CS540 Introduction to Artificial Intelligence
Lecture 20

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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Course Evaluation

Admin

- M11 is course evaluation on AEFIS, please submit a course evaluation, it is very important for me! Thanks!
- If you have no comments and suggestions, please write "none".

Coordination Game

Admin

Q1

- You are not allowed to discuss anything about this question in the public chat. There will be around 5 new questions on the final exam. I will post n of them before the exam (probably next Tuesday):

Stag Hunt Game.

- A: $n = 0$.
- B: $n = 1$ if more than 50 percent of you choose B.
- C: $n = 2$ if more than 75 percent of you choose C.
- D: $n = 3$ if more than 98 percent of you choose D.
- E: $n = 0$.
- I will repeat this question a second time. If you fail to coordinate both times, I will not post any of the new questions.

Coordination Game Repeat

Admin

Q3

- You are not allowed to discuss anything about this question in the public chat. There will be around 5 new questions on the final exam. I will post n of them before the exam (probably next Tuesday):
- A: $n = 0$.
- B: $n = 1$ if more than 50 percent of you choose B.
- C: $n = 2$ if more than 75 percent of you choose C.
- D: $n = 3$ if more than ~~98~~ 97 percent of you choose D.
- E: $n = 0$.

Course Evaluation

Admin

P1 – P6

7.29

due ~~7.28~~

- M11 is course evaluation on AEFIS, please submit a course evaluation, it is very important for me! Thanks!
- Thursday and Friday: Review Sessions.
- Tuesday and Wednesday: Public Office Hours.

Guess Average Game

Motivation

on M12

- Write down an integer between 0 and 100 that is the closest to two thirds (2/3) of the average of everyone's (including yours) integers.

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

payoff

$$br_{MAX}(s_{MIN}) = \arg\max_{s \in S_{MAX}} c(s, s_{MIN})$$

$$br_{MIN}(s_{MAX}) = \arg\min_{s \in S_{MIN}} c(s_{MAX}, s)$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c\left(s_i, s\right) > c\left(s_{i'}, s\right) \ \forall\, s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c\left(s, s_i\right) < c\left(s, s_{i'}\right) \ \forall\, s \in S_{MAX}$$

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Dominated Strategy Example 1

Quiz

- Fall 2005 Final Q6
- Both players are MAX players. What are the dominated strategies for the ROW player? Choose E if none of the strategies are dominated.

COL

ROW

–	A	B	C
A	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
C	(4, 1)	(2, 8)	(5, 3)
D	(3, 6)	(4, 0)	(1, 9)

NE

COL

$br_{ROW}(A) = C$

$br_{COL}(A) = B$

$br_{ROW}(B) = B$

$br_R(C) = C$

Dominated Strategy Example 2

Quiz

- Fall 2005 Final Q6
- Both players are MAX players. What are the dominated strategies for the COLUMN player? Choose E if none of the strategies are dominated.

IESDS

–	A	B	C
A	(2,4)	(3,7)	(4,5)
B	(1,2)	(5,4)	(2,3)
C	(4,1)	(2,8)	(5,3)
D	(3,6)	(4,0)	(1,9)

A is dominated by C.

D is dom by B

Rationalizable set

(B, B) → (5, 4)

↳ C is dom by B

Nash Equilibrium Example

Quiz

A, C dom by B

Q4

- Fall 2005 Final Q5, Fall 2006 Final Q4
- Find the value of the Nash equilibrium of the following zero-sum game.

MIN

MAX = ROW

–	I	II	III
I	(-4, 4)	(-7, 7)	(-3, 3)
II	(9, -9)	(1, -1)	(7, -7)
III	(-6, 6)	(-1, 1)	(5, -5)

- A: -7 , B: 9 , C: -3 , $\boxed{\text{D: 1,}}$ E: -4

Public Good Game

Discussion

- You received one free point for this question and you have two choices.
- A: Donate the point.
- B: Keep the point.
- Your final grade is the points you keep plus twice the average donation.

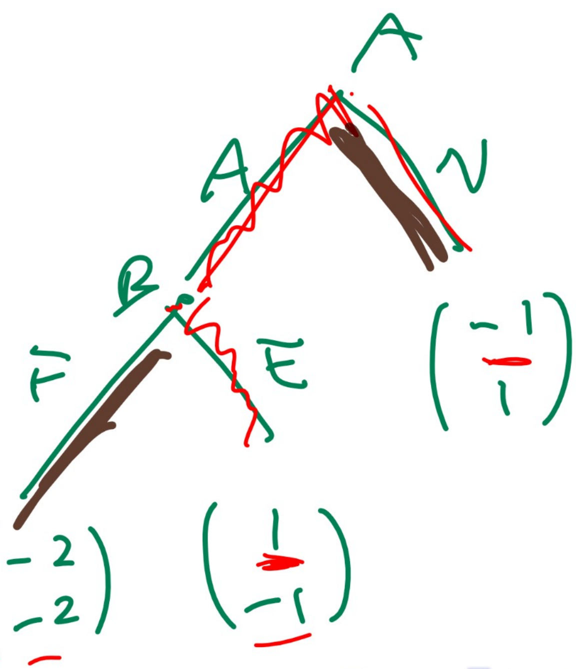
Non-credible Threat Example 1

Quiz

- Country A can choose to Attack or Not attack country B. If country A chooses to Attack, country B can choose to Fight back or Escape. The costs are the largest for both countries if they fight, but otherwise, A prefers attacking (and B escaping) and B prefers A not attacking. What are the Nash equilibria?
- A: (A, F)
- B: (A, E)
- C: (N, F)
- D: (N, E)
- E: (N)

Solution

SPE



extensive form

Non-credible Threat Example 1 Derivation

Quiz

normal form

	F	$\bar{E}$
A	$-2, -2$	$1, -1$
$\sim$	$-1, 1$	$-1, 1$

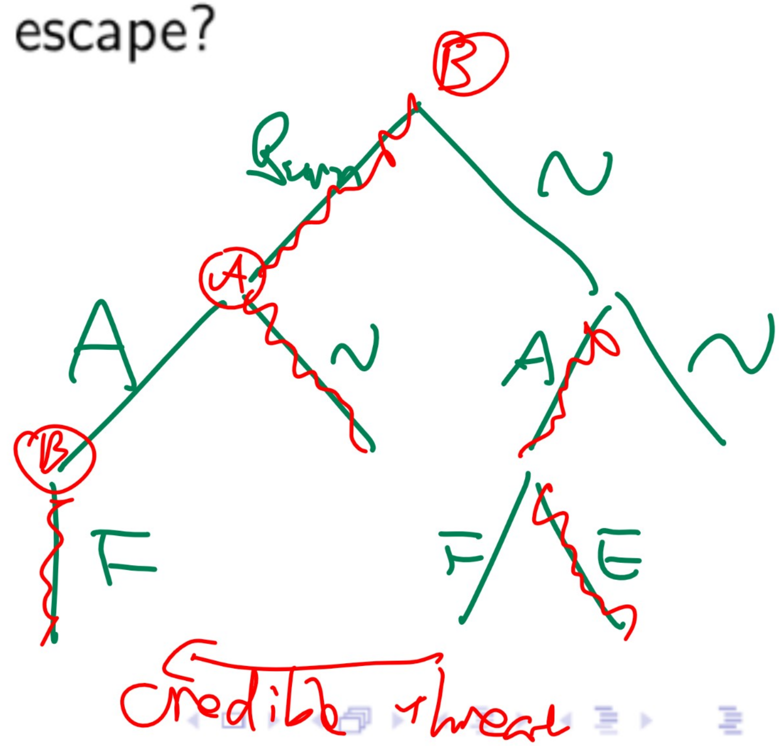
$\boxed{(N, \bar{E})}$, (A, E)

solution

Non-credible Threat Example 2

Quiz

- What if country B can burn the bridge at the beginning of the game so that it cannot choose to escape?



Mixed Strategy Nash Equilibrium

Definition

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Battle of the Sexes Example

Discussion

- Battle of the Sexes (BoS, also called Bach or Stravinsky) is a game that models coordination in which two players have different preferences in which alternative to coordinate on.

–	Bach	Stravinsky
Bach	A (x, y)	$B\,(0,0)$
Stravinsky	$C\,(0,0)$	D (y, x)

Romeo

Juliet

$y > x > 0$

Battle of the Sexes Example 1

Quiz

- Find all Nash equilibria of the following game.

–	I q	II $1-q$
p I	$A\,(3,5)$	$B\,(0,0)$
$1-p$ II	$C\,(0,0)$	$D\,(5,3)$

payoff if I   payoff if II

$$3q + 0(1-q) > 0q + 5(1-q) \Rightarrow q > \frac{5}{8}$$

$$\text{Row} = \begin{cases} \text{I} & q > \frac{5}{8} \\ \text{mix} & \boxed{q = \frac{5}{8}} \\ \text{II} & q < \frac{5}{8} \end{cases}$$

Battle of the Sexes Example 1 Derivation 1

Quiz

$5p + 0(1-p) \underset{<}{\overset{=}{>}} 0p + 3(1-p)$

$\boxed{p = \frac{3}{8}}$ $\Rightarrow p > \frac{3}{8}$

$p < \frac{3}{8}$

$$\text{Col} = \begin{cases} \text{I} \\ \text{mix} \\ \text{II} \end{cases}$$

- Find all mixed strategy Nash equilibria of the following game.

Row		–	I (q)	II ($1-q$)
	p	I	(3, 5)	(0, 0)
	$1-p$	II	(0, 0)	(5, 3)

Row Col

$$NE = \left(\frac{3}{8}, \frac{5}{8}\right)$$

$$= \left(\text{I}^{\left(\frac{3}{8}\right)} \text{II}^{\left(\frac{5}{8}\right)}, \ \text{I}^{\left(\frac{5}{8}\right)} \text{II}^{\left(\frac{3}{8}\right)}\right)$$

Battle of the Sexes Example 1 Derivation 2

Quiz

payoff I = payoff II

$$3q + 0(1-q) = 0q + 5(1-q)$$

	q	1-q
–	I	II
p: I	(3, 5)	(0, 0)
1-p: II	(0, 0)	(5, 3)

$$q = \frac{5}{8}$$

$$p = \frac{3}{8}$$

Mixed Strategy Example 1

Quiz

Q5 (last)

- Which ones of the following are mixed strategy Nash equilibria?

–	L	R
U	(3, 1)	(0, 0)
D	(0, 1)	(1, 1)

- ~~A~~: (always U , (L $\frac{1}{2}$ of the time , R $\frac{1}{2}$ of the time))
  L
- ~~B~~: (~~always D~~ U , (L $\frac{1}{2}$ of the time , R $\frac{1}{2}$ of the time))
  U → 1.5
  D → 0.5
- ~~C~~: ((U $\frac{1}{2}$ of the time , D $\frac{1}{2}$ of the time) U , always L)
- ~~D~~: ((U $\frac{1}{2}$ of the time , D $\frac{1}{2}$ of the time) D , always R)
- ~~E~~: ((U $\frac{1}{2}$ time , D $\frac{1}{2}$ time) U , (L $\frac{1}{2}$ time , R $\frac{1}{2}$ time))

Mixed Strategy Example 1 Derivation

Quiz

$1-p > 3p$

$\boxed{\text{payoff } D > \text{payoff } U}$

$p < \frac{1}{4}$

—	L (p)	R ($1-p$)
U	(3, 1)	(0, 0)
D	(0, 1)	(1, 1)

NE (always D, $(p, 1-p), p < \frac{1}{4}$)

(U, L)

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Fixed Point Nash Equilibrium

Algorithm

- Input: the payoff table $c(s_i, s_j)$ for $s_i \in S_{MAX}, s_j \in S_{MIN}$.
- Output: the Nash equilibria.
- Start with random state $s = (s_{MAX}, s_{MIN})$.
- Update the state by computing the best response of one of the players.

$$\begin{aligned} \text{either } s' &= \left(br_{MAX}\left(s_{MIN}\right), br_{MIN}\left(br_{MAX}\left(s_{MIN}\right)\right)\right) \\ \text{or } s' &= \left(br_{MAX}\left(br_{MIN}\left(s_{MAX}\right)\right), br_{MIN}\left(s_{MAX}\right)\right) \end{aligned}$$

- Stop when $s' = s$.