

CS540 Introduction to Artificial Intelligence

Lecture 18

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

July 27, 2021

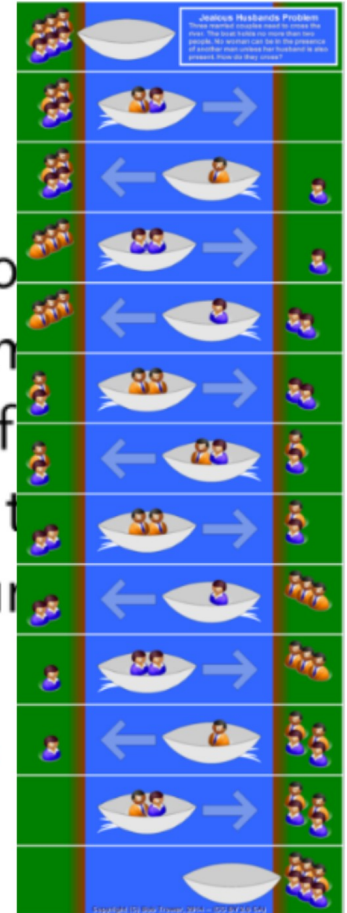
River Crossing Problem

Quiz

AI

- Three married couples need to cross the river. The boat requires at least one person to operate and holds no more than two people. No woman can be in the presence of a man unless her husband is also present (this is called the Jealous Husband Problem). What is the minimum number of times the boat needs to go across the river?

11



Remind Me to Start Recording

Admin

- The messages you send in chat will be recorded: you can change your Zoom name now before I start recording.

Breadth First Search Performance

Discussion

- BFS is complete.
- BFS is optimal with $c = 1$.

always finds a solution

cost shortest path solution.

Breadth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = \underline{1} + b + b^2 + \dots + \underline{b^d}$$



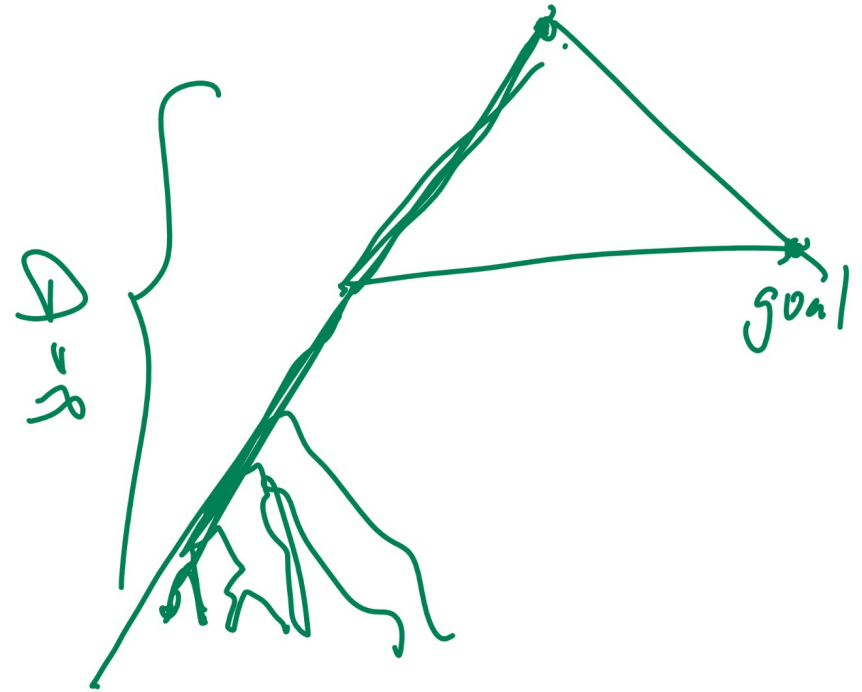
- Space complexity: the worst case is storing all vertices at depth d is in the frontier.

$$\underline{S = b^d}$$

Depth First Search Performance

Discussion

- DFS is incomplete if $D = \infty$.
- DFS is not optimal.

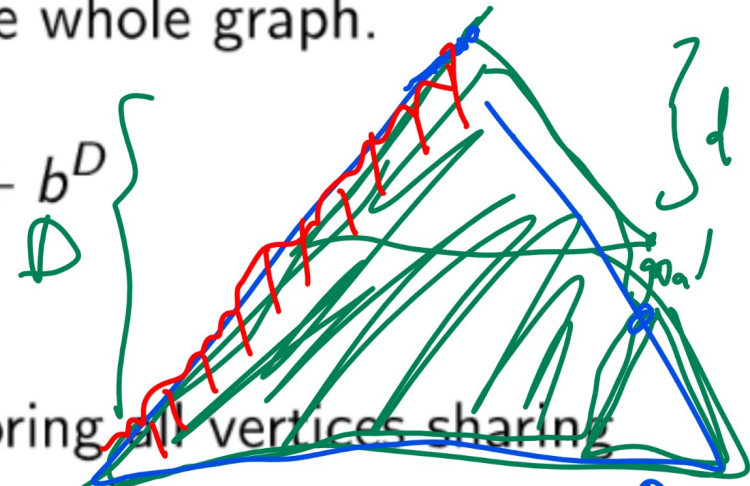


Depth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the root of the last subtree expanded in the whole graph.

$$T = b^{D-d+1} \dots + b^{D-1} + b^D$$



- Space complexity: the worst case is storing all vertices sharing the parents with vertices in the current path.

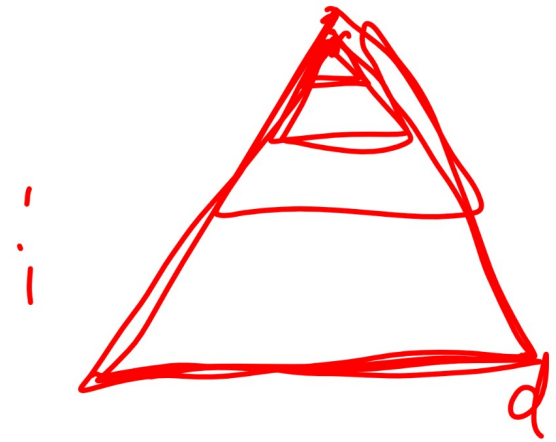
$$S = \underline{(b-1)D + 1}$$

$$1 + b + \dots + b^{D-d}$$

Iterative Deepening Search

Description

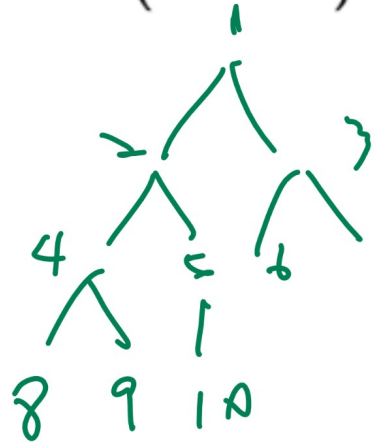
- DFS but stop if path length > 1
- repeat DFS but stop if path length > 2
- ...
- repeat DFS but stop if path length $> d$



IDS Example 1

Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a IDS expansion sequence?



depth 1: 1
 depth 2: 2, 3
 depth 3: 4, 5, 6, 7, 8, 9
 depth 4: 8, 9, 10, 11, 12, 13, 14

→ DFS

Iterative Deepening Search Performance

Discussion

- IDS is complete. ✓
- IDS is optimal with $c = 1$. ✓

Iterative Deepening Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d .

$$T = \overbrace{db + (d-1)b^2 + \dots + 3b^{d-2} + 2b^{d-1} + 1b^d}^{1 \cdot (d+1)}$$

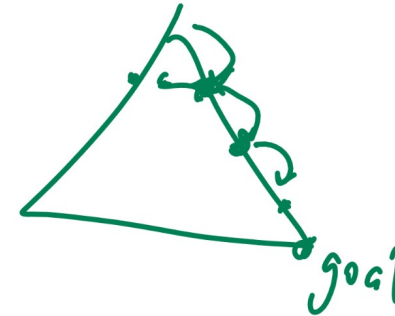
$$(1) + (\underline{1} + \underline{b}) + (\underline{1} + \underline{b} + \underline{b}^2) + (\underline{1} + \underline{b} + \underline{b}^2 + \underline{b}^3) + \dots + (1 + b + b^2 + \dots + b^d)$$

- Space complexity: it has the same space complexity as DFS.

$$S = (b-1)d + 1$$

Uniformed vs. Informed Search

Motivation



BFS, DFS, IDS.

- Uninformed search means only the goal G and the successor functions s' are given.
- Informed search means which non-goal states are better is also known.

Heuristic Diagram

Motivation



$\approx h(s)$
↳ approximate heuristic.

order Queue by

- $g(s) \rightarrow$ UCS ←
- $h(s) \rightarrow$ Greedy
- $g(s) + h(s) \rightarrow$ A

Uniform Cost Search

Description

- Expand the vertices with the lowest current path cost $g(s)$ first.
- It is BFS with a priority queue based on $g(s)$.
- It is equivalent to BFS if $c = 1$ is constant on all edges.
- It is also called Dijkstra's Algorithm.

UCS Example 1

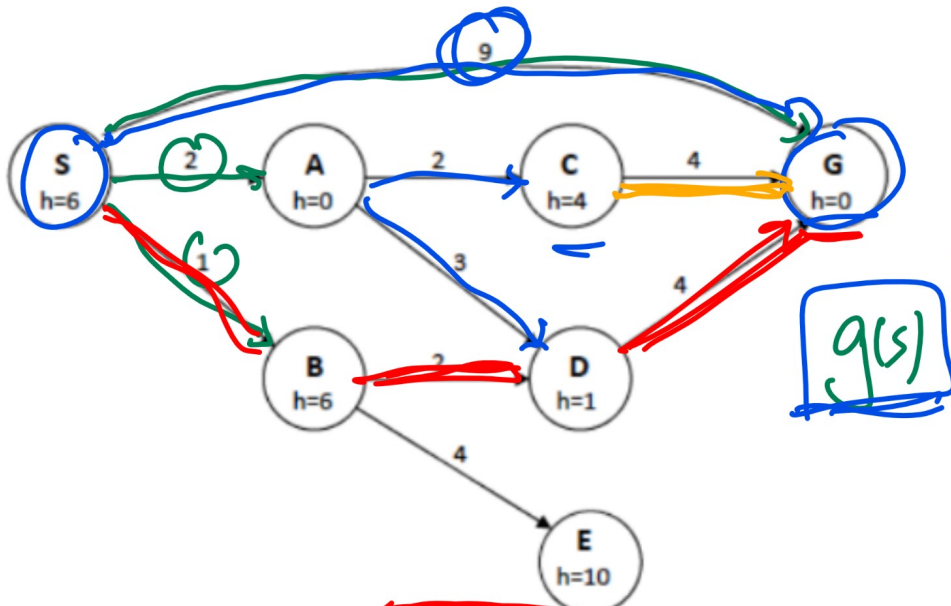
Quiz

- Spring 2017 Midterm Q1
- Given the following adjacency matrix. Find UCS expansion path.

—	S	A	B	C	D	E	G
S	$h = 6$	2	1	—	—	—	9
A	—	$h = 0$	—	2	3	—	—
B	—	—	$h = 6$	—	2	4	—
C	—	—	—	$h = 4$	—	—	4
D	—	—	—	—	$h = 1$	—	4
E	—	—	—	—	—	$h = 10$	—
G	—	—	—	—	—	—	$h = 0$

UCS Example 1 Diagram

Uniform Cost Quiz



Stop when expand G

Priority Queue



UCS Example 2

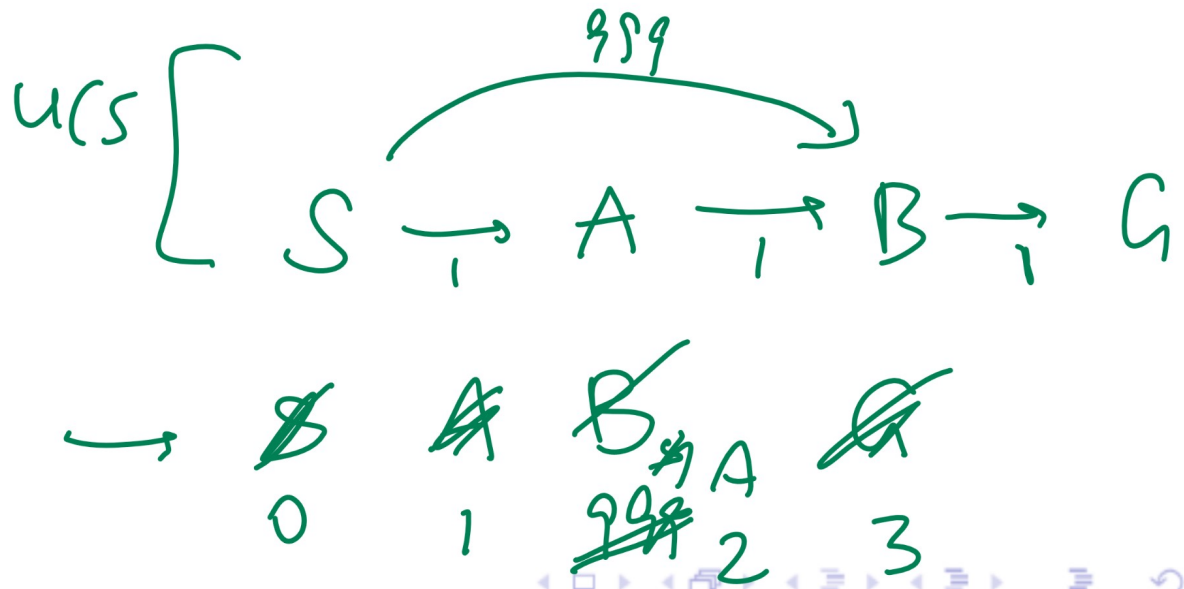
Quiz

- Find UCS expansion path

-	S	A	B	G
S	$h = 3$	1	999	-
A	-	$h = 1000$	1	-
B	-	-	$h = 1$	1
G	-	-	-	$h = 0$

Q2

- A: S, A, B, G
- B: S, B, G
- C: S, B, A, G
- D: S, B, A, B, G



Uniform Cost Search Performance

Discussion

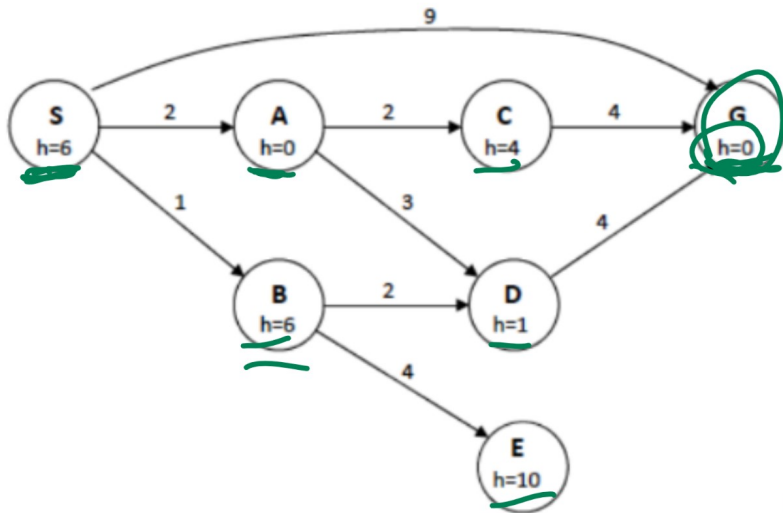
- UCS is complete. ✓
- UCS is optimal with any c . ✓

Best First Greedy Search

Description

~~BFS~~

- Expand the vertices with the lowest heuristic cost $h(s)$ first.
- Use a priority queue based on $h(s)$.



PQ:

~~A~~ ~~G~~ D C ~~S~~ B

$h(s)$ 0 0 1 4 6 6

S A G → solution S → G

Arbitrarily Chosen



PS
 → Euclidean
 manhattan

Greedy Example 1

Quiz

- Given the following adjacency matrix. Find Greedy Search expansion path.

—	S	A	B	C	D	E	G
S	$h = 6$	2	1	—	—	—	9
A	—	$h = 0$	—	2	3	—	—
B	—	—	$h = 6$	—	2	4	—
C	—	—	—	$h = 4$	—	—	4
D	—	—	—	—	$h = 1$	—	4
E	—	—	—	—	—	$h = 10$	—
G	—	—	—	—	—	—	$h = 0$

Greedy Example 2

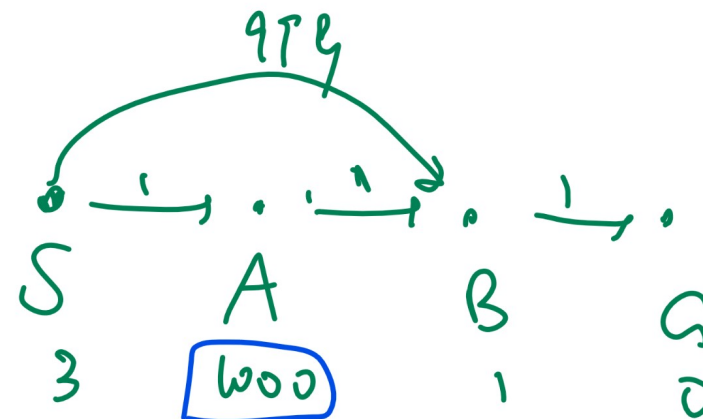
Quiz

- Find Greedy expansion path

—	S	A	B	G
S	$h = 3$	1	999	—
A	—	$h = 1000$	1	—
B	—	—	$h = 1$	1
G	—	—	—	$h = 0$

Q3

- A: S, A, B, G
- B: S, B, G**
- C: S, B, A, G
- D: S, B, A, B, G

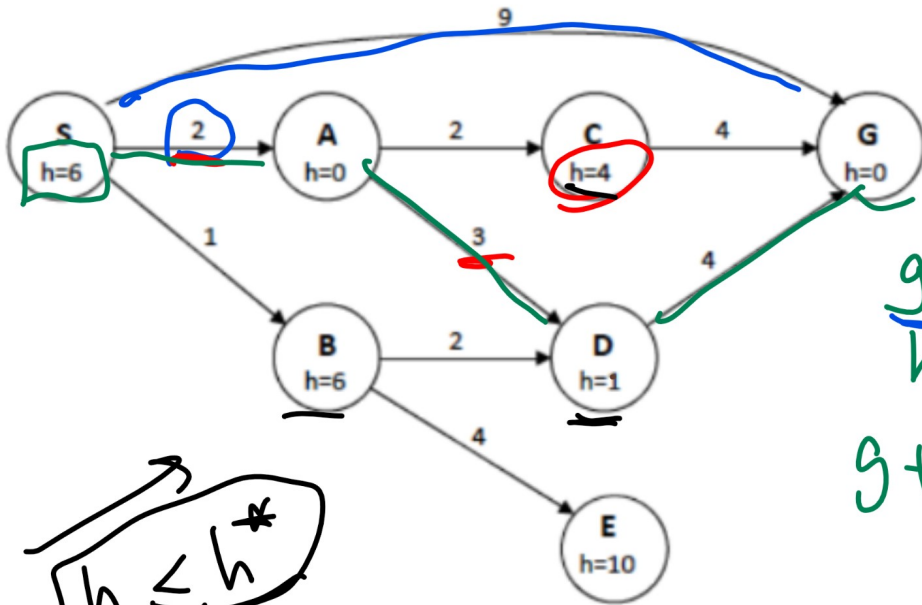


~~0~~
~~1~~
~~3~~

A
 1000

A Search Example 1 Diagram

Quiz



$h \leq h^*$
 $A^* \in Q \rightarrow B$

PQ

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
2	3	0	5	1	7	4	9	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95
0	1	6	1	6	0	4	9	4	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	
2	4	6	6	7	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	

$G_S \rightarrow 9$
 $G_D \rightarrow 9+0$

g/h
 $g+h$

P5

A	B	C	P
A	B		

predecessor.

stop

A Search Example 2

Quiz

- Find A search expansion path

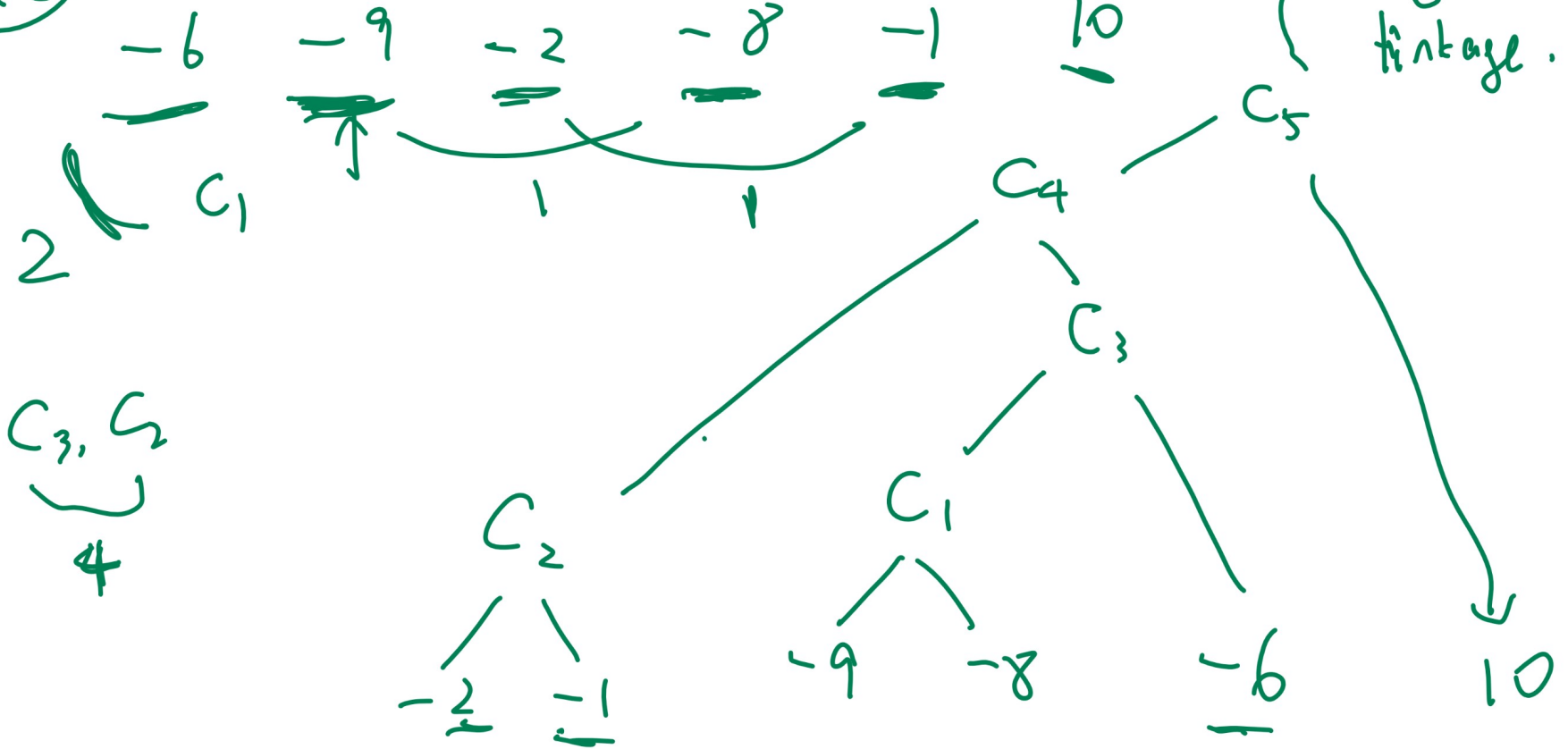
—	S	A	B	G
S	$h = 3$	1	999	—
A	—	$h = 1000$	1	—
B	—	—	$h = 1$	1
G	—	—	—	$h = 0$

- A: S, A, B, G
- B: S, B, G
- C: S, B, A, G
- D: S, B, A, B, G

A Search Example 2 Diagram

Quiz

MP
Q5



A Search Performance

Discussion

- A is complete. ✓
- A is not optimal. ✗