CS540 Introduction to Artificial Intelligence
Lecture 20

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Based on lecture slides by Jerry Zhu and Yingyu Liang

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Guess Average Game, Part I

Quiz (Participation)

- Write down an integer between 0 and 100 that is the closest to two thirds (2/3) of the average of everyone's (including yours) integers.
- A: 0 – 10
- B: 11 – 20
- C: 21 – 30
- D: 31 – 60
- E: 61 – 100

no matter what action other players pick

never (66, 100]

never best response

only if everyone's > 100

not nbr ⇒ 1 - rationalizable actions [0, 66]

Guess Average Game, Part II

Quiz (Participation)

$\underbrace{[0, 44]}_{\text{2 - rationalizable}}$

$\underbrace{(44, 66]}$

never best response to all actions other players if they only play 1-rationalizable action

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Solution concept

Traveler's Dilemma, Part I

Quiz (Participation)

- Two identical antiques are lost. The airline only knows that its value is at most 100 dollars, so the airline asks their owners (travelers) to report its value (nonnegative integers). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive 2 dollars in reward. What will be the reports?
- A: $(0, 0)$
- B: $(0, 1)$ or $(1, 0)$
- C: $(99, 99)$
- D: $(99, 100)$ or $(100, 99)$
- E: $(100, 100)$

Traveler's Dilemma, Part II

Quiz (Participation)

Normal Form Games

Definition

$s = (s_1, s_2 \ldots)$

- In a simultaneous move game, a state represents one action from each player.
- The costs or rewards, sometimes called payoffs, are written in a payoff table.
- The players are usually called the ROW player and the COLUMN player.
- If the game is zero-sum, the convention is: ROW player is MAX and COLUMN player is MIN.

Prisoner's Dilemma

COL MIN

ROW MIN

	Deny	Confess
Deny	2, 2	10, 1
Confess	1, 10	5, 5

Nash equilibrium

$br_{ROW}(COL, D) = C$

$br_{ROW}(COL, C) = C$

$br_{COL}(ROW, D) = C$

Best Response

$br_{COL}(ROW, C) = C$

Definition

Deny is a dominated strategy

Confess happens to be dominant strategy.

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}(s_{MIN}) = \arg\max_{s \in S_{MAX}} c(s, s_{MIN})$$

$$br_{MIN}(s_{MAX}) = \arg\min_{s \in S_{MIN}} c(s_{MAX}, s)$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c(s_i, s) > c(s_{i'}, s) \ \forall s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c(s, s_i) < c(s, s_{i'}) \ \forall s \in S_{MAX}$$

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Weakly Dominated and Dominant Strategy

Definition

- An action s_i weakly dominates another $s_{i'}$ if it leads to a better state or a state with the same payoff no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c(s_i, s) \geqslant c(s_{i'}, s) \ \forall\, s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c(s, s_i) \leqslant c(s, s_{i'}) \ \forall\, s \in S_{MAX}$$

- The action $s_{i'}$ is called weakly dominated.

Dominated Strategy Example, Part I

Quiz (Graded)

no mixed strategy

- Fall 2005 Final Q6
- Both players are MAX players. What are the dominated strategies for the ~~ROW~~ COL player? Choose E if none of the strategies are donimated.

Col

rewards ⇒ MAX players.

Q13 E

Q15 A

ROW

–	A	B	C
A	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
C	(4, 1)	(2, 8)	(5, 3)
D	(3, 6)	(4, 0)	(1, 9)

Dominated Strategy Example, Part II

Quiz (Graded)

- Fall 2005 Final Q6
- Both players are MAX players. What are the dominated strategies for the COLUMN player? Choose E if none of the strategies are donimated.

–	~~A~~	B	~~C~~
~~A~~	~~(2, 4)~~	~~(3, 7)~~	~~(4, 5)~~
B	(1, 2)	(5, 4)	(2, 3)
~~C~~	~~(4, 1)~~	~~(2, 8)~~	~~(5, 3)~~
~~D~~	~~(3, 6)~~	~~(4, 0)~~	~~(1, 9)~~

S = (B, B)

iterated elimination of Strictly dominated strategies

IESDS ≠ rationalizable. (IENBR)

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example

Quiz (Graded)

- Fall 2005 Final Q5, Fall 2006 Final Q4
- Find the value of the Nash equilibrium of the following zero-sum game.

–	I	II	III
I	-4	-7	-3
II	9	1	7
III	-6	-1	5

max along columns
min along rows.

- A: -7 , B: 9 , C: -3 , D: 1, E: -4

Properties of Nash Equilibrium

Definition

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- All Nash equilibria ~~can be found be~~ are contained in iterated elimination of strictly dominated actions.
- The above statements are not true for weakly dominated actions.

example on final

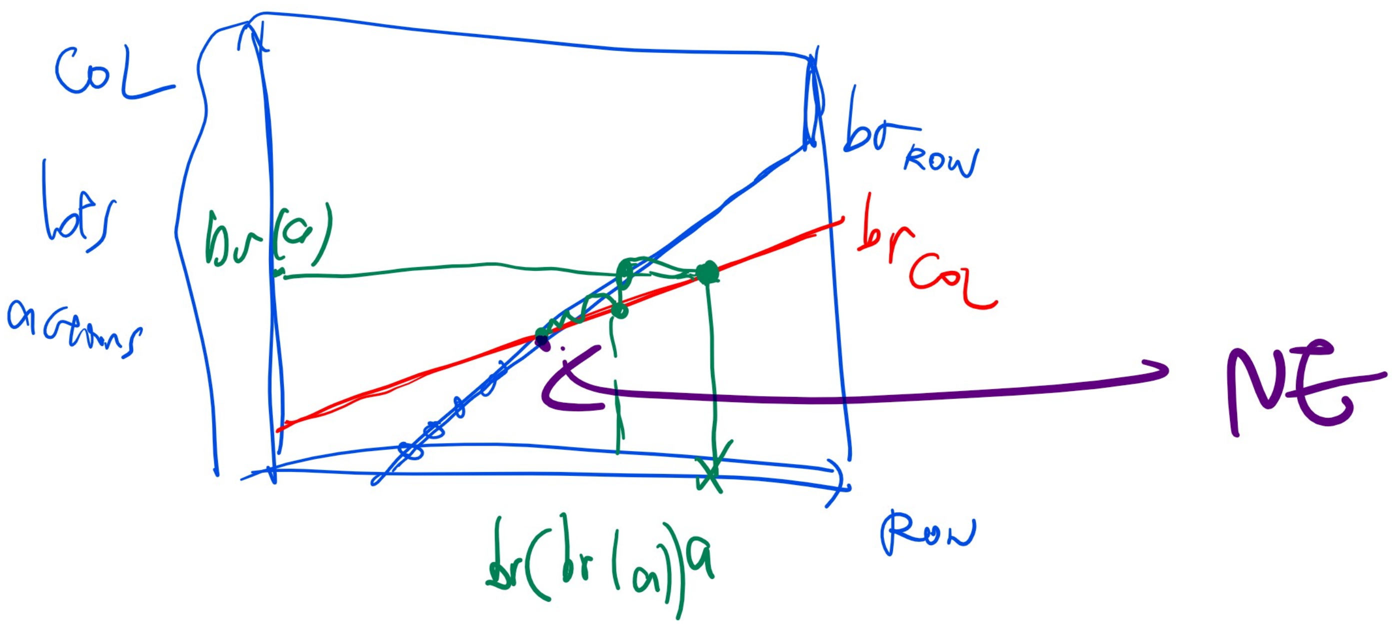
Fixed Point Algorithm

Description

- For small games, it is possible to find all the best responses. The states that are best responses for all players are the solutions of the game.
- For large games, start with a random action, find the best response for each player and update until the state is not changing.

Fixed Point Diagram

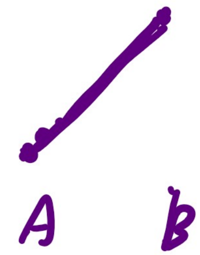
Definition



Mixed Strategy Nash Equilibrium

Definition

to make set of actions continuous (convex)



- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Battle of the Sexes Example, Part I

Quiz (Graded)

$\left(\left(\tfrac{1}{2} \mathrm{I}, \tfrac{1}{2} \mathrm{II} \right), \left(\tfrac{1}{2} \mathrm{I}, \tfrac{1}{2} \mathrm{II} \right) \right)$

$br_{Row}\left(\tfrac{1}{2} \mathrm{I}, \tfrac{1}{2} \mathrm{II} \right) = \begin{cases} \mathrm{I} \rightarrow 1.5 \\ \mathrm{II} \rightarrow 2.5 \end{cases} = \mathrm{II} \neq$

- Find all Nash equilibria of the following game.

girl

boy

–	I	II
I	$A\,(3,5)$	$B\,(0,0)$
II	$C\,(0,0)$	$D\,(5,3)$

A, D Nash equilibria

$\left(\left(\tfrac{3}{8} \mathrm{I}, \tfrac{5}{8} \mathrm{II} \right), \left(\tfrac{5}{8} \mathrm{I}, \tfrac{3}{8} \mathrm{II} \right) \right) \rightarrow$ guess NE

$br_{Row}\left(\tfrac{5}{8} \mathrm{I}, \tfrac{3}{8} \mathrm{II} \right) = \begin{cases} \mathrm{I} & 3 \cdot \tfrac{5}{8} + 0 \cdot \tfrac{3}{8} = \tfrac{15}{8} \\ \mathrm{II} & 5 \cdot \tfrac{3}{8} + 0 \cdot \tfrac{5}{8} = \tfrac{15}{8} \end{cases}$

Battle of the Sexes Example, Part II

Quiz (Graded)

$br_{Row}(\frac{5}{8} I, \frac{3}{8} II) =$ any mixed strategy between I, II

in particular $(\frac{3}{8} I, \frac{5}{8} II)$

- Find all mixed strategy Nash equilibria of the following game.

–	I	II
I	(3, 5)	(0, 0)
II	(0, 0)	(5, 3)
	p	$1-p$

select p such that Row will mix $\Rightarrow$ Row gets same expected payoff from I and II

Row $\to$ I: $3 \cdot p + 0(1-p)$

$\parallel$

Row $\to$ II: $0 \cdot p + 5(1-p)$ $\longrightarrow$ solve to get $p = \frac{5}{8}$

Mixed Strategy Example, Part I

Quiz (Graded)

- Fall 2008 Final Q6
- What is a mixed strategy Nash equilibrium of Rock Paper Scissors?
- A: $\left(\left(\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S\right), \left(\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S\right)\right)$
- B: $\left(\left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S\right), \left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S\right)\right)$
- C: $\left(\left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S\right), \left(\frac{1}{2}R, \frac{1}{4}P, \frac{1}{4}S\right)\right)$
- D: $\left(\left(\frac{1}{4}R, \frac{1}{4}P, \frac{1}{2}S\right), \left(\frac{1}{4}R, \frac{1}{2}P, \frac{1}{4}S\right)\right)$

Mixed Strategy Example, Part II

Quiz (Graded)

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Fixed Point Nash Equilibrium

Algorithm

- Input: the payoff table $c(s_i, s_j)$ for $s_i \in S_{MAX}, s_j \in S_{MIN}$.
- Output: the Nash equilibria.
- Start with random state $s = (s_{MAX}, s_{MIN})$.
- Update the state by computing the best response of one of the players.

$$\begin{aligned} \text{either } s' &= \left(br_{MAX}\left(s_{MIN}\right), br_{MIN}\left(br_{MAX}\left(s_{MIN}\right)\right)\right) \\ \text{or } s' &= \left(br_{MAX}\left(br_{MIN}\left(s_{MAX}\right)\right), br_{MIN}\left(s_{MAX}\right)\right) \end{aligned}$$

- Stop when $s' = s$.

Normal Form of Sequential Games

Discussion

- Sequential games can have normal form too, but the solution concept is different.
- Nash equilibria of the normal form may not be a solution of the original sequential form game.

Non-credible Threat Example, Part I

Quiz (Graded)

- Country A can choose to Attack or Not attack country B. If country A chooses to Attack, country B can choose to Fight back or Escape. The costs are the largest for both countries if they fight, but otherwise, A prefers attacking (and B escaping) and B prefers A not attacking. What are the Nash equilibria?
- A: (A, F)
- B: (A, E)
- C: (N, F)
- D: (N, E)
- E: (N)

Non-credible Threat Example, Part II

Quiz (Graded)

- What if country B can burn the bridge at the beginning of the game so that it cannot choose to escape?

Wage Competition Example

Quiz (Graded)

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

–	C	D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.