

CS540 Introduction to Artificial Intelligence

Lecture 15

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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Bridge and Torch Game 1

Motivation

Q 1

- Four people with one flashlight (torch) want to go across a river. The bridge can hold two people at a time, and they must cross with the flashlight. The time it takes for each person to cross the river:

A	B	C	D
1	2	3	4

- What is the minimum total time required for everyone to cross the river?

- A: 10, B: 11, C: 12, D: 13, E: 14

AB → 2

A ← 1

AC → 3

A ← 1

AD → 4

Bridge and Torch Game 2

Motivation

Q2

- Four people with one flashlight (torch) want to go across a river. The bridge can hold two people at a time, and they must cross with the flashlight. The time it takes for each person to cross the river:

A	B	C	D
1	2	4	5

- What is the minimum total time required for everyone to cross the river?

- A: 10, B: 11, C: 12, D: 13, E: 14

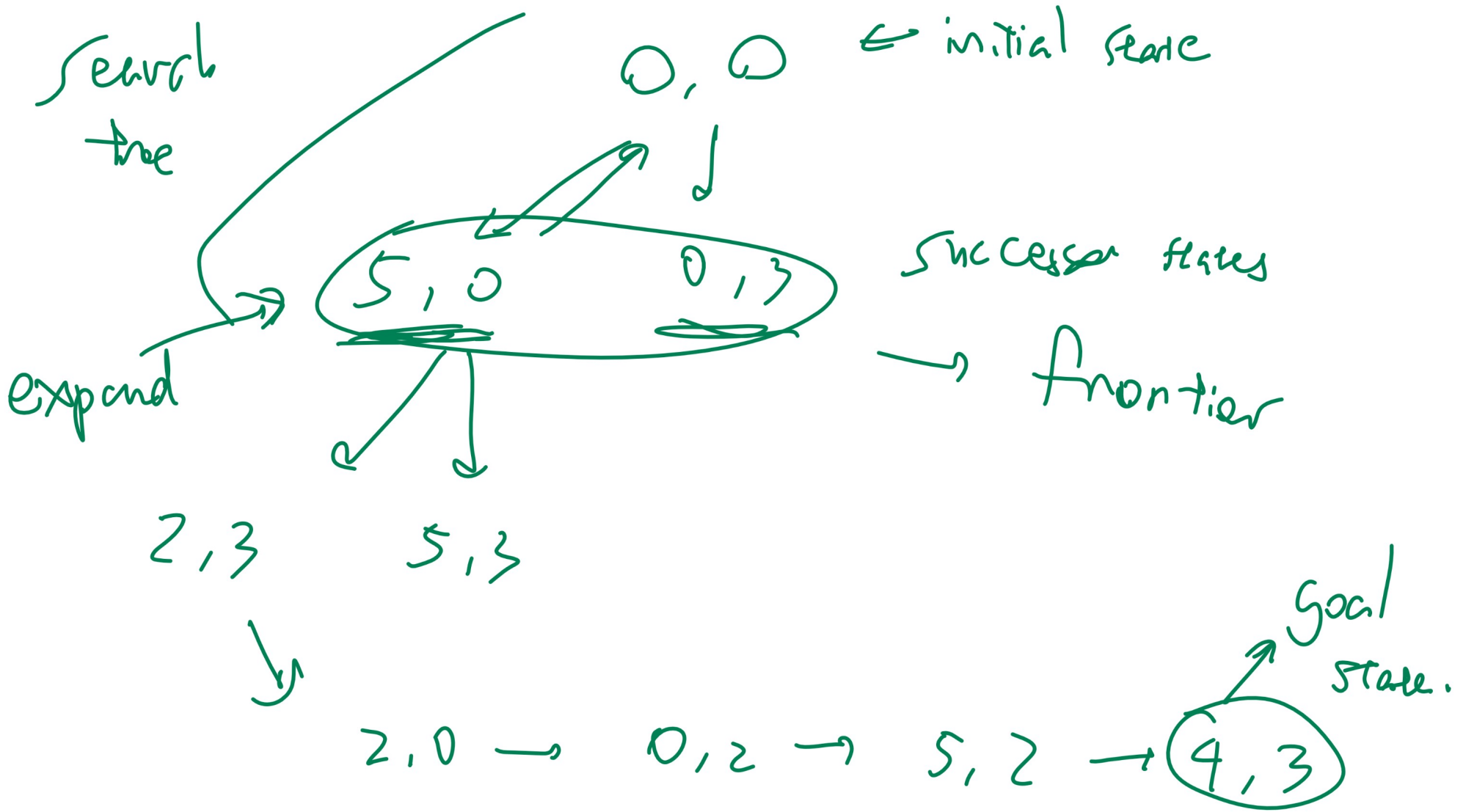
$$2 + 1 + 4 + 1 + 5 = \cancel{13}$$

AB → 2

AB → 2
A ← 1
CD → 5
B ← 2

Water Jugs Example

Definition



Performance

Definition

- A search strategy is complete if it finds at least one solution.
- A search strategy is optimal if it finds the optimal solution.
- For uninformed search, the costs are assumed to be 1 for all edges $c = 1$.

Complexity

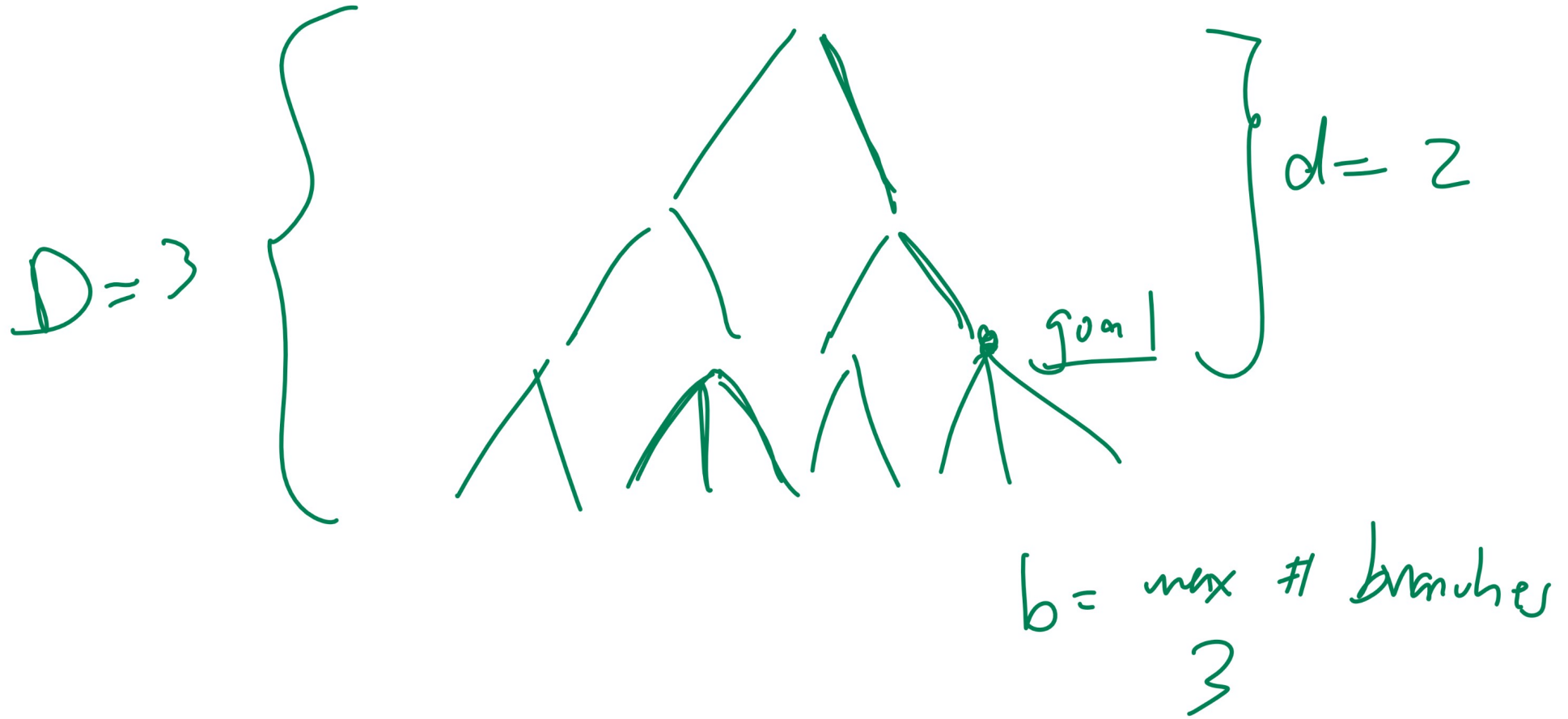
Definition

- The time complexity of a search strategy is the worst case maximum number of vertices expanded.
- The space complexity of a search strategy is the worst case maximum number of states stored in the frontier at a single time.
- Notation: the goals are d edges away from the initial state. This means assuming a constant cost of 1, the optimal solution has cost d . The maximum depth of the graph is D .
- Notation: the branching factor is b , the maximum number of actions associated with a state.

$$b = \max_{s \in V} |s'(s)|$$

Search Tree Diagram

Definition



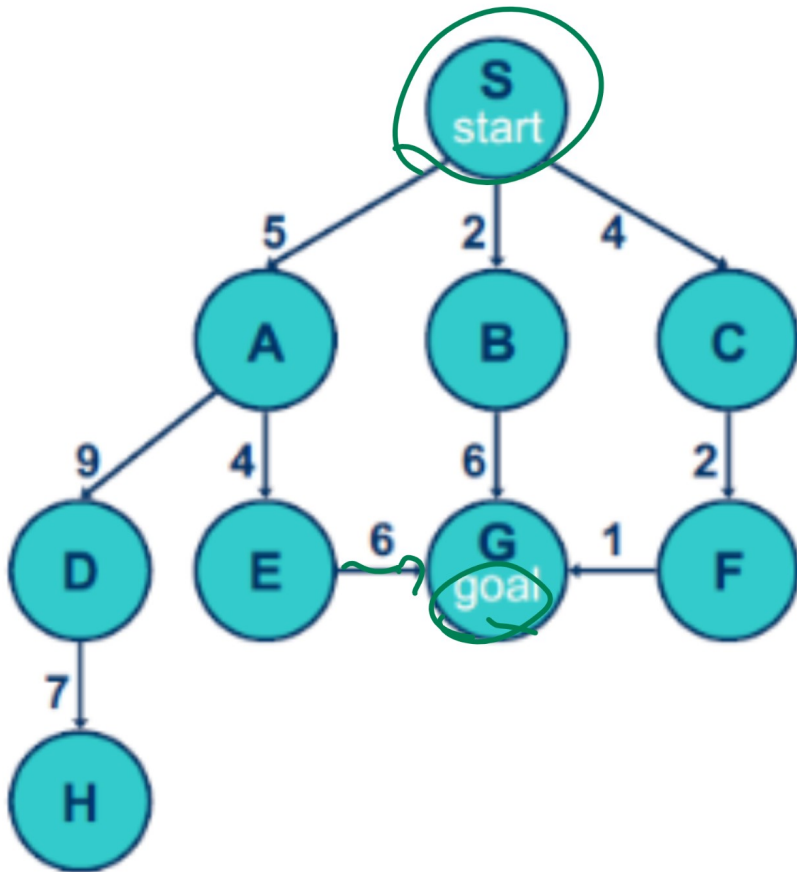
Breadth First Search

Description

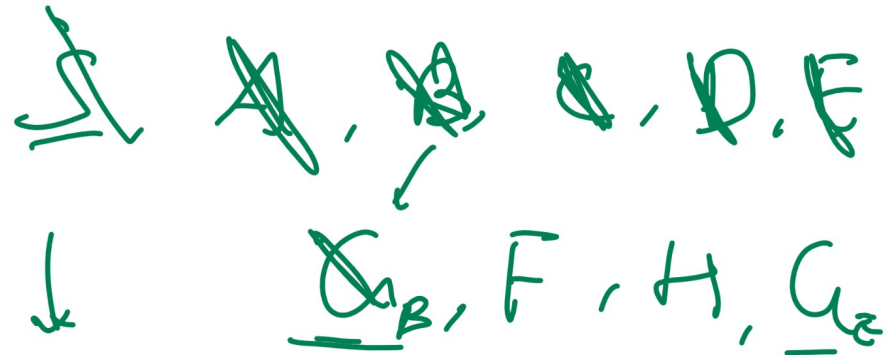
- Use Queue (FIFO) for the frontier.
- Remove from the front, add to the back.

BFS Simple Example

Quiz



Queue:



de Queue ⇒ expand this
 ↓ en Queue successors
 Stop when de Queue G (goal)
 Goal Check

list of expanded nodes:

S, A, B, C, D, E, G

solution: S, B, G

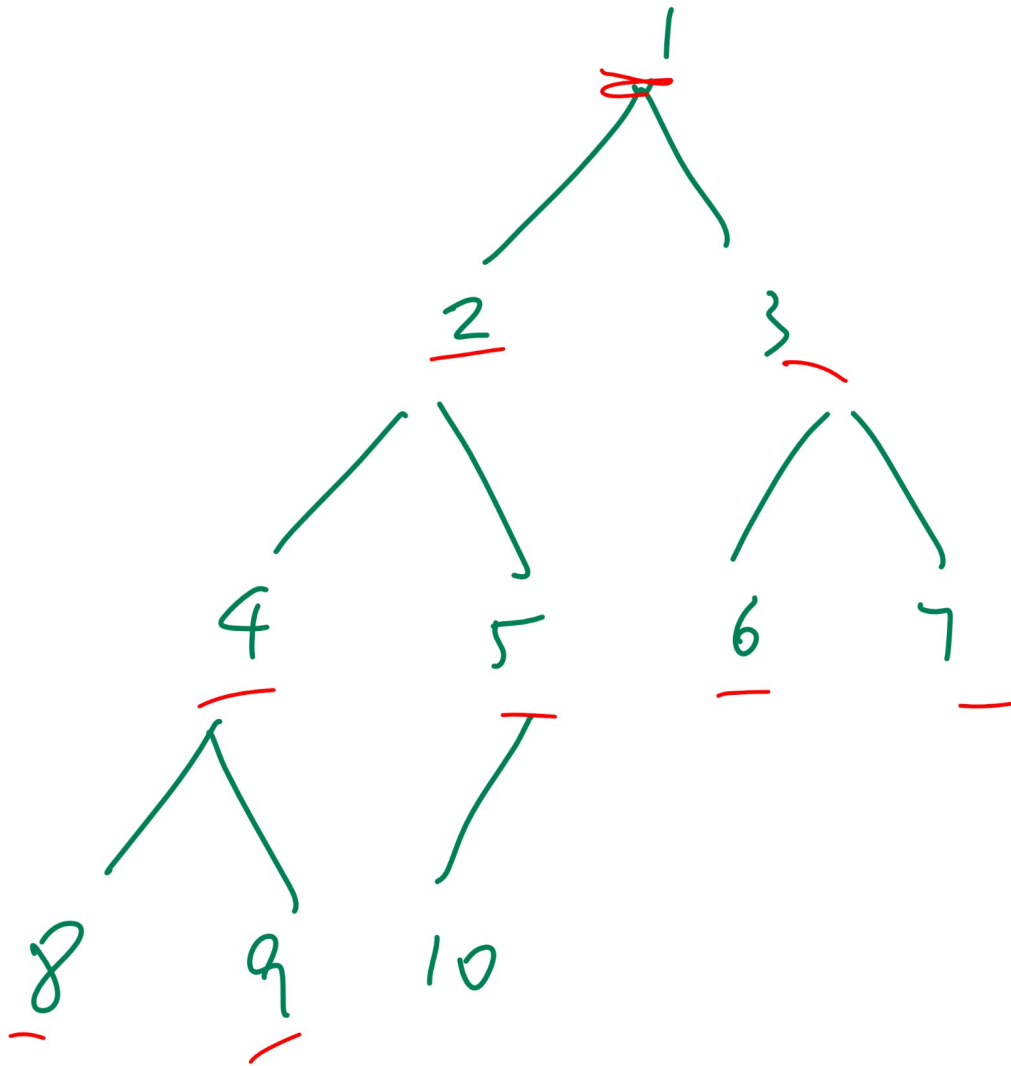
BFS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a BFS expansion sequence?

BFS Example 1 Diagram

Quiz



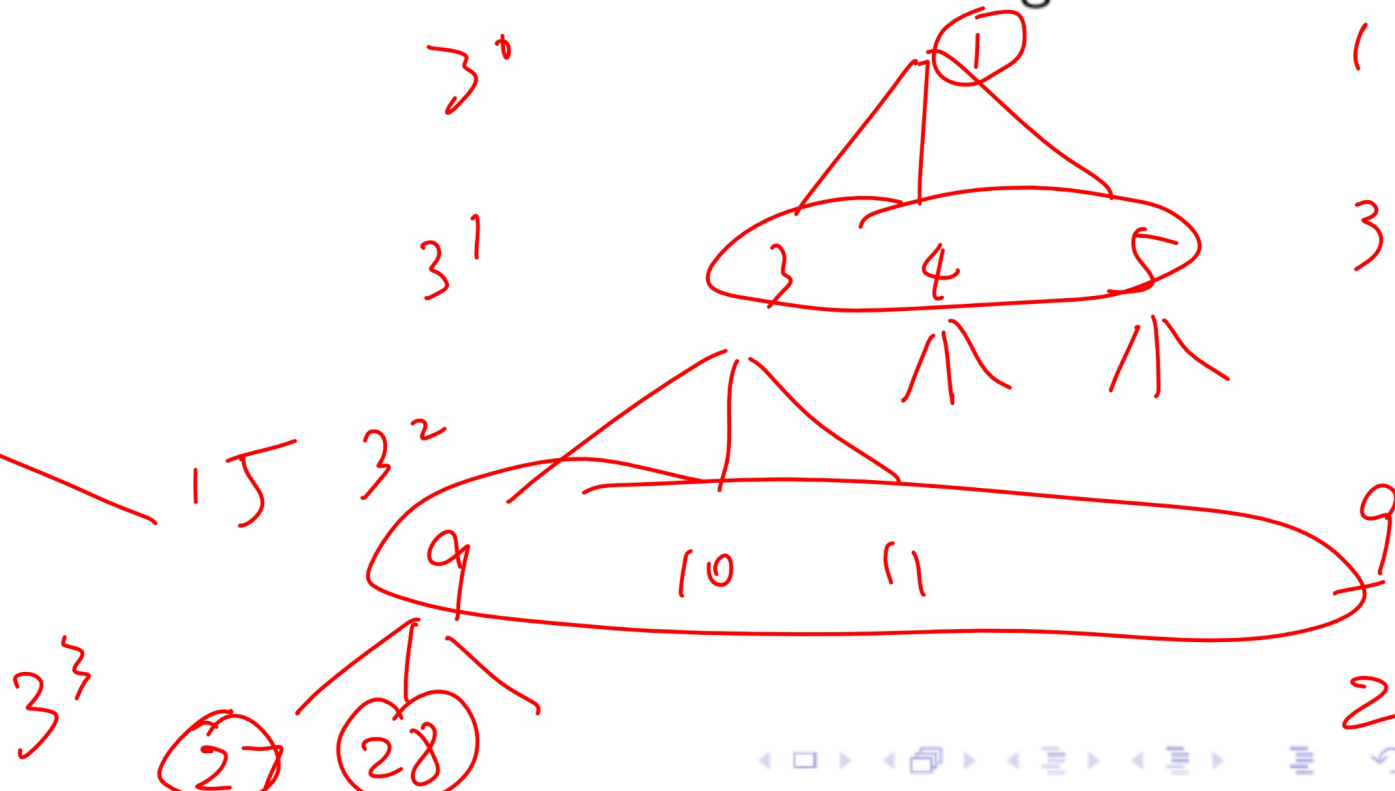
Expansion
sequence:
1, 2, 3,
4, 5, 6, 7, 8, 9
↓
Goal

BFS Example 2

Quiz

Q3

- Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a BFS search? Include both the initial and goal states.
- A: 4
- B: 5
- C: 14
- **D: 15**
- E: 28



BFS Example 3

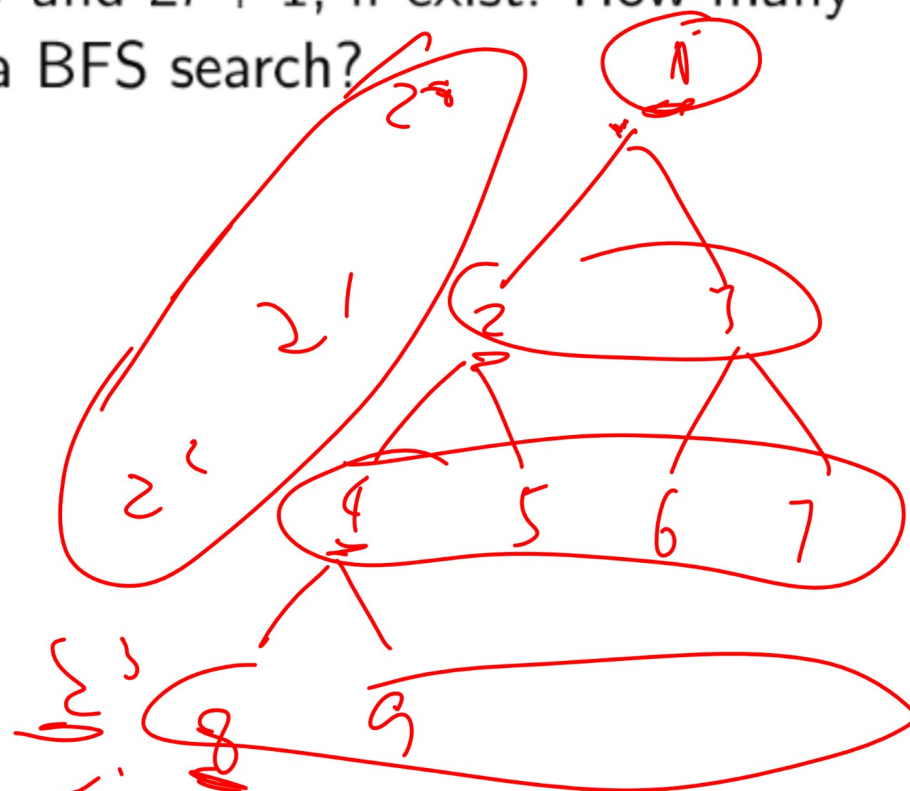
Quiz

goal check
= # expanded.

Q4

• Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- **E: 1024**



1024

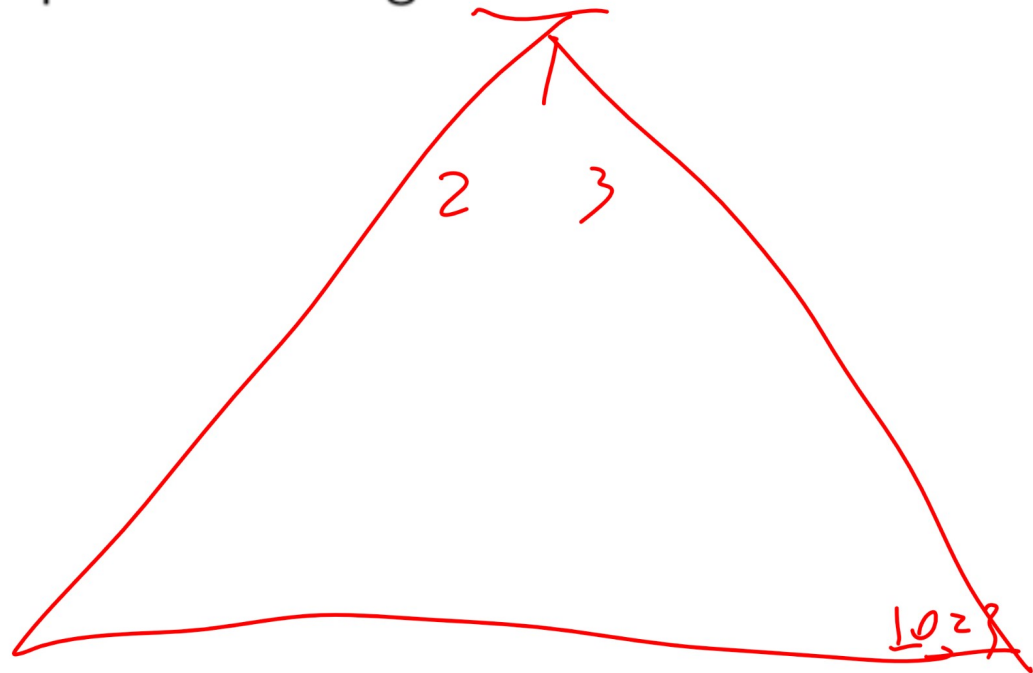
BFS Example 4

Quiz

Q5

• Suppose the states are integers between 1 and $2^{10} - 1 = 1023$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12
- **D: 1023**
- E: 1024



Breadth First Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- EnQueue initial states.

$$Q = I$$

- While Q is not empty and goal is not deQueued, deQueue Q and enQueue its successors.

$$s = Q_0$$

$$Q = Q + s'(s)$$

Breadth First Search Performance

Discussion

- BFS is complete.
- BFS is optimal with $c = 1$.

Breadth First Search Complexity

Discussion

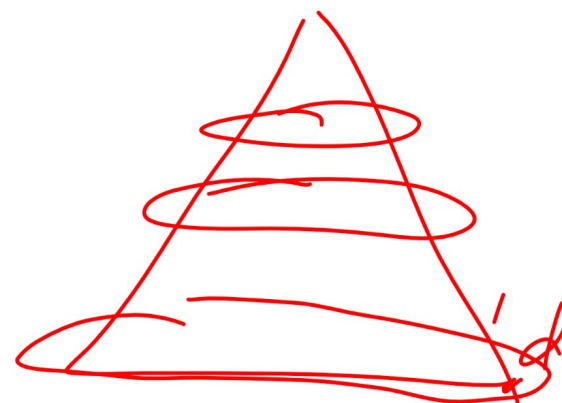
- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = b + b^2 + \dots + b^d$$

17

- Space complexity: the worst case is storing all vertices at depth d is in the frontier.

$$S = \underline{b^d}$$



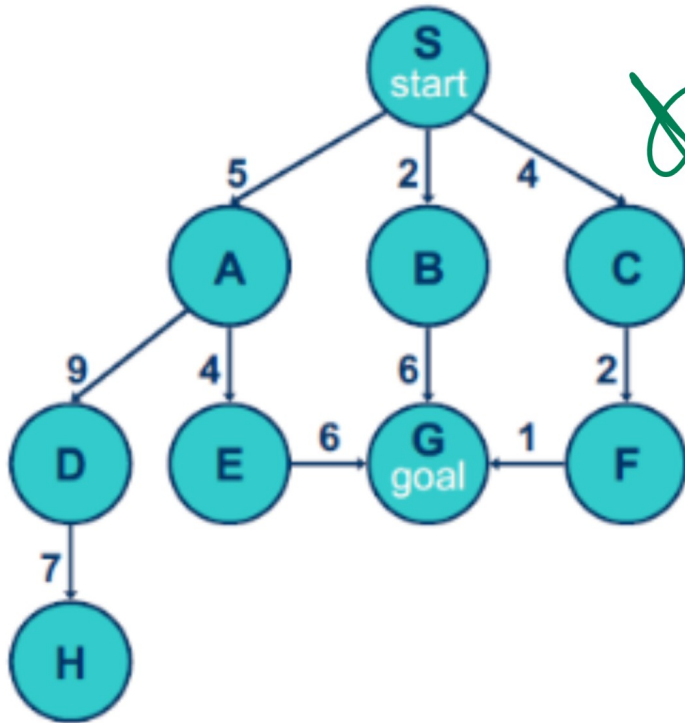
Depth First Search

Description

- Use Stack (LIFO) for the frontier.
- Remove from the front, add to the front.

DFS Simple Example

Quiz



Stack

~~C~~
~~H~~, ~~D~~, ~~A~~

~~A~~, B, C, \$

goal
)

expansion path: S, A, D, H, E, G_E

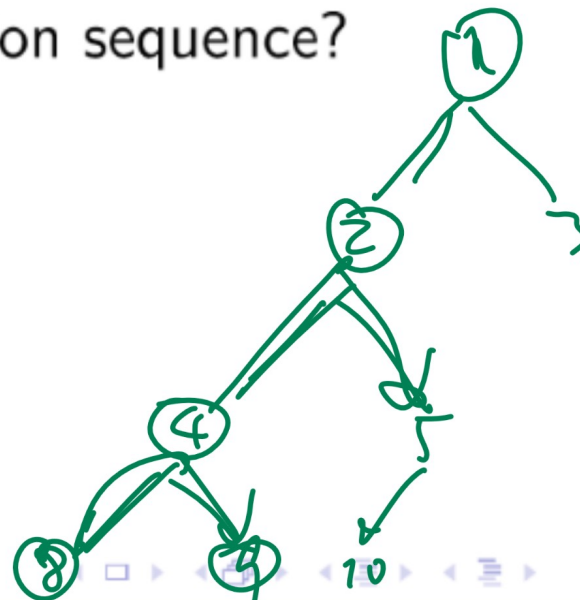
Solution: S, A, E, G

DFS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a DFS expansion sequence?

1, 2, 4, 8, 9



DFS Example 1 Diagram

Quiz

DFS Example 2

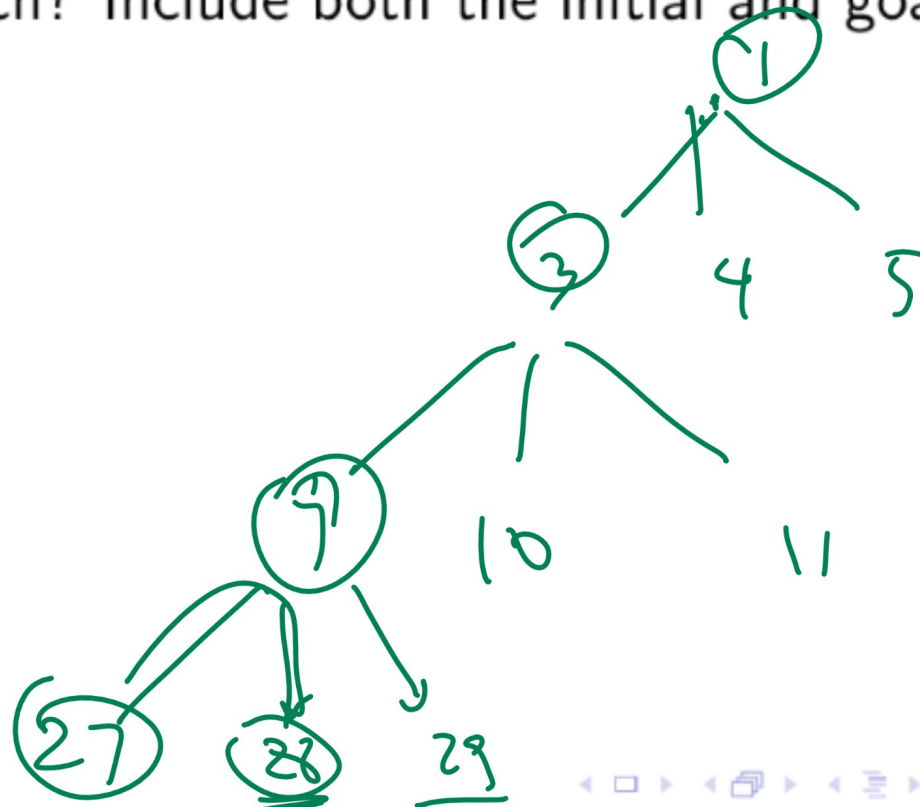
Q6

PS



Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a DFS search? Include both the initial and goal states.

- A: 4
- B: 5
- C: 14
- D: 15
- E: 28



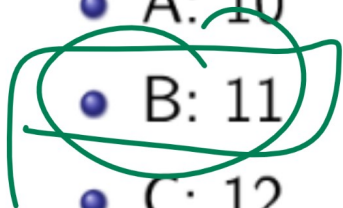
DFS Example 3

Quiz

Q7

• Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- E: 1024



DFS Example 4

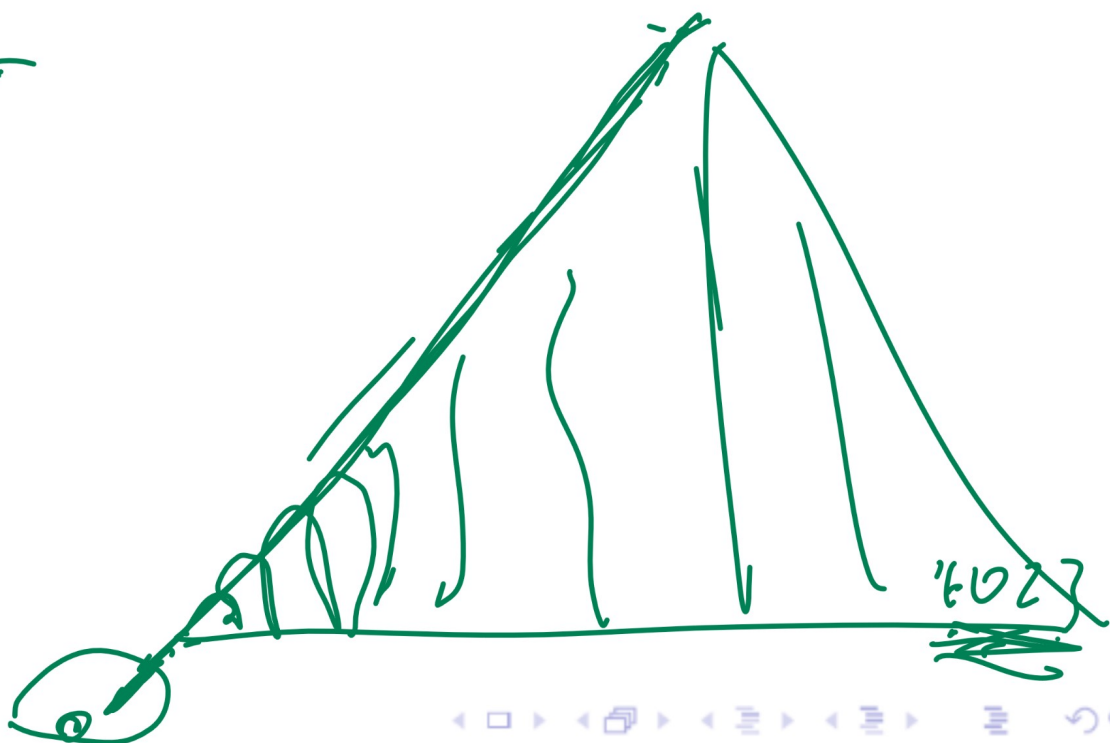
Quiz

Q8

• Suppose the states are integers between 1 and $2^{10} - 1 = \text{1023}$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?

- A: 10
- B: 11
- C: 12
- **D: 1023**
- E: 1024

1024



Depth First Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- Push initial states.

$$S = I$$

- While S is not empty and goal is not popped, pop S and push its successors.

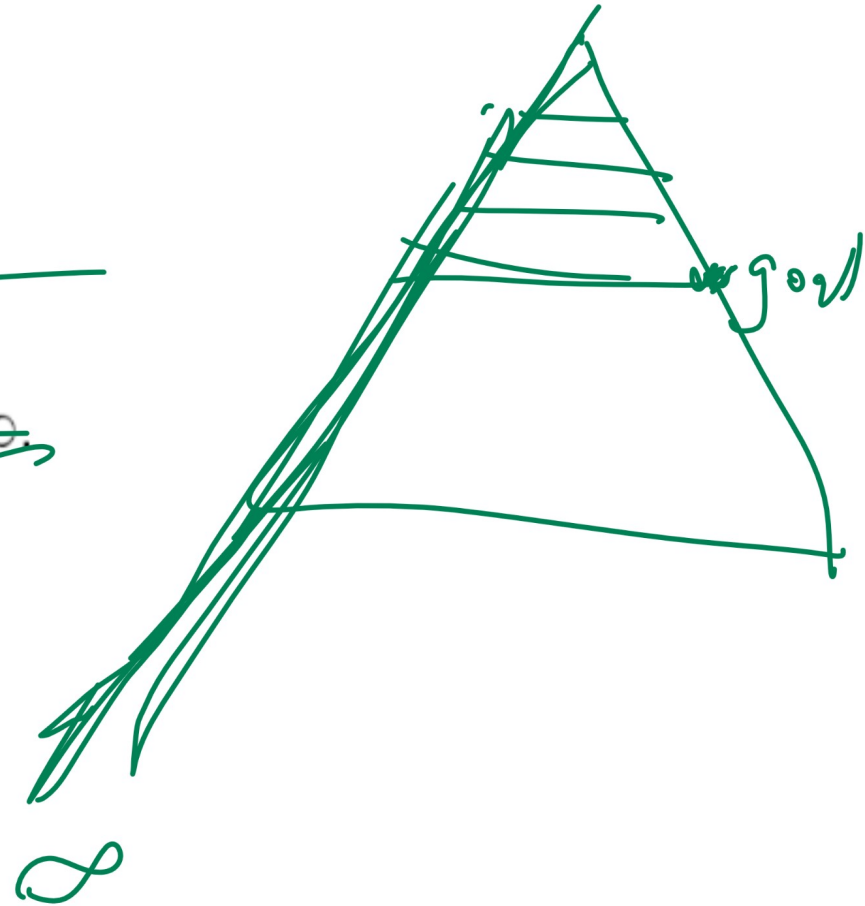
$$s = S_0$$

$$S = s'(s) + S$$

Depth First Search Performance

Discussion

- DFS is incomplete if $D = \infty$.
- DFS is not optimal.

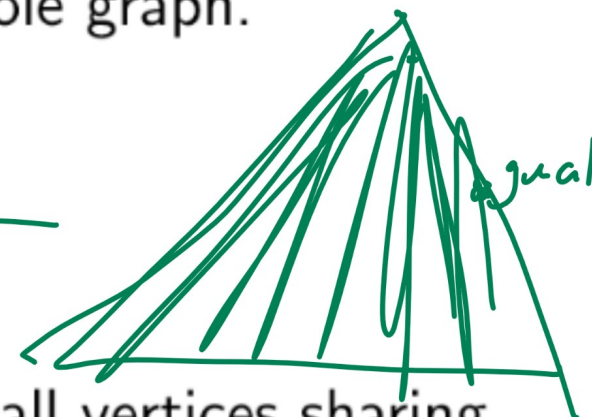


Depth First Search Complexity

Discussion

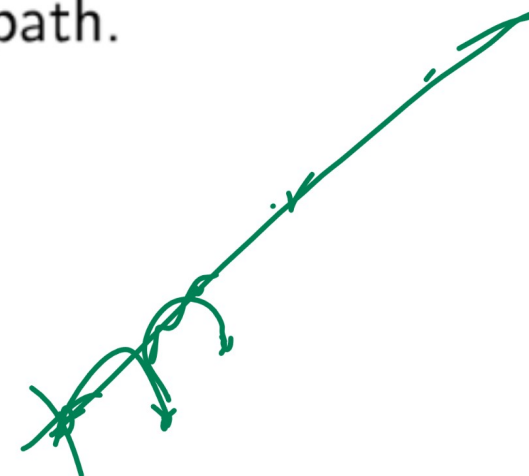
- Time complexity: the worst case occurs when the goal is the root of the last subtree expanded in the whole graph.

$$T = b^{D-d+1} \dots + b^{D-1} + b^D$$



- Space complexity: the worst case is storing all vertices sharing the parents with vertices in the current path.

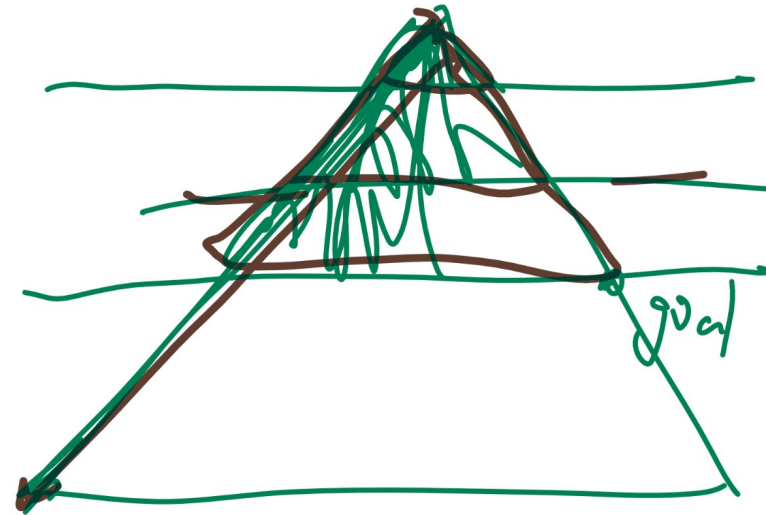
$$S = (b - 1) D + 1$$



Iterative Deepening Search

Description

- DFS but stop if path length > 1
- repeat DFS but stop if path length > 2
- ...
- repeat DFS but stop if path length > d



Uninformed Search
○○○○○○

BFS
○○○○○○○○○○

DFS
○○○○○○○○○○●○○○○○○○○

IDS Simple Example

Quiz

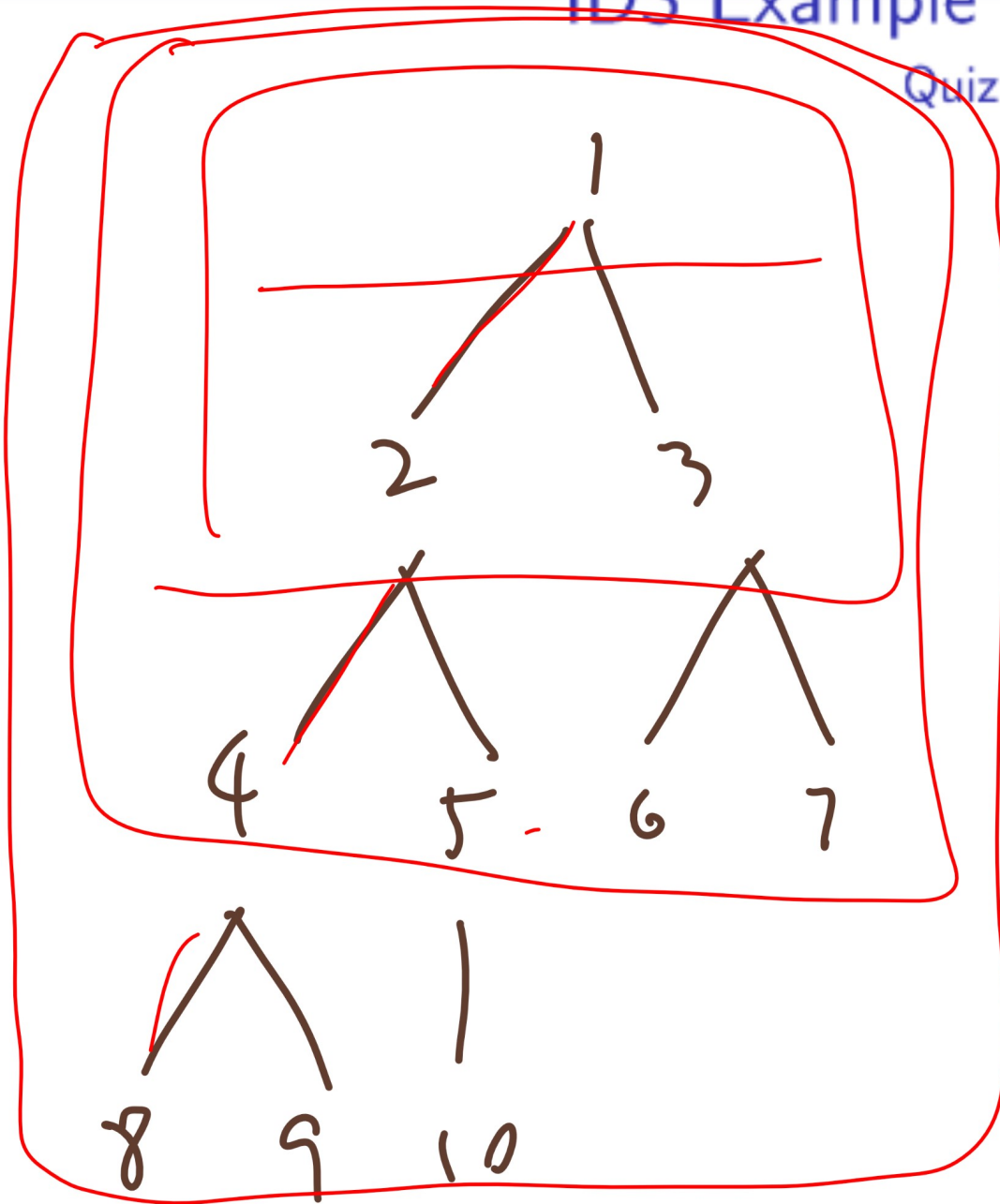
IDS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a IDS expansion sequence?

IDS Example 1 Diagram

Quiz



depth 1: root

1 → 1

2 → 1, 2, 3

3 → 1, 2, 4, 5, 3, 6, 7

4 → 1, 2, 4, 8, 9

IDS Example 2

Quiz

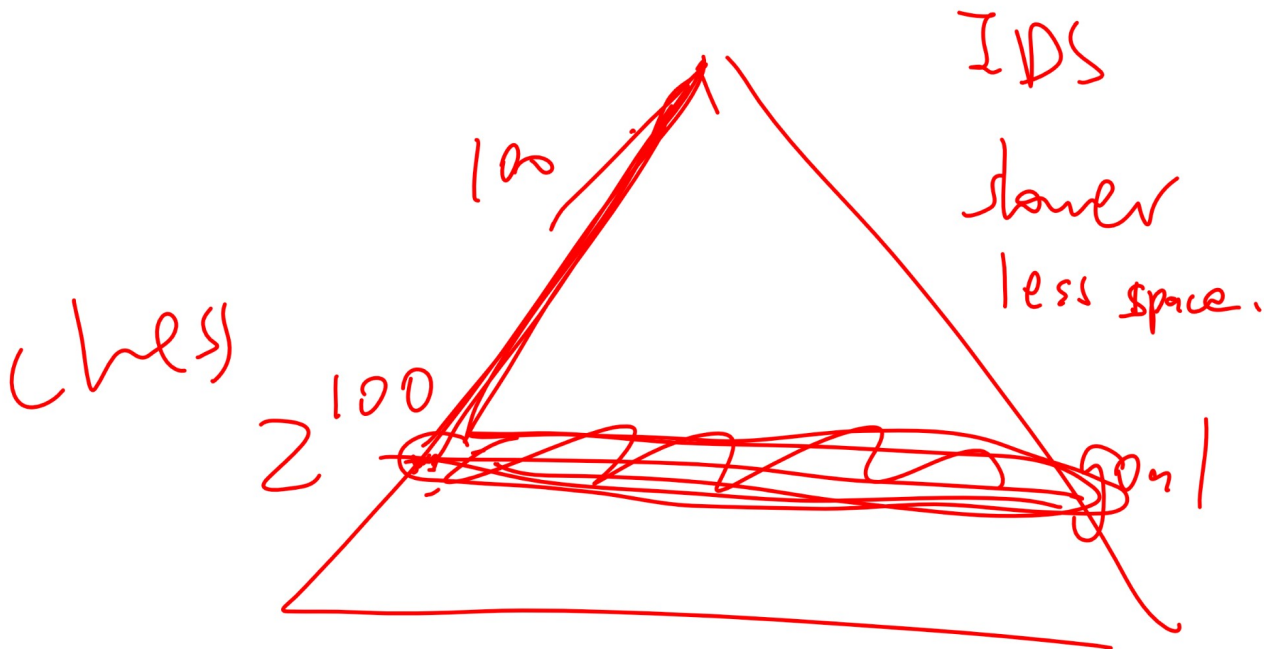
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Q9

• Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many unique states are expanded during a IDS search? Include both the initial and goal states.

- A: 4
- B: 5
- C: 14
- D: 15
- E: 28



BFS Example 3

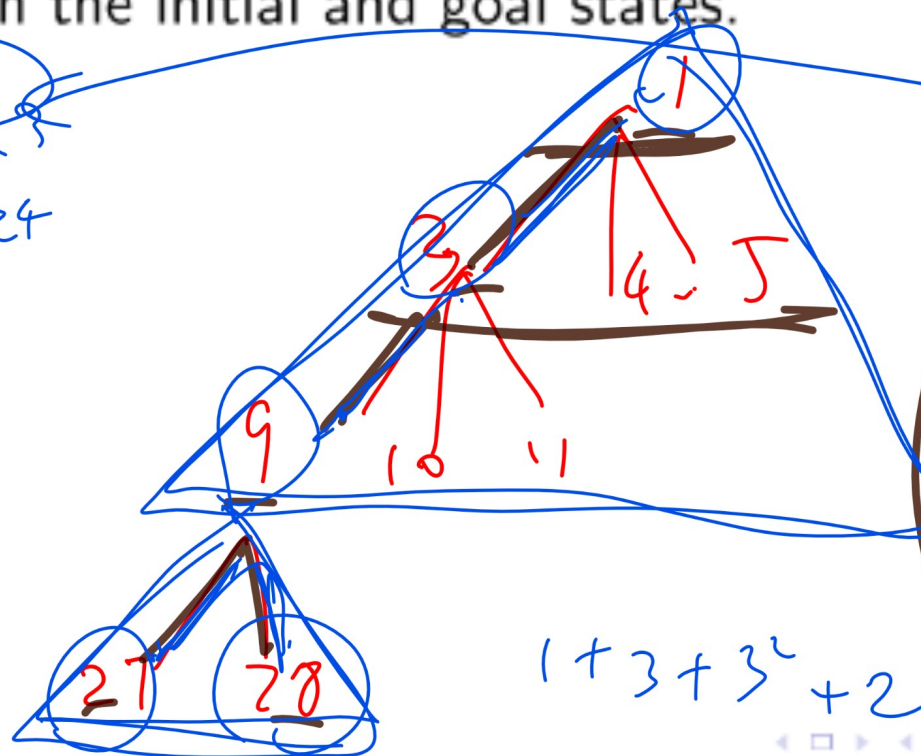
Quiz

Back @
7:10

Q VO

Suppose the states are integers between 1 and 28. The initial state is 1, and the goal state is 28. The successors of a state i are $3i, 3i + 1, 3i + 2$, if exist. How many states are expanded during a IDS search in total (counting repeated ones)? Include both the initial and goal states.

- A: 15 + 8
- B: 15 + 9
- C: 15 + 13
- D: 15 + 17
- E: 15 + 18



1
 $+ 1 + 3$
 $+ 1 + 3 + 3^2$
 $+ 5$

$1 + 3 + 3^2 + 2$

Iterative Deepening Search

Algorithm

- Input: a weighted digraph (V, E, c) , initial states I and goal states G .
- Output: a path from I to G .
- Perform DFS on the digraph restricted to vertices with depth ≤ 1 from the initial state.
- Perform DFS on the digraph restricted to vertices with depth ≤ 2 from the initial state.
- Repeat until the goal is deQueued.

Iterative Deepening Search Performance

Discussion

- IDS is complete.
- IDS is optimal with $c = 1$.

Iterative Deepening Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = \underline{db + (d - 1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d}$$

- Space complexity: it has the same space complexity as DFS.

$$S = (b - 1) d$$

Configuration Space

Discussion