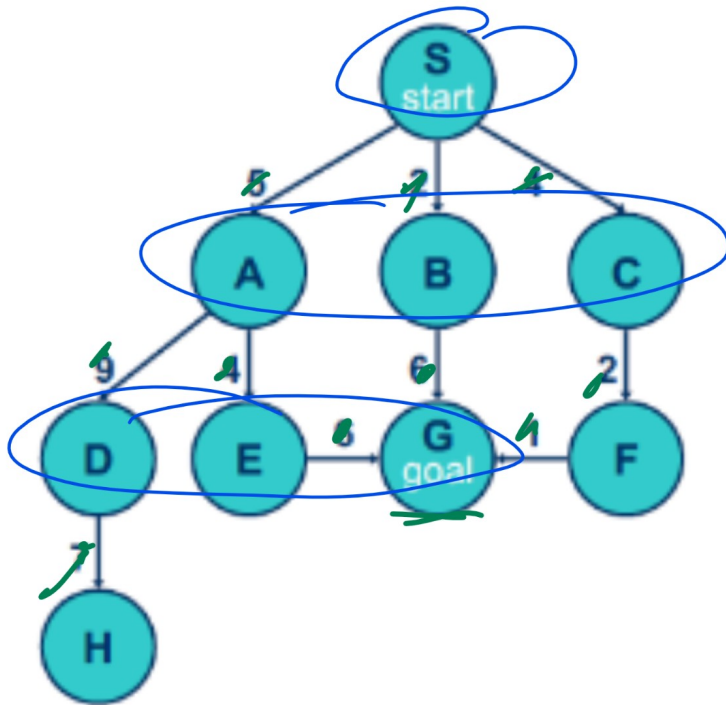
goal

d



BFS Simple Example

Quiz



Queue: ~~S~~, ~~A~~, ~~B~~, ~~C~~, ~~D~~, ~~E~~,

de Q from the queue
en Q successors.

~~S~~, F
H, G,

Stop when de Q G

expansion path: S, A, B, C, D, E, G

BFS Example 3

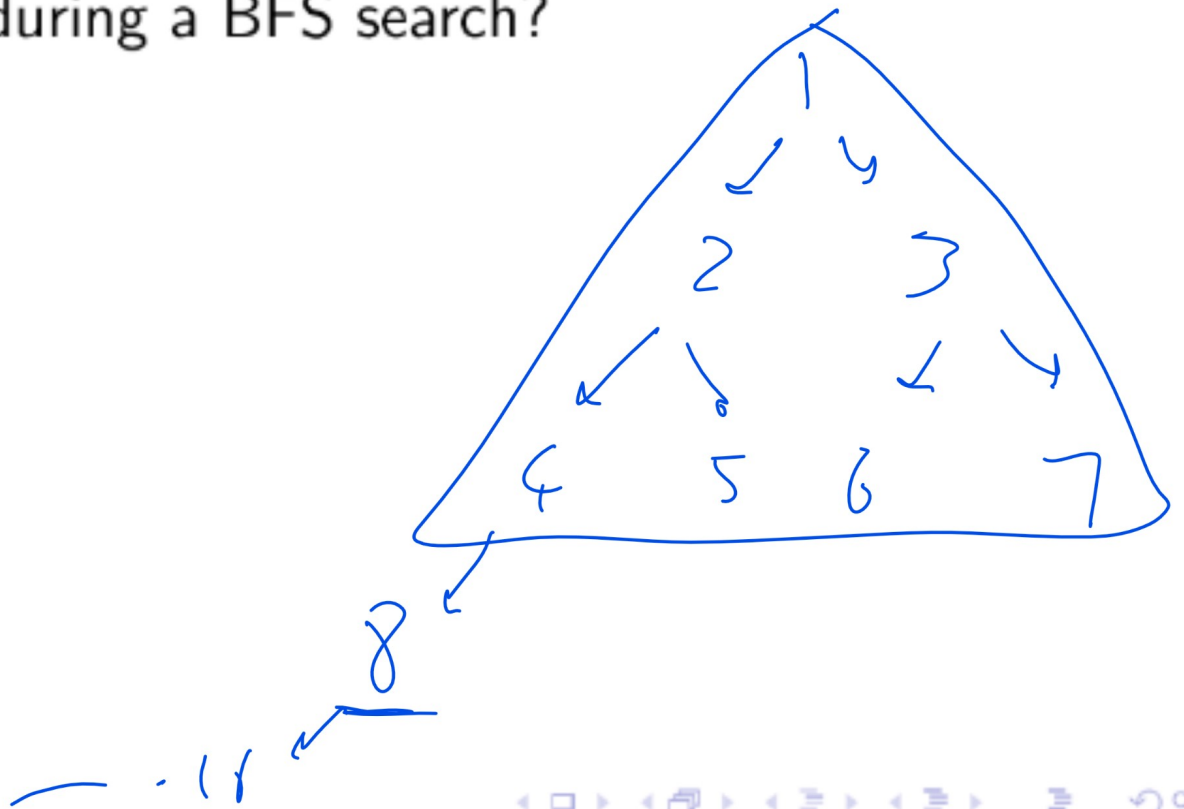
Quiz

$$2^3 = 8$$

Q4

• Suppose the states are integers between 1 and $2^{10} = \underline{1024}$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12
- ~~C~~: 1023
- ~~D~~: 1024



E

BFS Example 4

Quiz

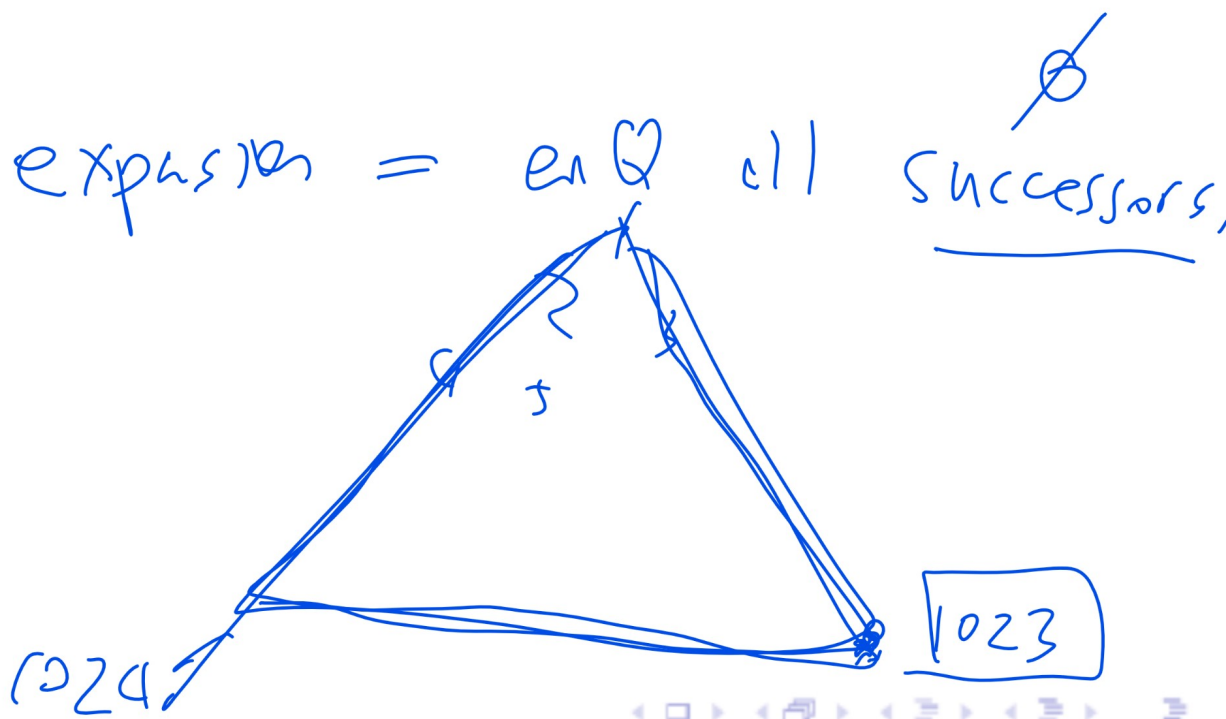
Q5 (last)

D }



• Suppose the states are integers between 1 and $2^{10} - 1 = \underline{1023}$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- E: 1024



Breadth First Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- EnQueue initial states.

$$Q = I$$

- While Q is not empty and goal is not deQueued, deQueue Q and enQueue its successors.

$$s = Q_0$$

$$Q = Q + s'(s)$$

Breadth First Search Performance

Discussion

- BFS is complete.
- BFS is optimal with $c = 1$.

Breadth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = b + b^2 + \dots + b^d$$

- Space complexity: the worst case is storing all vertices at depth d is in the frontier.

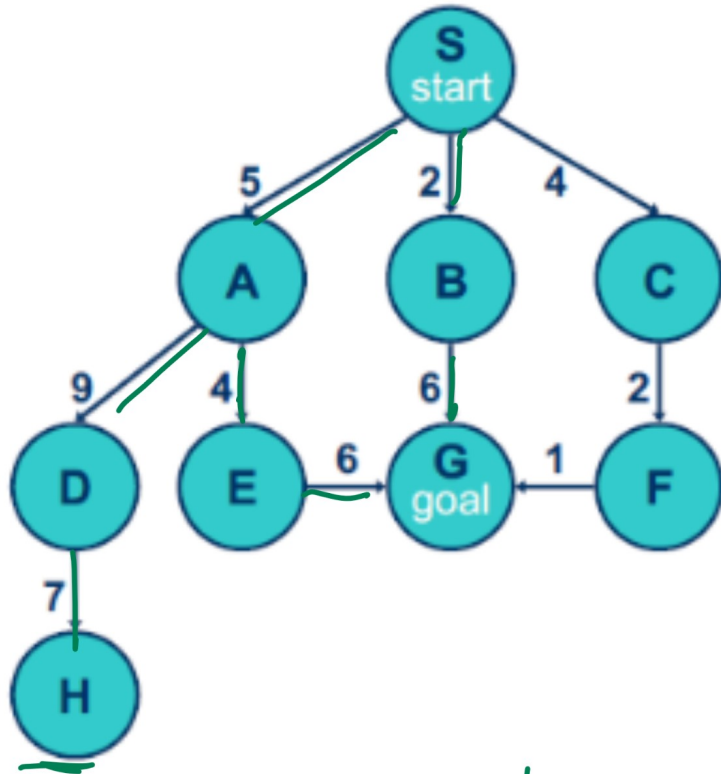
$$S = b^d$$

Depth First Search

Description

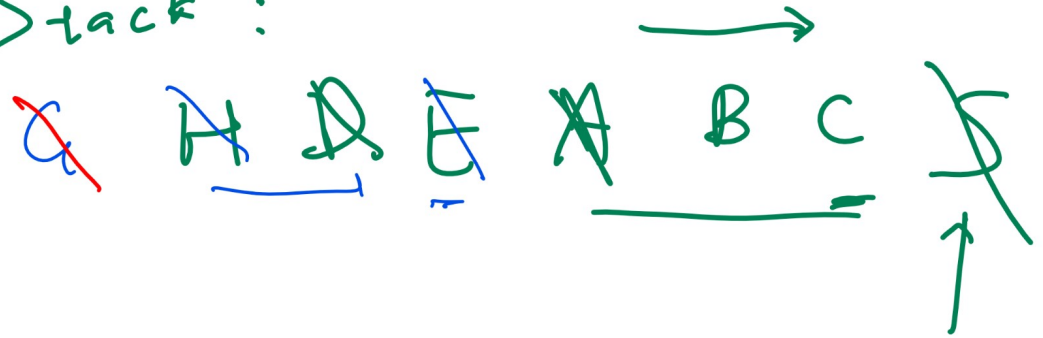
- Use Stack (LIFO) for the frontier.
- Remove from the front, add to the front.

DFS Simple Example



expansion path:

Quiz
Stack:



note
BFS

~~S~~ A B C

Stop when pop G

S, A, D, H, E, G

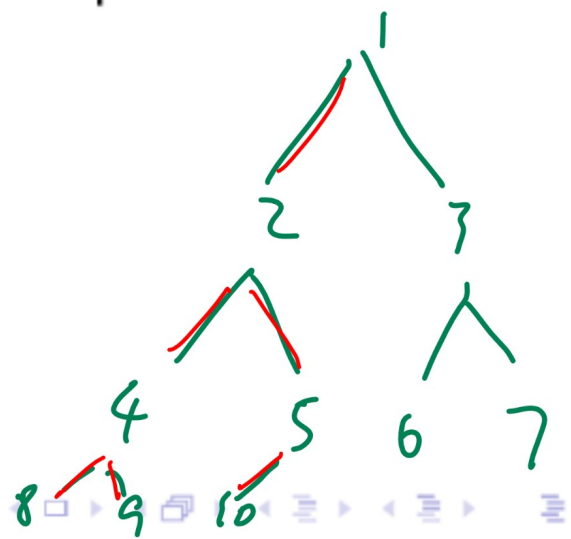
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DFS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a DFS expansion sequence?

1, 2, 4, 8, 9, 5, 10



DFS Example 1 Diagram

Quiz

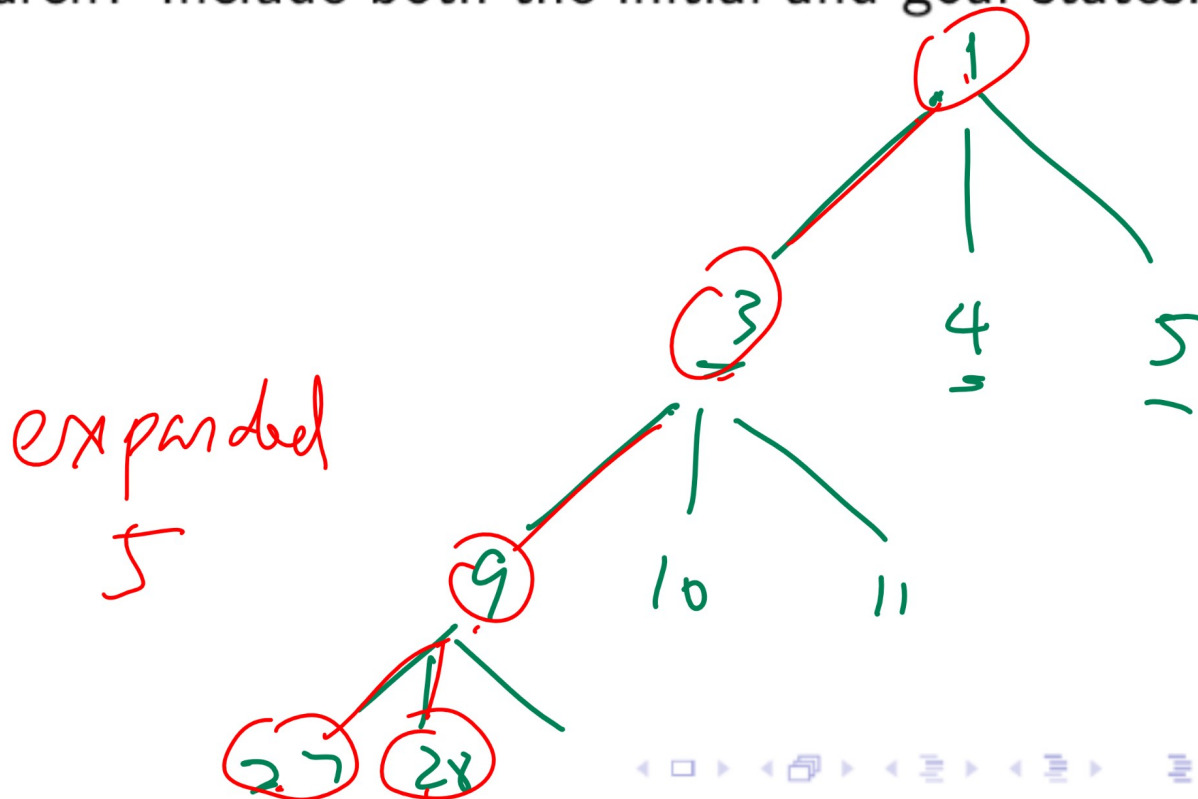
DFS Example 2

Quiz

Q 1

- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$ if exist. How many states are expanded during a DFS search? Include both the initial and goal states.

- A: 4
- B: 5
- C: 14
- D: 15
- E: 28



DFS Example 3

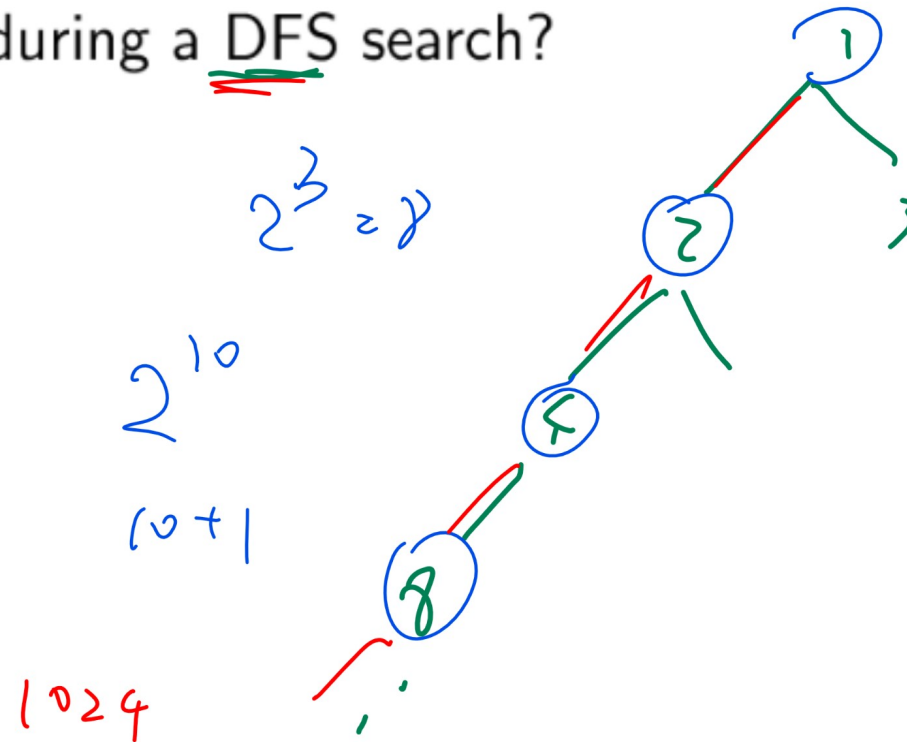
Quiz

Q2

Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- E: 1024

D
E



DFS Example 4

Quiz

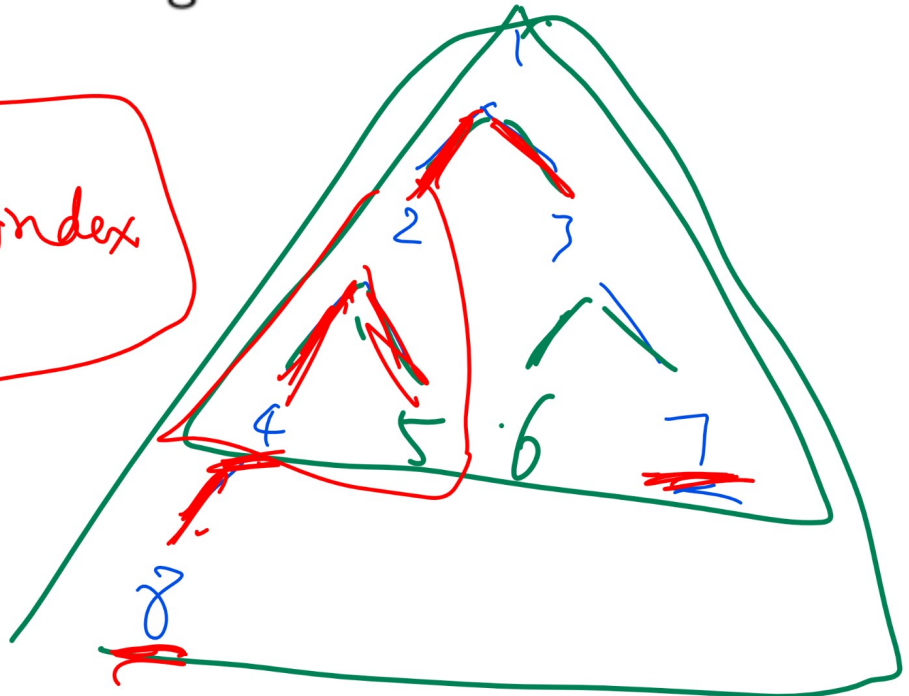
Q3

put on final

- Suppose the states are integers between 1 and $2^{10} - 1 = 1023$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- E: 1024

push nodes with larger index first



$2^4 - 1$

if 1024 is a state,

Depth First Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- Push initial states.

$$S = I$$

- While S is not empty and goal is not popped, pop S and push its successors.

$$s = S_0$$

$$S = s'(s) + S$$

Depth First Search Performance

Discussion

- DFS is incomplete if $D = \infty$.
- DFS is not optimal.

Depth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the root of the last subtree expanded in the whole graph.

$$T = b^{D-d+1} \dots + b^{D-1} + b^D$$


- Space complexity: the worst case is storing all vertices sharing the parents with vertices in the current path.

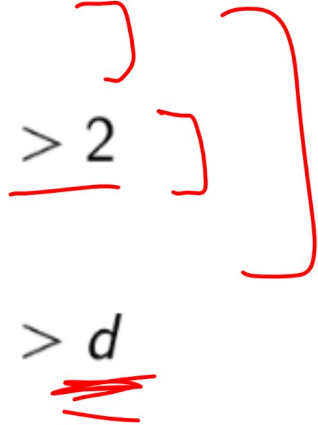
$$S = (b - 1) D + 1$$

Iterative Deepening Search

Description

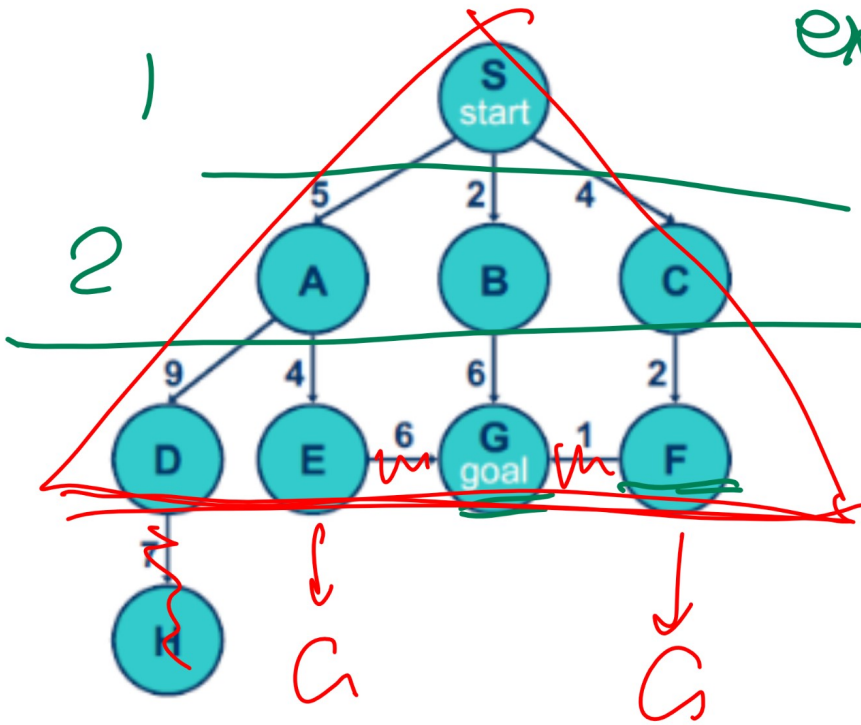
} BFS slow
} DFS bad



- DFS but stop if path length > 1
 - repeat DFS but stop if path length > 2
 - ...
 - repeat DFS but stop if path length > d
- 

IDS Simple Example

Quiz



expansion path DFS level 2

DFS on ~~S~~, S, A, B, C
or limited or level 1, 2

S, A, D, E, B, G
DFS level 3

C is put on stack, not expanded.

IDS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a IDS expansion sequence?

Same.

IDS Example 1 Diagram

Quiz

IDS Example 2

Quiz

- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many unique states are expanded during a IDS search? Include both the initial and goal states.
- A: 4
- B: 5
- C: 14
- D: 15
- E: 28

~~BFS~~ Example 3

IDS Quiz

Q4

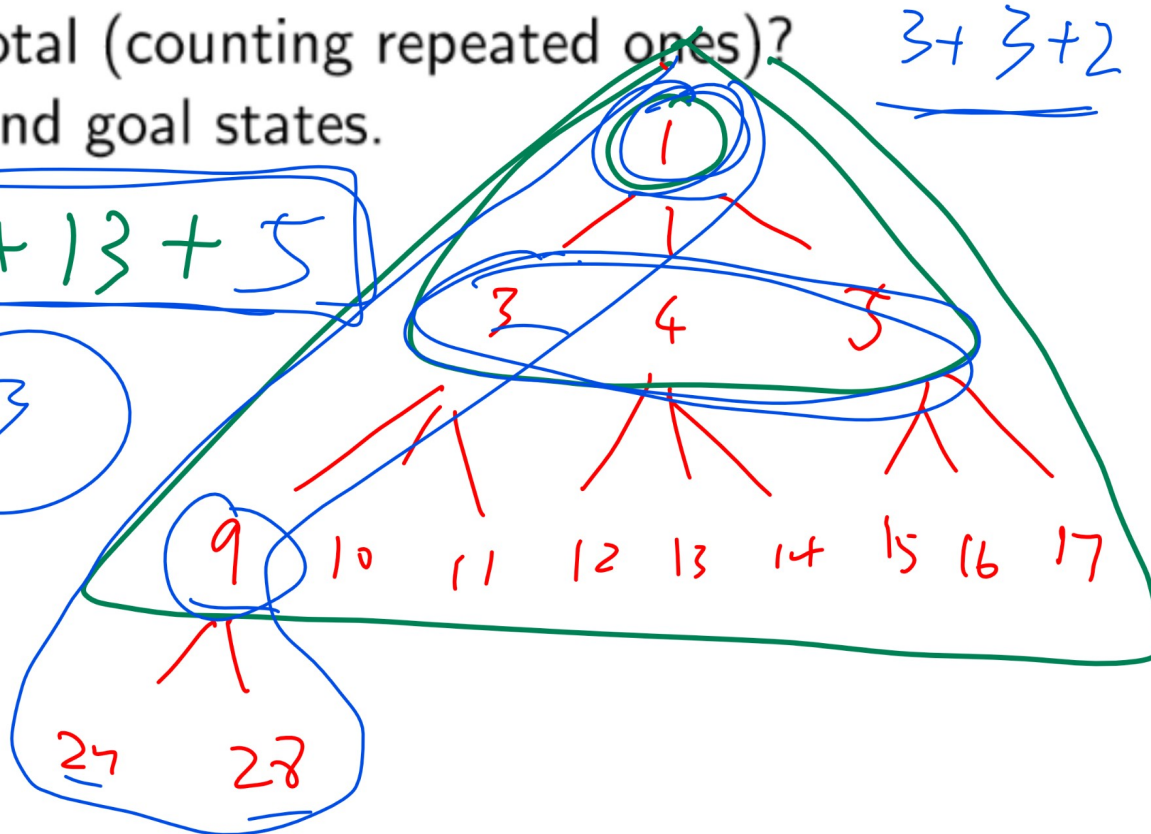
- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a IDS search in total (counting repeated ones)? Include both the initial and goal states.

$3 + 3 + 2$

- A: 15 + 8
- B: 15 + 9
- C: 15 + 13
- D: 15 + 17
- E: 15 + 18

$1 + 4 + 13 + 5$

23



Iterative Deepening Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- Perform DFS on the digraph restricted to vertices with depth ≤ 1 from the initial state.
- Perform DFS on the digraph restricted to vertices with depth ≤ 2 from the initial state.
- Repeat until the goal is deQueued.

Iterative Deepening Search Performance

Discussion

- IDS is complete.
- IDS is optimal with $c = 1$.

Iterative Deepening Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = db + (d - 1) b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- Space complexity: it has the same space complexity as DFS.

$$S = (b - 1) d$$

Configuration Space

Discussion