

CS540 Introduction to Artificial Intelligence

Lecture 20

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Based on lecture slides by Jerry Zhu and Yingyu Liang

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Guess Average Game, Part I

Quiz (Participation)

- Write down an integer between 0 and 100 that is the closest to two thirds ($2/3$) of the average of everyone's (including yours) integers.
- A: 0 – 10
- B: 11 – 20
- C: 21 – 30
- D: 31 – 60
- E: 61 – 100

| | | | |

{0, 1} are rationalizable

Guess Average Game, Part II

Quiz (Participation)

$$S = (S_1, S_2, \dots, S_{25})$$

$$S_i = [0, \dots, 66] , (66, \dots, 100)$$

1 - rationalizable

$$\frac{2}{3} \left(\frac{S_1 + S_2 + \dots + S_{25}}{25} \right) \leq 66$$

never best response,

Suppose people only choose 1 - rationalizable action.

$$S_i = [0, \dots, 44] , (44, \dots, 66]$$

2 - rationalizable,

never best response

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞ -rationalizable.

Traveler's Dilemma, Part I

Quiz (Participation)

- Two identical antiques are lost. The airline only knows that its value is at most 100 dollars, so the airline asks their owners (travelers) to report its value (nonnegative integers). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive 2 dollars in reward. What will be the reports?

- A: (0, 0)
- B: (0, 1) or (1, 0)
- C: (99, 99)
- D: (99, 100) or (100, 99)
- E: (100, 100)

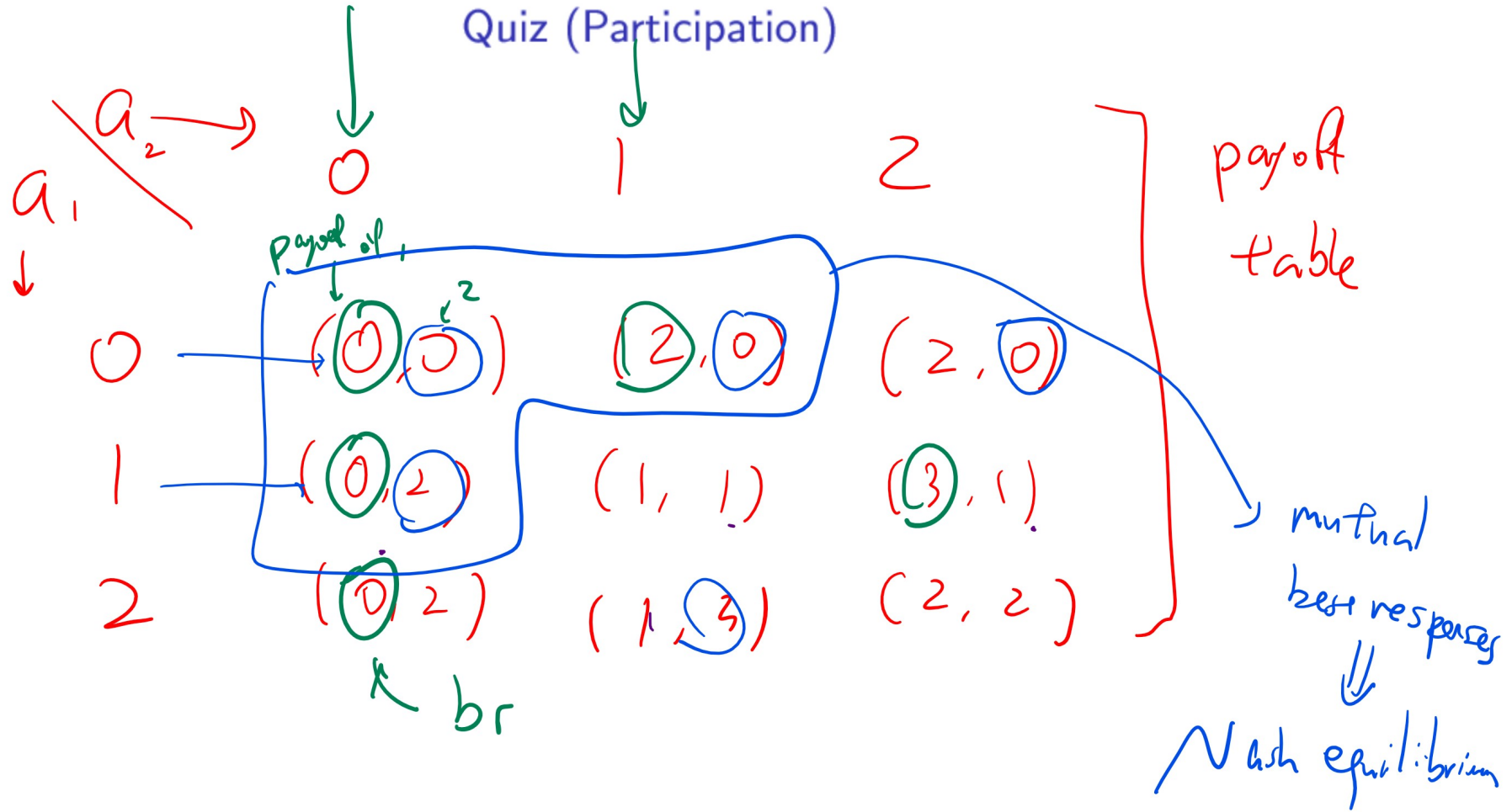
100 is not rationalizable — —

action of opponent	action	payment
100	100	100
	99	101
	98	100
	97	99

99	100	99
	99	99
	98	100

Traveler's Dilemma, Part II

Quiz (Participation)



Normal Form Games

Definition

- In a simultaneous move game, a state represents one action from each player.
- The costs or rewards, sometimes called payoffs, are written in a payoff table.
- The players are usually called the ROW player and the COLUMN player.
- If the game is zero-sum, the convention is: ROW player is MAX and COLUMN player is MIN.

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}(s_{MIN}) = \arg \max_{s \in S_{MAX}} c(s, s_{MIN})$$

$$br_{MIN}(s_{MAX}) = \arg \min_{s \in S_{MIN}} c(s_{MAX}, s)$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c(s_i, s) > c(s_{i'}, s) \quad \forall s \in S_{MIN}$$

$$s_i \succ_{MIN} s_{i'} \text{ if } c(s, s_i) < c(s, s_{i'}) \quad \forall s \in S_{MAX}$$

bad action \rightarrow dominated

good action may NOT
be dominant.

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Weakly Dominated and Dominant Strategy

Definition

- An action s_i weakly dominates another $s_{i'}$ if it leads to a better state or a state with the same payoff no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c(s_i, s) \geq c(s_{i'}, s) \quad \forall s \in S_{MIN}$$

$$s_i \succ_{MIN} s_{i'} \text{ if } c(s, s_i) \leq c(s, s_{i'}) \quad \forall s \in S_{MAX}$$

- The action $s_{i'}$ is called weakly dominated.

Dominated Strategy Example, Part I

Quiz (Graded)

- Fall 2005 Final Q6
- Both players are MAX players. What are the dominated strategies for the ROW player? Choose E if none of the strategies are dominated.

no mixed strategy.

—	A	B	C
A	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
C	(4, 1)	(2, 8)	(5, 3)
D	(3, 6)	(4, 0)	(1, 9)

Q4

not dominated.
for original game.

Q7

D

IESDS

≠ rationalizable (IENBR)

Dominated Strategy Example, Part II

Quiz (Graded)

- Fall 2005 Final Q6
- Both players are MAX players. What are the dominated strategies for the COLUMN player? Choose E if none of the strategies are dominated.

Q6

—	A	B	C
A	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
C	(4, 1)	(2, 8)	(5, 3)
D	(3, 6)	(4, 0)	(1, 9)

$C > A$
 no matter
 what Row
 is chosen.

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example

Quiz (Graded)

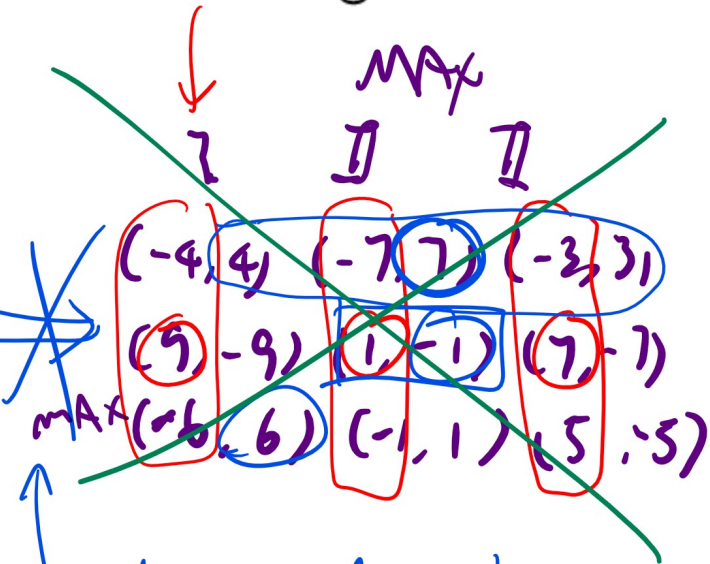
Q 10

- Fall 2005 Final Q5, Fall 2006 Final Q4
- Find the value of the Nash equilibrium of the following zero-sum game.

MIN

	I	II	III
I	-4	-7	-3
II	9	1	7
III	-6	-1	5

MAX →



- A: -7 , B: 9 , C: -3 , D: 1, E: -4

circle or
cell with 2 circles ⇒ NE.

Properties of Nash Equilibrium

Definition

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- Nash equilibrium can be found by iterated elimination of strictly dominated actions. *contains*
- The above statements are not true for weakly dominated actions.

↘ an example on final exam.

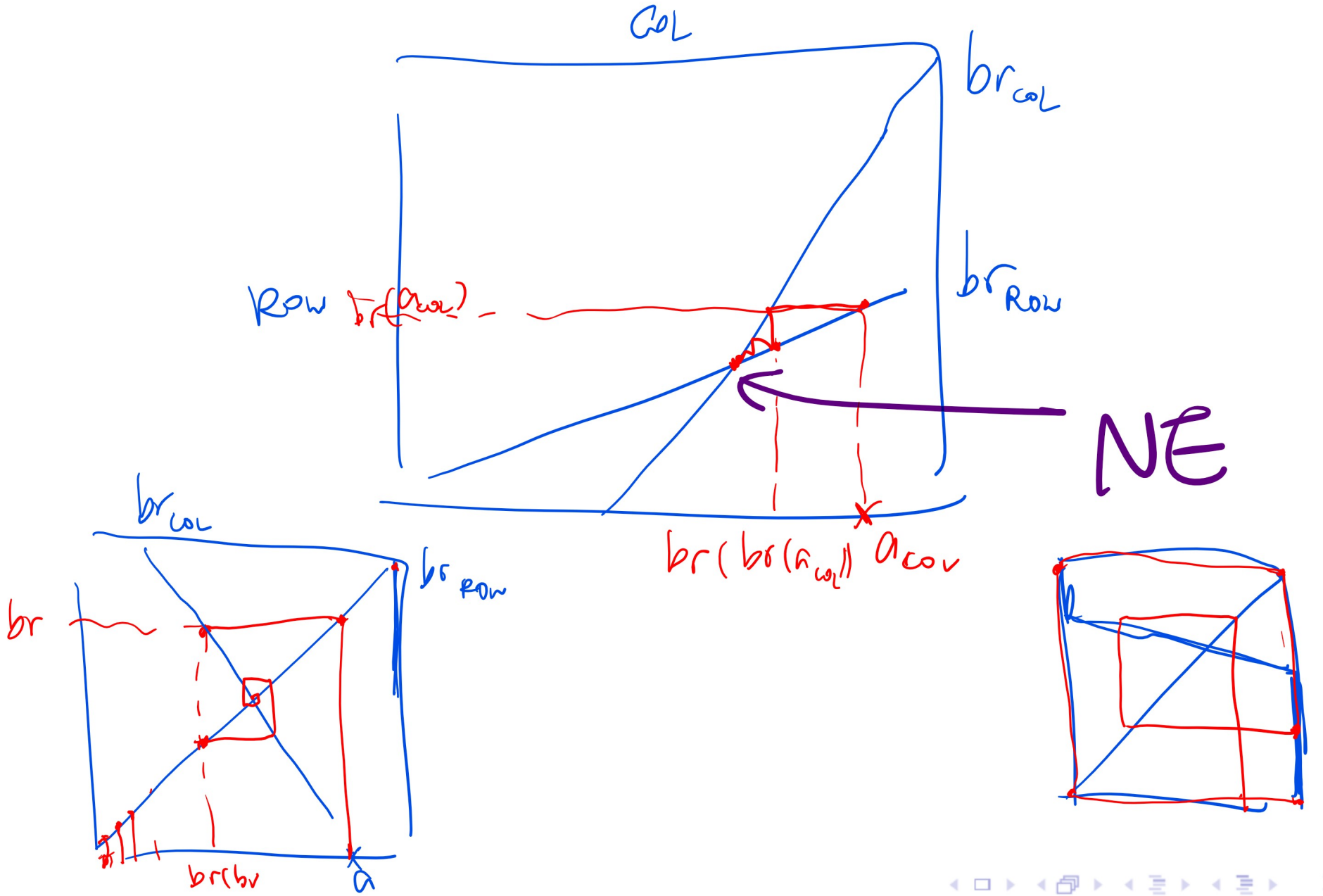
Fixed Point Algorithm

Description

- For small games, it is possible to find all the best responses. The states that are best responses for all players are the solutions of the game.
- For large games, start with a random action, find the best response for each player and update until the state is not changing.

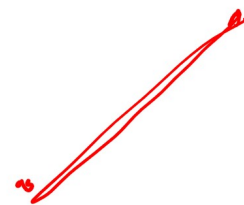
Fixed Point Diagram

Definition



Mixed Strategy Nash Equilibrium

Definition



- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Battle of the Sexes Example, Part I

Quiz (Graded)

- Find all Nash equilibria of the following game.

—	I	II
I	$A(3, 5)$	$B(0, 0)$
II	$C(0, 0)$	$D(5, 3)$

Battle of the Sexes Example, Part II

Quiz (Graded)

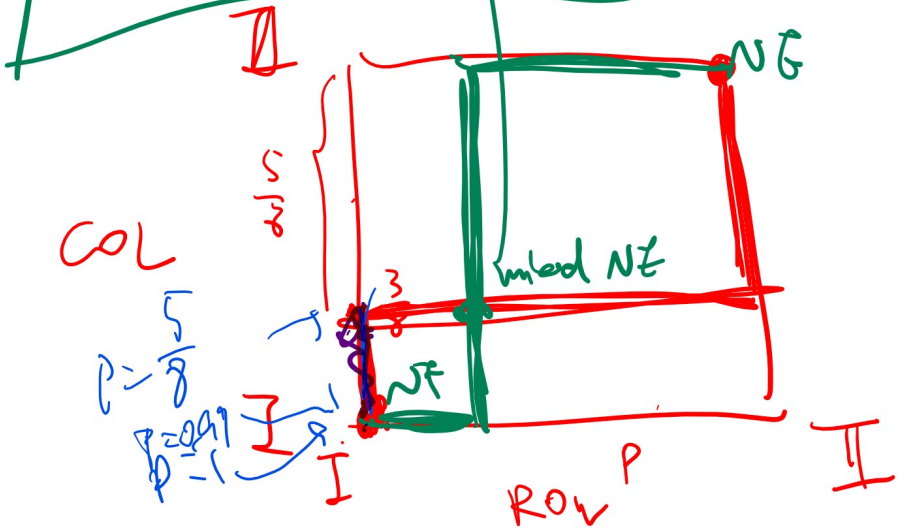
$$\left(\left(\frac{3}{8} I, \frac{5}{8} II \right), \left(\frac{5}{8} I, \frac{3}{8} II \right) \right) = NE$$

- Find all mixed strategy Nash equilibria of the following game.

		MAX	
		I	II
I		(3, 5)	(0, 0)
II		(0, 0)	(5, 3)

Row, COL
plays I
with prob $\frac{5}{8}$
II $\frac{3}{8}$ MAX

(I, I) = NE
(II, II) = NE
given Row $q, 1-q$
I II



P for Row
1-P for COL

given COL $p, 1-p$
 $p > \frac{5}{8} \Rightarrow I > II$

I $\rightarrow 3p + 0(1-p)$
 II $\rightarrow 0p + 5(1-p)$
 expected payoffs,
 $3p = 5 - 5p \Rightarrow p = \frac{5}{8} \quad 1-p = \frac{3}{8}$

Mixed Strategy Example, Part I

Quiz (Graded)

- Fall 2008 Final Q6
- What is a mixed strategy Nash equilibrium of Rock Paper Scissors?

R	→	0
P	→	0
S	→	0

- A: $\left(\left(\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S \right), \left(\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S \right) \right)$
- B: $\left(\left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S \right), \left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S \right) \right)$
- C: $\left(\left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S \right), \left(\frac{1}{2}R, \frac{1}{4}P, \frac{1}{4}S \right) \right)$
- D: $\left(\left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S \right), \left(\frac{1}{4}R, \frac{1}{2}P, \frac{1}{4}S \right) \right)$

MAX

	R	P	S
R	0	-1	1
P	-1	0	-1
S	1	1	0

for MAX

R → $\frac{1}{4} \cdot 0 + (-1) \cdot \frac{1}{4} + 1 \cdot \frac{1}{2}$
 $= \frac{1}{4}$

P → $\frac{1}{4} \cdot 1 + 0 \cdot \frac{1}{4} + (-1) \cdot \frac{1}{2}$
 $= -\frac{1}{4}$

S → $(-1) \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} + 0 \cdot \frac{1}{2} = 0$

Mixed Strategy Example, Part II

Quiz (Graded)

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Fixed Point Nash Equilibrium

Algorithm

- Input: the payoff table $c(s_i, s_j)$ for $s_i \in S_{MAX}, s_j \in S_{MIN}$.
- Output: the Nash equilibria.
- Start with random state $s = (s_{MAX}, s_{MIN})$.
- Update the state by computing the best response of one of the players.

$$\text{either } s' = (br_{MAX}(s_{MIN}), br_{MIN}(br_{MAX}(s_{MIN})))$$

$$\text{or } s' = (br_{MAX}(br_{MIN}(s_{MAX})), br_{MIN}(s_{MAX}))$$

- Stop when $s' = s$.

Normal Form of Sequential Games

Discussion

- Sequential games can have normal form too, but the solution concept is different.
- Nash equilibria of the normal form may not be a solution of the original sequential form game.

Non-credible Threat Example, Part I

Quiz (Graded)

- Country A can choose to Attack or Not attack country B. If country A chooses to Attack, country B can choose to Fight back or Escape. The costs are the largest for both countries if they fight, but otherwise, A prefers attacking (and B escaping) and B prefers A not attacking. What are the Nash equilibria?
- A: (A, F)
- B: (A, E)
- C: (N, F)
- D: (N, E)
- E: (N)

Non-credible Threat Example, Part II

Quiz (Graded)

- What if country B can burn the bridge at the beginning of the game so that it cannot choose to escape?

Wage Competition Example

Quiz (Graded)

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

–	C	D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.