CS540 Introduction to Artificial Intelligence
Lecture 16

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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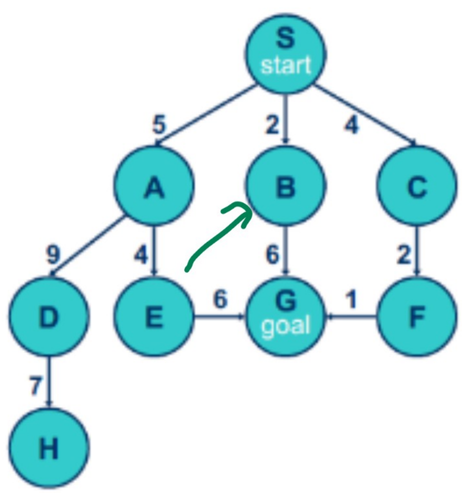
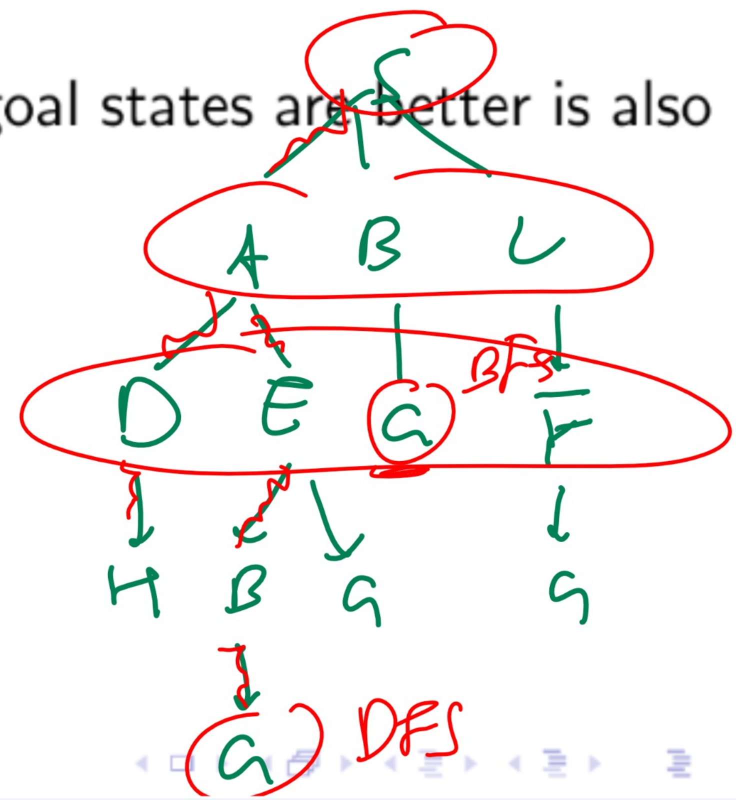
Uniformed vs. Informed Search

Motivation

- Uninformed search means only the goal G and the successor functions s' are given.
- Informed search means which non-goal states are better is also known.



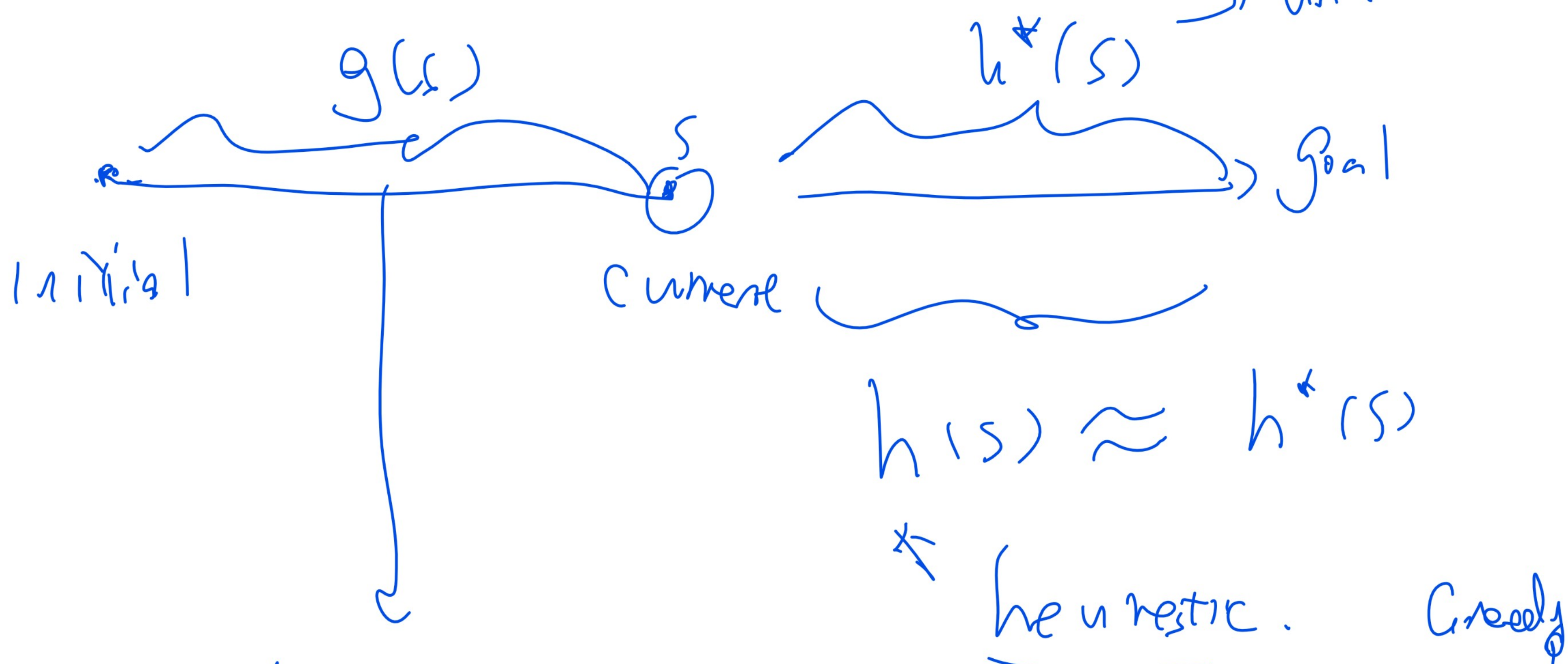


Heuristic Diagram

Motivation

A search

⇒ expand smallest g+h first.



expand smallest g first → UCS

expand smallest h first → BFGS

Uniform Cost Search

Description

- Expand the vertices with the lowest current path cost $g(s)$ first.
- It is BFS with a priority queue based on $g(s)$.
- It is equivalent to BFS if $c = 1$ is constant on all edges.
- It is also called Dijkstra's Algorithm.

UCS Example 1

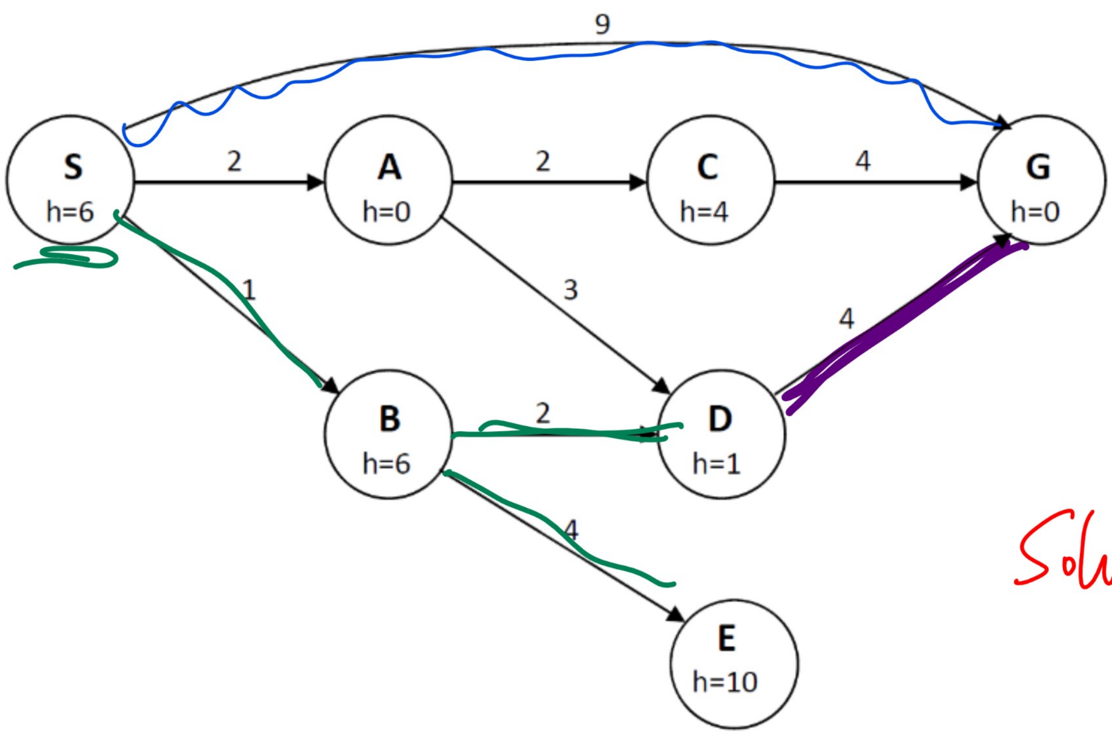
Quiz

- Spring 2017 Midterm Q1
- Given the following adjacency matrix. Find UCS expansion path.

–	S	A	B	C	D	E	G
S	$h = 6$	2	1	–	–	–	9
A	–	$h = 0$	–	2	3	–	–
B	–	–	$h = 6$	–	2	4	–
C	–	–	–	$h = 4$	–	–	4
D	–	–	–	–	$h = 1$	–	4
E	–	–	–	–	–	$h = 10$	–
G	–	–	–	–	–	–	$h = 0$

UCS Example 1 Diagram

Quiz



expansion path:

S, B, A, D

C, D, E, G

Solution S, B, D, G

Priority Queue: S B_S A_S D_B C_A D_A E_B G_D G_C G_D G_S

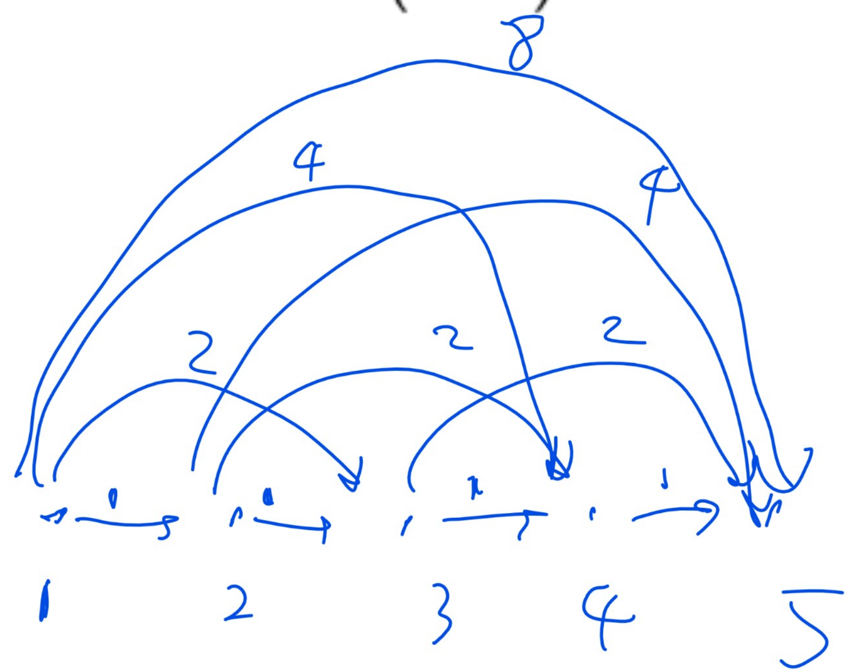
g 0 1 2 1+2 2+2 2+3 1+4 3+4 4+4 4+4 9

UCS Example 2

Quiz

Q 11

- Given that cost from state i to j is 2^{j-i-1} for $j > i$. The initial state is 1 and goal state is 5. What is a vertex expansion sequence if Uniform Cost Search (UCS) is used?
- A: 1, 5
- B: 1, 2, 3, 4, 5
- C: 1, 2, 3, 4, 4, 5
- D: 1, 2, 3, 3, 4, 4, 5
- E: 1, 2, 3, 3, 4, 4, 4, 5



if c = 1 constant, BFS = UCS.

UCS Example 2 Diagram

Quiz

Queue :

selecting smallest to deQueue;

S_1, S_2, S_3, S_3

0 1 2 4 8 1+1 1+2 1+4 2+1 2+2, 2+1, 2+2

S_4, S_4, S_4

3+1 3+1, 3+1, 4+1

1, 2, 3, 4, 4, 4, 4, 5

post video later

Uniform Cost Search

Algorithm

- Input: a weighted digraph (V, E, c), initial states I and goal states G.
- Output: a path from I to G.
- EnQueue initial states into a priority queue Q. Here, Q is ordered by $g(s)$ for $s \in Q$.

$$Q = I$$

- While Q is not empty and goal is not deQueued, deQueue Q and enQueue its successors.

$$s = Q_{(0)} = \arg\min_{s \in Q} g(s)$$

$$Q = Q + s'(s)$$

Uniform Cost Search Performance

Discussion

- UCS is complete.
- UCS is optimal with any c.

Best First Greedy Search

Description

- Expand the vertices with the lowest heuristic cost $h(s)$ first.
- Use a priority queue based on $h(s)$.

Greedy Example 1

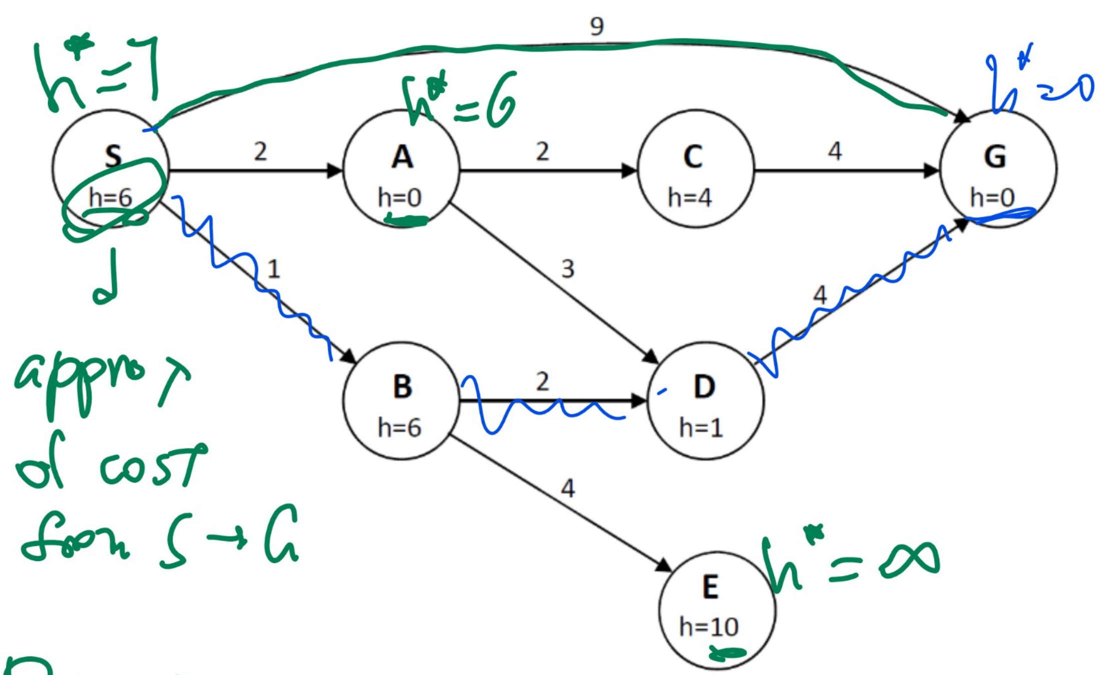
Quiz

- Given the following adjacency matrix. Find Greedy Search expansion path.

–	S	A	B	C	D	E	G
S	$h = 6$	2	1	–	–	–	9
A	–	$h = 0$	–	2	3	–	–
B	–	–	$h = 6$	–	2	4	–
C	–	–	–	$h = 4$	–	–	4
D	–	–	–	–	$h = 1$	–	4
E	–	–	–	–	–	$h = 10$	–
G	–	–	–	–	–	–	$h = 0$

Greedy Example 1 Diagram

Quiz



expansion: S, A, G

Solution S → G

Priority Queue: A G D C S B

h 0 0 1 4 6 6

BFGS

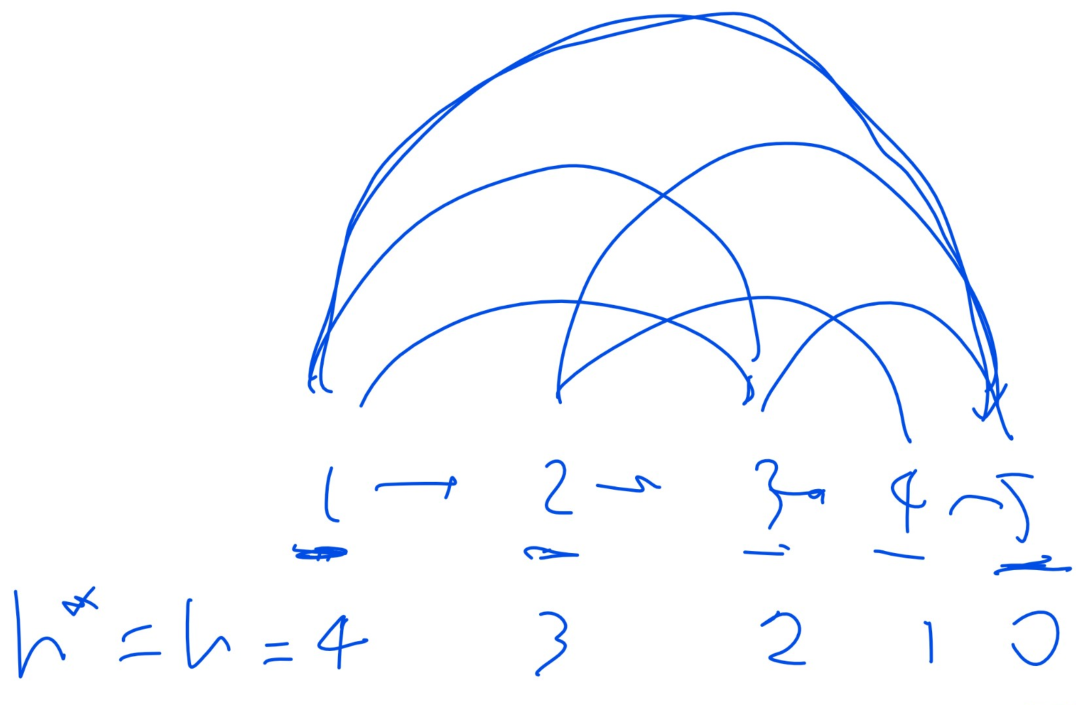
Greedy is wrong in general!

Greedy Example 2

Quiz

Q12

- Given that cost from state i to j is 2^{j-i-1} for $j > i$. The heuristic is $h(i) = 5 - i$, The initial state is 1 and goal state is 5. What is a vertex expansion sequence if Best First Greedy Search is used?
- A: 1, 5
- B: 1, 2, 3, 4, 5
- C: 1, 2, 3, 4, 4, 5
- D: 1, 2, 3, 3, 4, 4, 5
- E: 1, 2, 3, 3, 4, 4, 4, 5



Greedy Example 2 Diagram

Quiz

Best First Greedy Search

Algorithm

- Input: a weighted digraph (V, E, c), initial states I and goal states G, and the heuristic function $h(s), s \in V$.
- Output: a path from I to G.
- EnQueue initial states into a priority queue Q. Here, Q is ordered by $h(s)$ for $s \in Q$.

$$Q = I$$

- While Q is not empty and goal is not deQueued, deQueue Q and enQueue its successors.

$$s = Q_{(0)} = \arg\min_{s \in Q} h(s)$$

$$Q = Q + s'(s)$$

Best First Greedy Search Performance

Discussion

- Greedy is incomplete.
- Greedy is not optimal.

A Search

Description

- Expand the vertices with the lowest total cost $g(s) + h(s)$ first.
- Use a priority queue based on $g(s) + h(s)$.
- A stands for Always be optimistic?

A Search Example 1

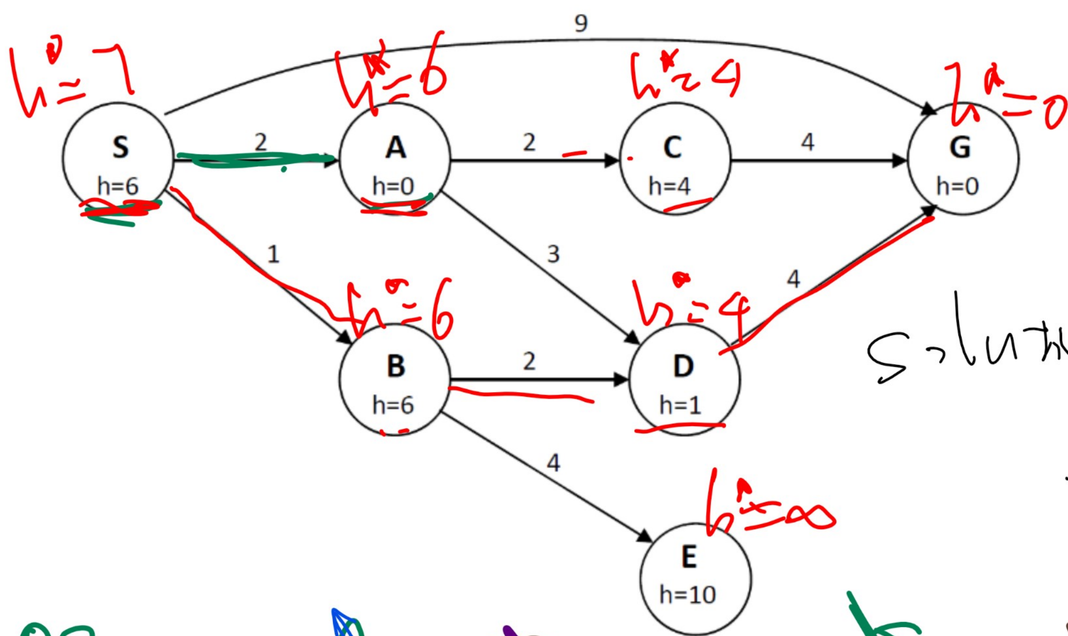
Quiz

- Given the following adjacency matrix. Find *A* Search expansion path.

–	S	A	B	C	D	E	G
S	$h = 6$	2	1	–	–	–	9
A	–	$h = 0$	–	2	3	–	–
B	–	–	$h = 6$	–	2	4	–
C	–	–	–	$h = 4$	–	–	4
D	–	–	–	–	$h = 1$	–	4
E	–	–	–	–	–	$h = 10$	–
G	–	–	–	–	–	–	$h = 0$

A Search Example 1 Diagram

Quiz



expansion

S, A, D, B, D, G

solution:

S ← B ← D ← G

pq	A_S	D_B	D_A	S	B_S	G_D	C_A	G_S	G_D	E_B
g	2	3	5	0	1	7	4	9	9	5
h	0	1	1	6	6	0	4	0	0	10
g+h	2	4	6	6	7	7	8	9	9	15

A Search

Algorithm

- Input: a weighted digraph (V, E, c), initial states I and goal states G, and the heuristic function $h(s), s \in V$.
- Output: a path from I to G.
- EnQueue initial states into a priority queue Q. Here, Q is ordered by $g(s) + h(s)$ for $s \in Q$.

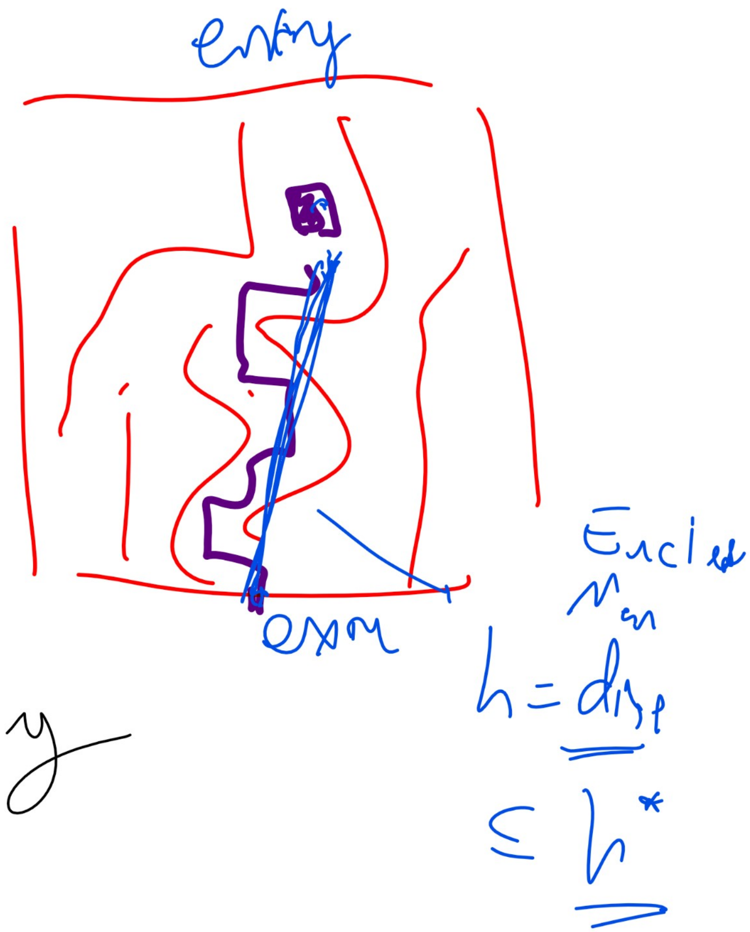
$$Q = I$$

- While Q is not empty and goal is not deQueued, deQueue Q and enQueue its successors.

$$s = Q_{(0)} = \arg\min_{s \in Q} g(s) + h(s)$$
$$Q = Q + s'(s)$$

A Search Performance

Discussion



- A is complete.
- A is not optimal.

h can be wrong

A Star Search

Description

$0 \le h \le h^*$

- $A^\star$ search is A search with an admissible heuristic.

Admissible Heuristic

Definition

- A heuristic is admissible if it never over estimates the true cost.

$$0 \leqslant h(s) \leqslant h^{\star}(s)$$

Dominated Heuristic

Definition

- One heuristic, h_1, is dominated by another, h_2, if:

$$h_1(s) \leqslant h_2(s) \leqslant h^\star(s), \ \forall\, s \in S$$

- If h_2 dominates h_1, then h_2 is better than h_1 since $A^\star$ using h_1 expands at least as many states (or more) than $A^\star$ using $h_{2.}$
- If h_2 dominated h_1, $A^\star$ with h_2 is better informed than $A^\star$ with $h_{1.}$

Admissible Heuristic 8 Puzzle Example

Quiz

- Which ones (select multiple) of the following are admissible heuristic function for the 8 Puzzle?
- A: $h(s)$ = number of tiles in the wrong position.
- B: $h(s) = 0$.
- C: $h(s) = 1$.
- D: $h(s)$ = sum of Manhattan distance between each tile and its goal location.
- E: $h(s)$ = sum of Euclidean distance between each tile and its goal location.

Admissible Heuristic General Example 1

Quiz

- Which ones (select multiple) of the following are admissible heuristic function?
- A: $h(s) = h^{\star}(s)$.
- B: $h(s) = \max\{2, h^{\star}(s)\}$.
- C: $h(s) = \min\{2, h^{\star}(s)\}$.
- D: $h(s) = h^{\star}(s) - 2$.
- E: $h(s) = \sqrt{h^{\star}(s)}$.

$0 \le h \le h^{*}$

$h^{*}(goal) = 0$

$h(goal) = \max(2, 0) = 2 > 0$

$\le h^{*}(s)$

$h^{*}(goal) = 0$

$h(goal) = 0 - 2 < 0$

$h^{*}(s) = 0.25$, $h(s) = \sqrt{0.25} = 0.5 > 0.25 = h^{*}$.

A Star Search Example 2

Quiz

- Given that cost from state i to j is 2^{j-i-1} for $j > i$. How many of the following heuristic functions are admissible? For $i = 1, 2, 3, 4, 5$:

$$h(i) = 5 - i$$

$$h(i) = \sqrt{5 - i}$$

$$h(i) = \log_2(6 - i)$$

$$h(i) = 1 - \mathbb{1}_{i=5}$$

$$h(i) = 0$$

- A: 1, B: 2, C: 3, D: 4, E: 5

A Star Search Example 2

Quiz

- Given that cost from state i to j is 2^{j-i-1} for $j > i$. Which one of the following heuristic functions is not dominated (among the admissible ones)? For $i = 1, 2, 3, 4, 5$:
- A: $h(i) = 5 - i$
- B: $h(i) = \sqrt{5 - i}$
- C: $h(i) = \log_2(6 - i)$
- D: $h(i) = 1 - \mathbb{1}_{i=5}$
- E: $h(i) = 0$

A Star Search with Revisit, Part I

Algorithm

- Input: a weighted digraph (V, E, c), initial states I and goal states G, and the heuristic function $h(s), s \in V$.
- Output: a path with minimum cost from I to G.
- EnQueue initial states into a priority queue Q. Here, Q is ordered by $g(s) + h(s)$ for $s \in Q$.

$$Q = I$$
$$g(I) = 0$$
$$g(s) = \infty, \text{ for } s \notin I$$

- Initialize the list of visited vertices, P.

$$P = \varnothing$$

A Star Search with Revisit, Part II

Algorithm

- While Q is not empty and goal is not deQueued, deQueue Q, put it on P and enQueue its successors to Q, and update the cost functions.

$$s = Q_{(0)} = \arg\min_{s \in Q} g(s) + h(s)$$

$$P = P + s$$

$$Q = Q + s'(s), \text{ update } g(s') = \min\left\{g(s'), g(s) + c(s, s')\right\}$$

A Search Performance

Discussion

A with $h \leq h^*$

- $A^\star$ is complete.
- $A^\star$ is optimal.

Iterative Deepening A Star Search

Discussion

- $A^{\star}$ can use a lot of memory.
- Do path checking without expanding any vertex with $g(s) + h(s) > 1$.
- Do path checking without expanding any vertex with $g(s) + h(s) > 2$.
- ...
- Do path checking without expanding any vertex with $g(s) + h(s) > d$.

IDA*

Iterative Deepening A Star Search Performance

Discussion

- IDA* is complete.
- IDA* is optimal.
- IDA* is more costly than $A^{\star}$.

Beam Search

Discussion

- Version 1: Keep a priority queue with fixed size k. Only keep the top k vertices and discard the rest.
- Version 2: Only keep the vertices that are at most ε worse than the best vertex in the queue. ε is called the beam width.

$g + h \le \varepsilon$

Beam Search Performance

Discussion

- Beam is incomplete.
- Beam is not optimal.