Uninformed Search
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BFS
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DFS
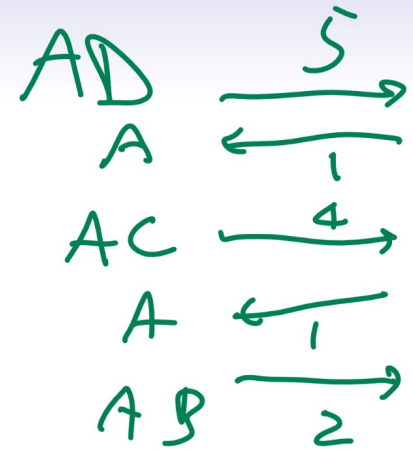ooooooooooooooo

CS540 Introduction to Artificial Intelligence
Lecture 17

Young Wu

Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

July 26, 2021

Uninformed Search
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BFS
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DFS
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Bridge and Torch Game

Motivation

- Four people with one flashlight (torch) want to go across a river. The bridge can hold two people at a time, and they must cross with the flashlight. The time it takes for each person to cross the river:

A	B	C	D
1	2	4	5

- What is the minimum total time required for everyone to cross the river?

- A: 10, B: 11, C: 12, D: 13, E: 14

Uninformed Search
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Remind Me to Start Recording
Admin

- The messages you send in chat will be recorded: you can change your Zoom name now before I start recording.

Uninformed Search
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Reduced Space Example 2

Quiz

go through on Friday

- $\hat{\Sigma} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. If one original data is $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. What is the new representation using only the first two principal components?

- A: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, B: $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, C: $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, D: $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$, E: $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Uninformed Search
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Reduced Space Example 3
Quiz

- $\hat{\Sigma} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. If one original data is $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. What is the reconstructed vector using only the first two principal components?
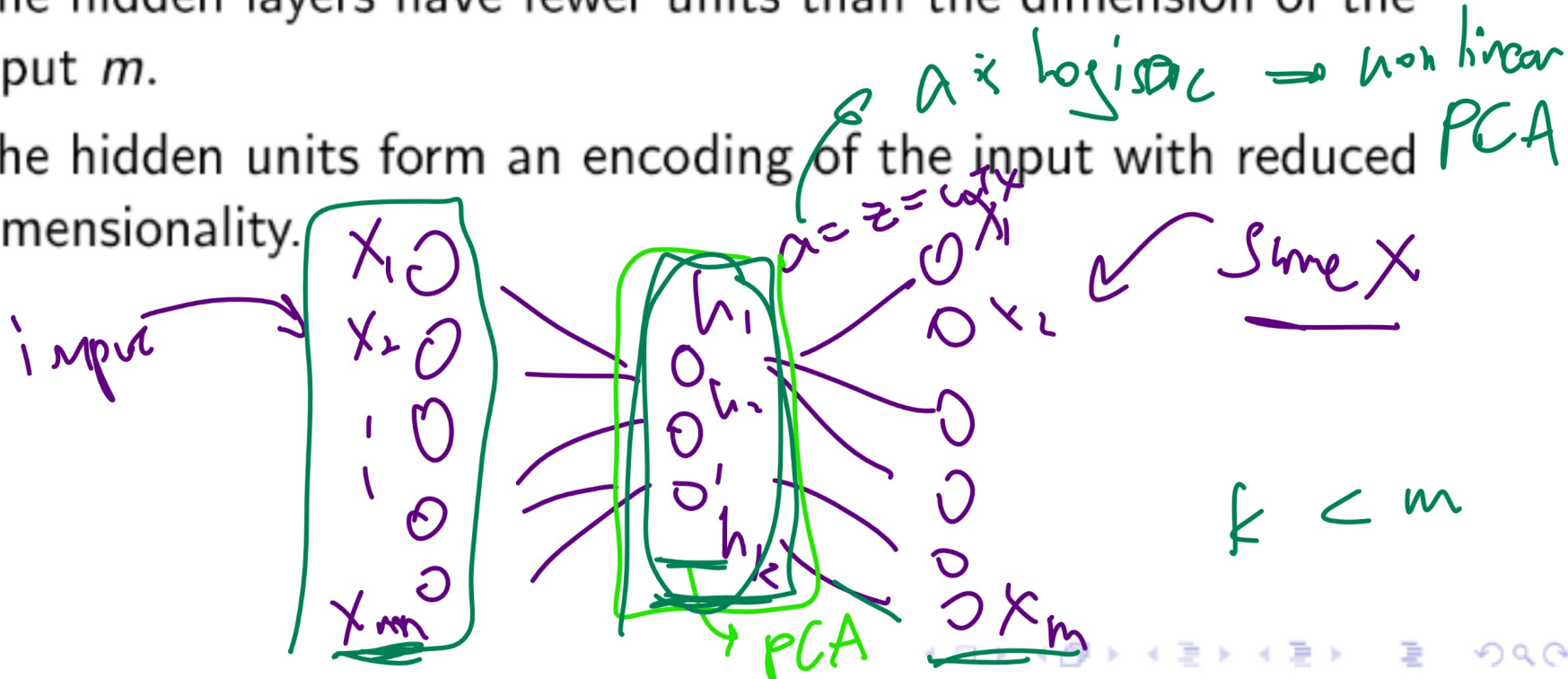
- A: $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, B: $\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, C: $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$, D: $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$, E: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

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Autoencoder

Discussion
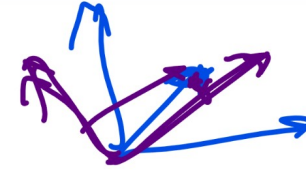
non linear
PCA

- A multi-layer neural network with the same input and output $y_i = x_i$ is called an autoencoder.

- The hidden layers have fewer units than the dimension of the input m.

- The hidden units form an encoding of the input with reduced dimensionality.

input

a is logistic $\Rightarrow$ non linear

PCA

$a = z = \omega^T x$

Same X

PCA

$k < m$

Uninformed Search
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Kernel PCA

Discussion

SVM

- A kernel can be applied before finding the principal components.

$$\hat{\Sigma} = \frac{1}{n-1} \sum_{i=1}^{n} \varphi(x_i) \varphi(x_i)^T$$

K kernel matrix

var for transformed feature

- The principal components can be found without explicitly computing $\varphi(x_i)$, similar to the kernel trick for support vector machines.

- Kernel PCA is a non-linear dimensionality reduction method.

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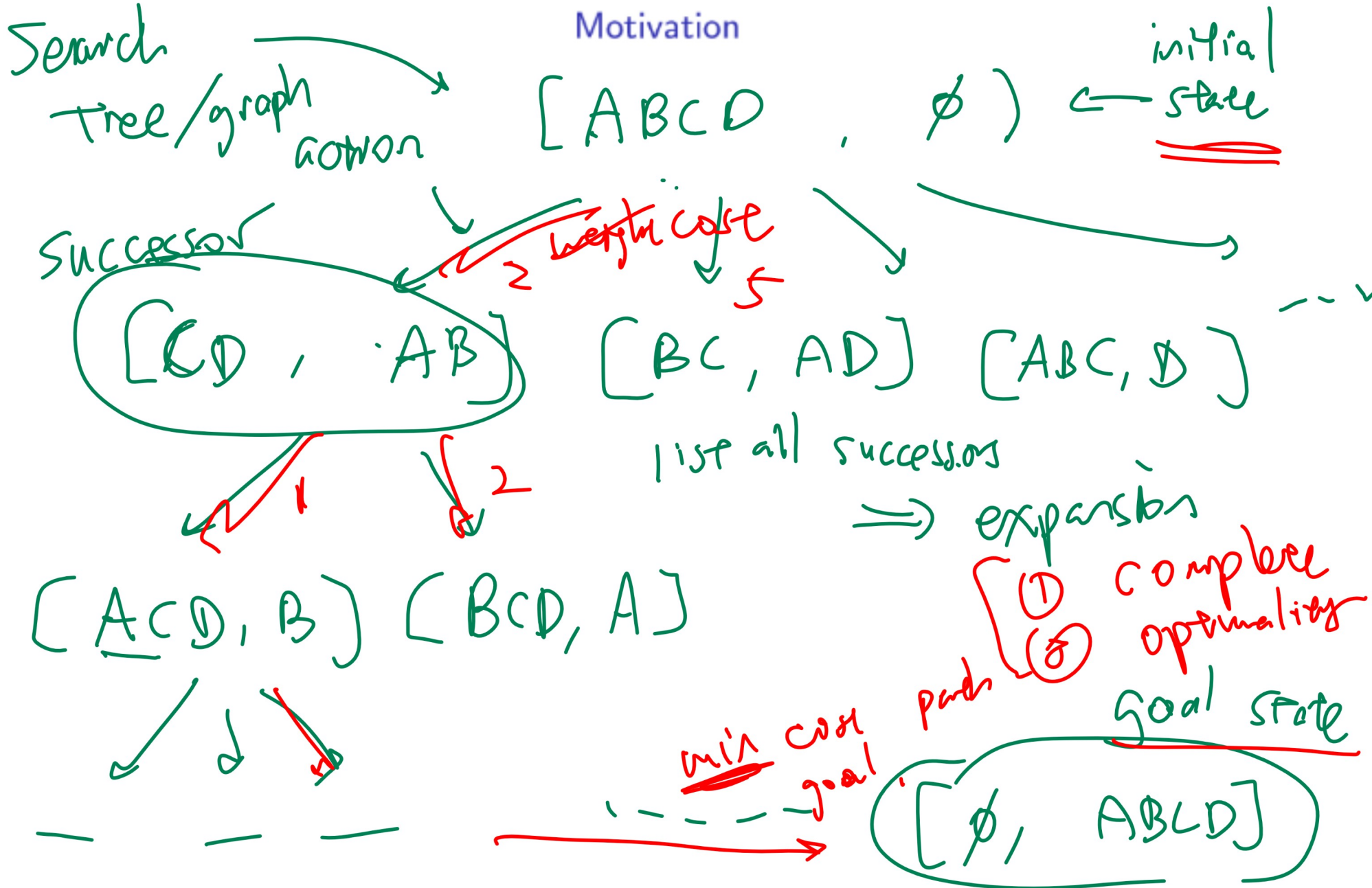
Learning vs Search
Motivation

Q table

- In reinforcement learning, the reward and state transition need to be learned by taking actions.

- In search problems, the reward and state transitions are given.

- The problem is to find a sequence of actions that lead to the goal with minimum cost.

Uninformed Search
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DFS
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Bridge and Torch Game Again
Motivation

- Four people with one flashlight (torch) want to go across a river. The bridge can hold two people at a time, and they must cross with the flashlight. The time it takes for each person to cross the river:

A	B	C	D
1	2	4	5

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Bridge and Torch Game States 1

Motivation

Search Tree/graph

$[ABCD, \emptyset)$ ← initial state

gowon

successor

$[CD, AB]$ length cost

$[BC, AD]$ $[ABC, D]$

list all successors

$\Rightarrow$ expansion

$[ACD, B]$ $[BCD, A]$

① complete

② optimality

min cost path goal

goal state

$[\emptyset, ABCD]$

Uninformed Search
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Bridge and Torch Game States 2
Motivation

Uninformed Search
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BFS
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DFS
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Sizes of State Space
Motivation

adversarial search.

(next week)

- Tic Tac Toe: 10^3

- Checkers: 10^{20}

- Chess: 10^{50}

- Go: 10^{170}

Uninformed Search
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BFS
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DFS
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Performance

Definition

- A search strategy is complete if it finds at least one solution.
- A search strategy is optimal if it finds the optimal solution.
- For uninformed search, the costs are assumed to be 1 for all edges $c = 1$.

Uninformed Search
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Complexity
Definition

- The time complexity of a search strategy is the worst case maximum number of vertices expanded.
- The space complexity of a search strategy is the worst case maximum number of states stored in the frontier at a single time.

- Notation: the goals are d edges away from the initial state. This means assuming a constant cost of 1, the optimal solution has cost d. The maximum depth of the graph is D.

- Notation: the branching factor is b, the maximum number of actions associated with a state.

$$b = \max_{s \in V} |s'(s)|$$

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Search Tree Diagram
Definition

$$max = b$$

\# Successors

goal

d

D

Uninformed Search
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BFS
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DFS
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Breadth First Search

Description

- Use Queue (FIFO) for the frontier.
- Remove from the front, add to the back.

nodes expanded not closed

Uninformed Search
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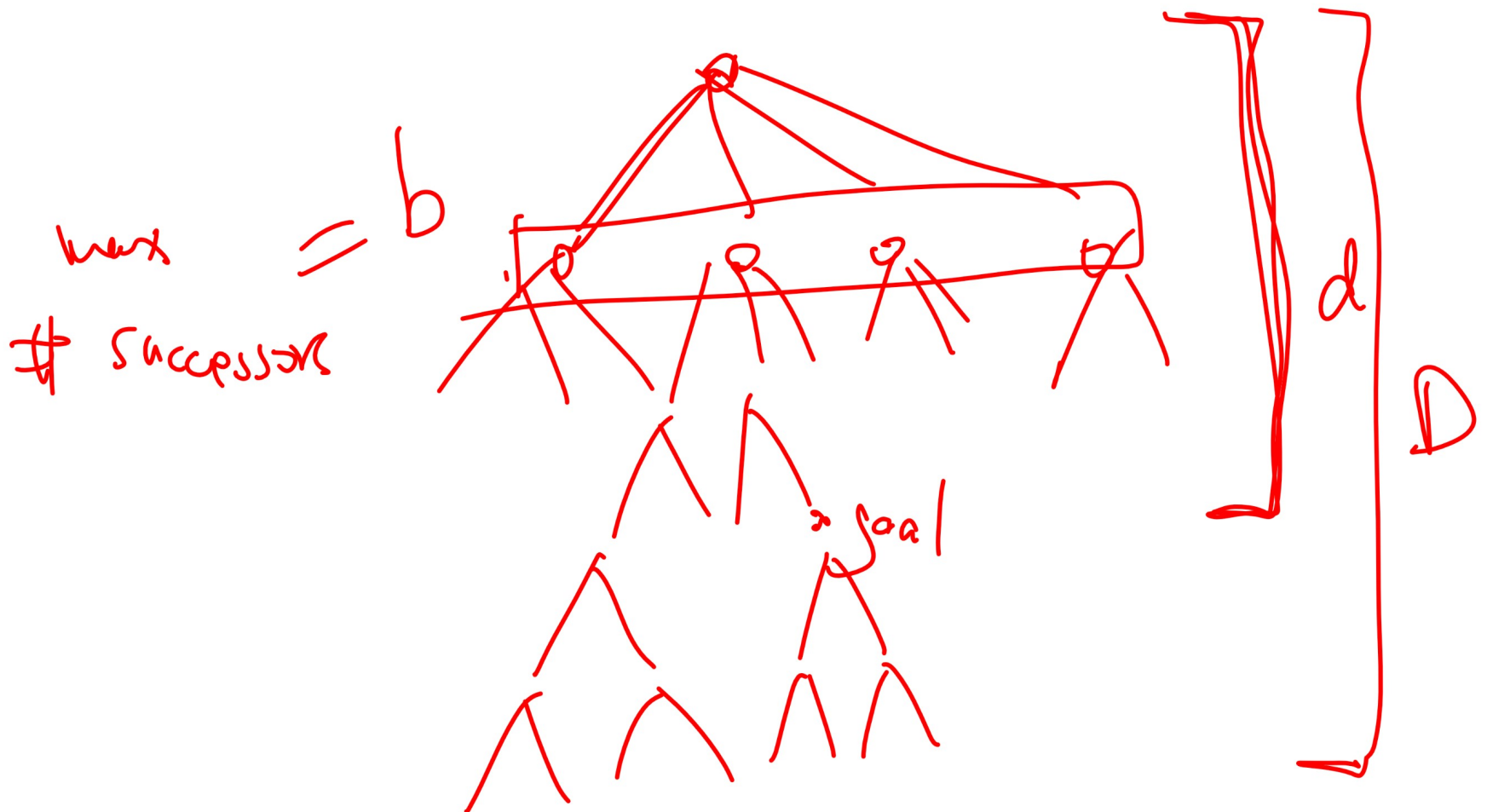
BFS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2

- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a BFS expansion sequence?

may goal

a

shortest path

Costs = 1

Queue

remove a state
add successors!

$2i$ $2i+1$

stop when we expand goal (9)

10

2

4   5   6   7

8   9   10

Uninformed Search
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BFS
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DFS
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BFS Example 1 Diagram

Quiz

$\cancel{1}$ $\cancel{2}$ 3   4   5

space of 3

frontier.

Uninformed Search
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BFS
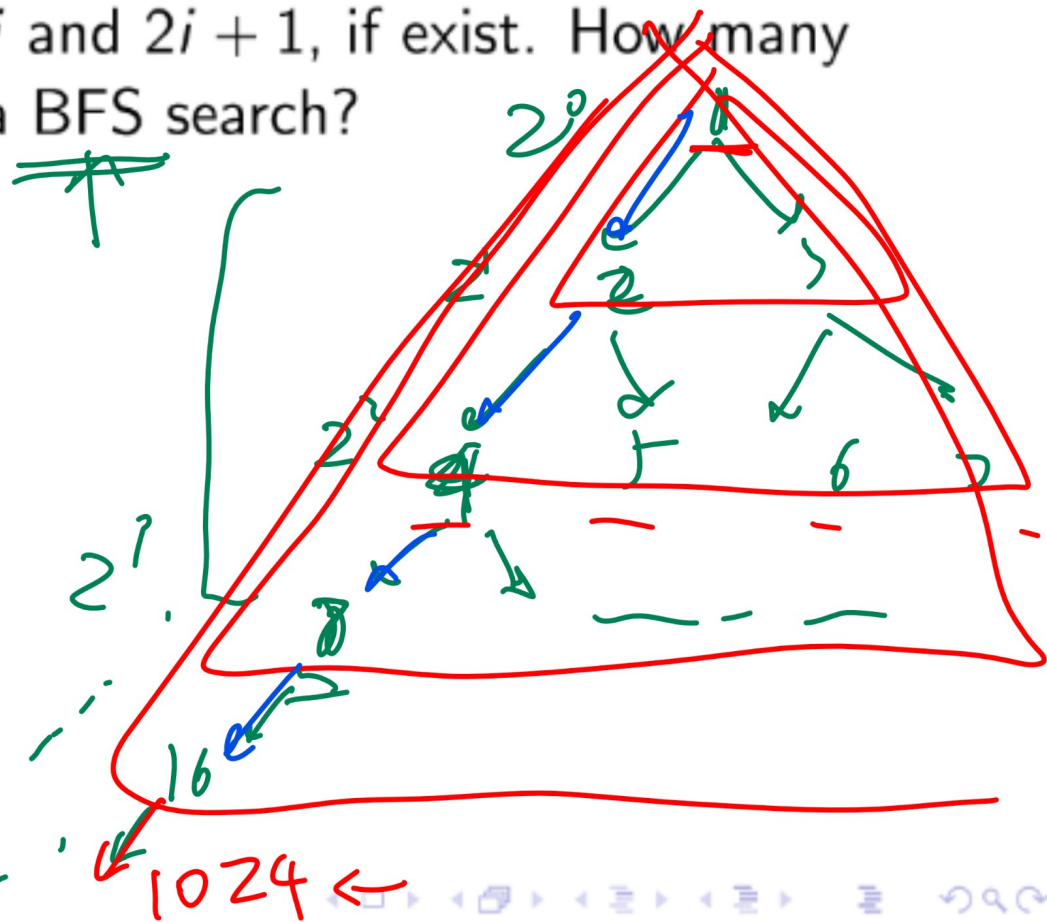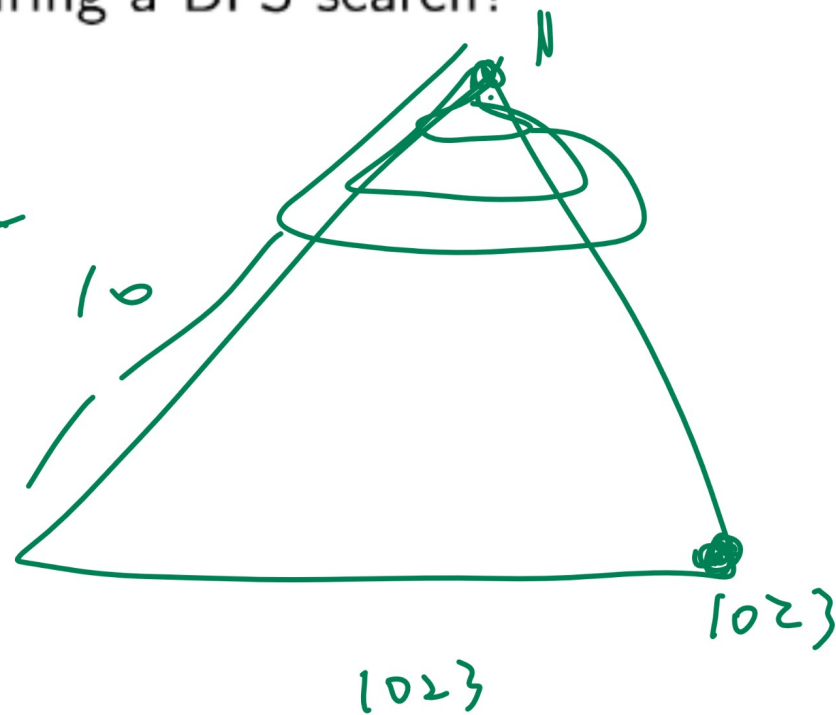○○○○●○○○

DFS
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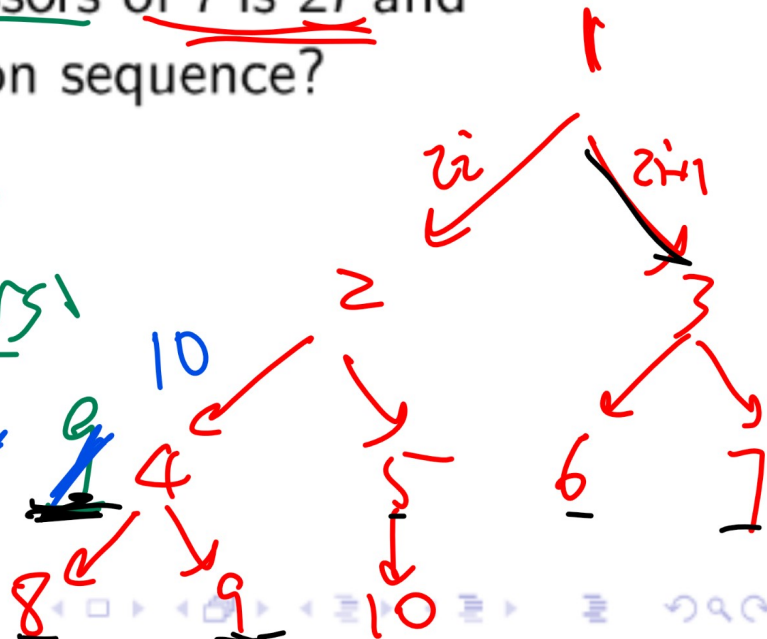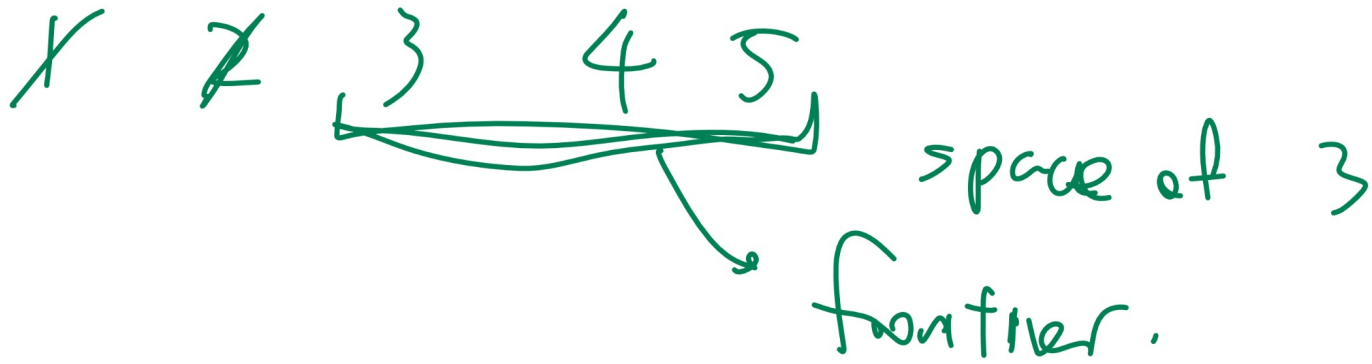
BFS Example 2

Quiz

- Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

- A: 10
- B: 11
- C: 12
- D: 1023
- E: 1024

Uninformed Search
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BFS
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DFS
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BFS Example 3

Quiz

- Suppose the states are integers between 1 and $2^{10} - 1 = 1023$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?

  - A: 10
  - B: 11
  - C: 12
  - D: 1023
  - E: 1024

Handwritten annotations: Q 3, include expand goal, 10, 1023, 1023

Uninformed Search
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BFS
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DFS
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Breadth First Search Performance

Discussion

- BFS is complete.
- BFS is optimal with $c = 1$.

Uninformed Search
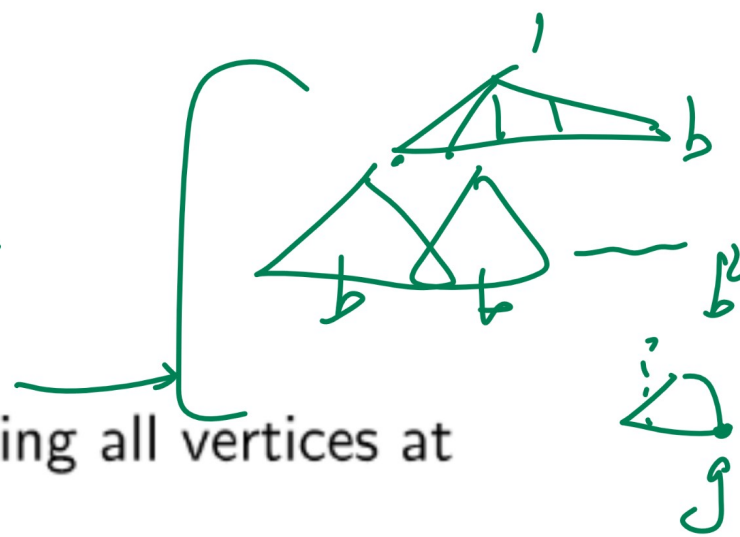OOOOOOOOOOOOOOO

BFS
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DFS
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Breadth First Search Complexity

Discussion

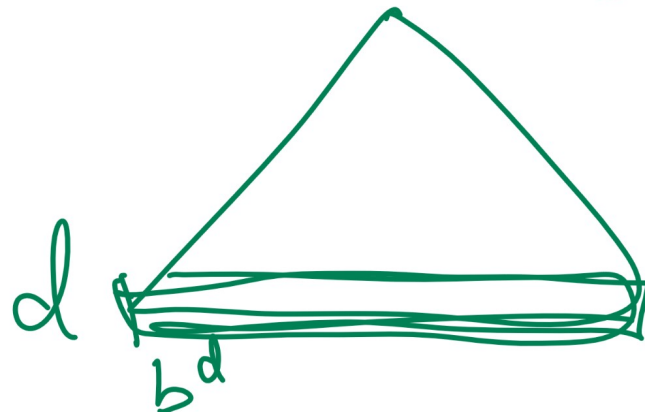- Time complexity: the worst case occurs when the goal is the last vertex at depth d.

$$T = b + b^2 + ... + b^d$$

$$O(b^d)$$

- Space complexity: the worst case is storing all vertices at depth d is in the frontier.

$$S = b^d$$

Uninformed Search
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BFS
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DFS
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Depth First Search

Description

- Use Stack (LIFO) for the frontier.
- Remove from the front, add to the front.

Uninformed Search
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BFS
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DFS
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DFS Example 1
Quiz

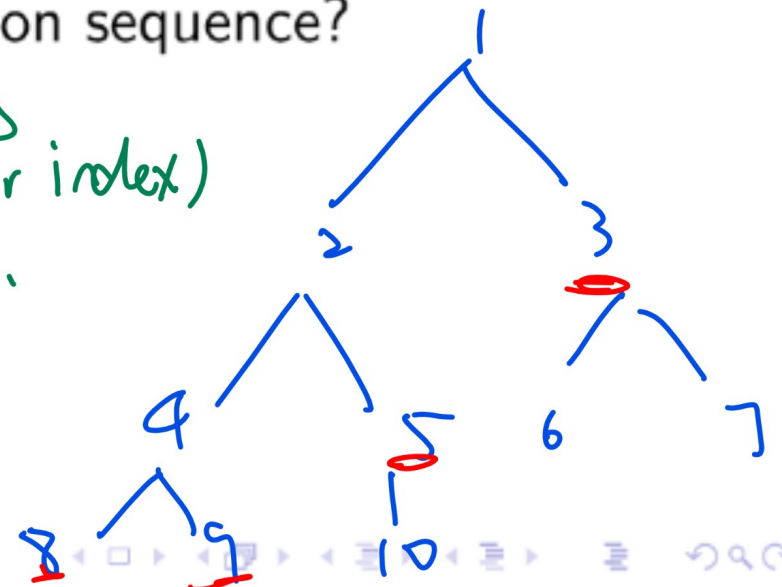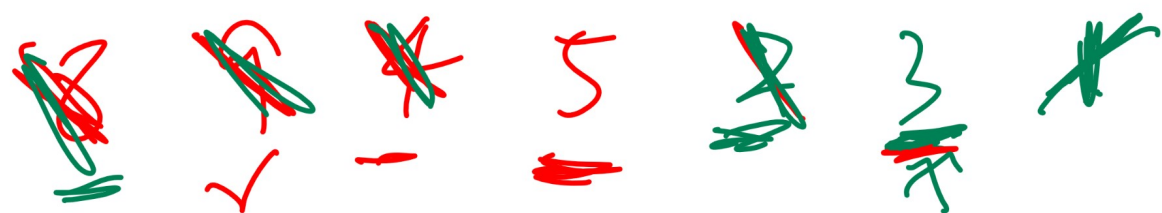Space. 3. 5, 8 9 , time : 1, 2, 4, 8, 9

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2 (4) (5)

- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a DFS expansion sequence?

Convention: put nodes on the higher (larger index) first.

Stack

8 6 4 5 2 3 1

1
2   3
4   5 6 7
8 9 10

DFS Example 1 Diagram
Quiz

Uninformed Search
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BFS
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DFS
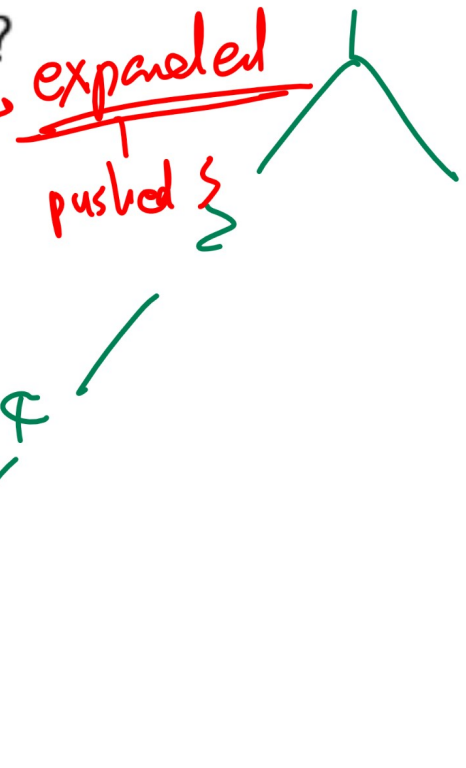○○○●○○○○○○○○○○○

DFS Example 2

Quiz

Q4

- Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?
  - A: 10
  - B: 11
  - C: 12
  - D: 1023
  - E: 1024

8, 9, 4, 5, 2, 1, 3 → expanded

pushed

2^3 2^2

2^3 2^8

Goal is expanded 1024

Uninformed Search
oooooooooooooo

BFS
ooooooo

DFS
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DFS Example 3

Quiz

- Suppose the states are integers between 1 and $2^{10} - 1 = 1023.$ The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?
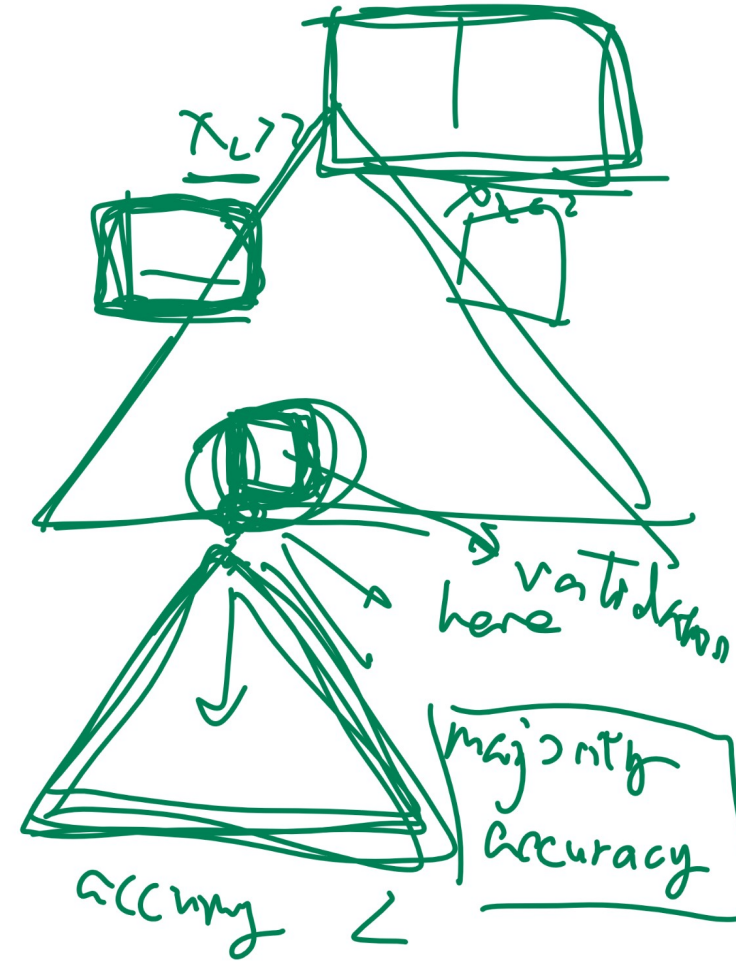
- A: 10

- B: 11

- C: 12

- D: 1023

- E: 1024

Uninformed Search
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BFS
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DFS
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Depth First Search Performance

Discussion

- DFS is incomplete if $D = \infty$.
- DFS is not optimal.

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DFS
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Depth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the root of the last subtree expanded in the whole graph.

$$T = b^{D-d+1}... + b^{D-1} + b^D$$

- Space complexity: the worst case is storing all vertices sharing the parents with vertices in the current path.

$$S = (b - 1) D + 1$$

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Iterative Deepening Search
Description

- DFS but stop if path length > 1
- repeat DFS but stop if path length > 2
- ...
- repeat DFS but stop if path length $> d$

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BFS
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IDS Example 1
Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2

- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a IDS expansion sequence?

IDS Example 1 Diagram

Quiz

Uninformed Search
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BFS
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DFS
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Iterative Deepening Search
Algorithm

- Input: a weighted digraph (V, E, c), initial states I and goal states G.

- Output: a path from I to G.

- Perform DFS on the digraph restricted to vertices with depth $\leqslant 1$ from the initial state.

- Perform DFS on the digraph restricted to vertices with depth $\leqslant 2$ from the initial state.

- Repeat until the goal is deQueued.

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DFS
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Iterative Deepening Search Performance
Discussion

- IDS is complete.
- IDS is optimal with $c = 1$.

Uninformed Search
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BFS
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DFS
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Iterative Deepening Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d.

$$T = db + (d-1)\, b^2 + \ldots + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- Space complexity: it has the same space complexity as DFS.

$$S = (b-1)\, d$$

Uninformed Search

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Configuration Space

Discussion