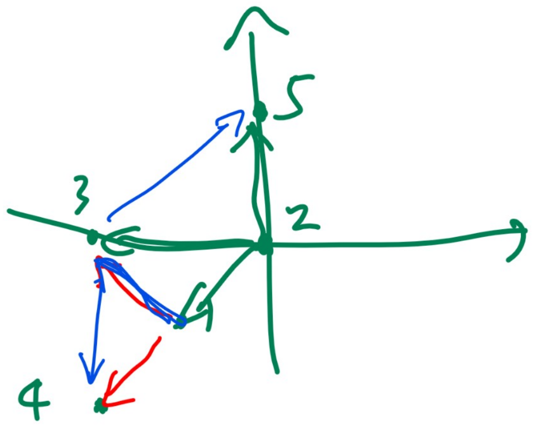
19. Let the states be the following five cities: Fitchburg, Madison, Middleton, Verona, Waunakee. The locations on the map are given in the following table. In the search digraph, the edge costs are the Euclidean distance between the cities corresponding to the state. The initial state is Madison, and the goal state is Verona. What is a vertex expansion sequence if Uniform Cost Search (UCS) is used? Reminder: when there are ties, the state with a smaller index has priority.

19A

Q19, 20

| Index | City | X-coordinate | Y-coordinate | Successors |
|---|---|---|---|---|
| 1 | Fitchburg | -1 | -1 | Middleton, Verona |
| 2 | Madison | 0 | 0 | Fitchburg, Middleton, Waunakee |
| 3 | Middleton | -2 | 0 | Verona |
| 4 | Verona | -2 | -2 | – |
| 5 | Waunakee | 0 | 2 | Middleton |

last year we combined the two 3's and kept only the one with smaller g.




Queue: ~~2~~ ~~1~~$_2$, ~~3~~$_2$, $5_2$, ~~3~~$_1$, ~~4~~$_1$

g: $0$, $\sqrt{2}$, $2$, $2$, $\sqrt{2}+\sqrt{2}$, $2\sqrt{2}$

$4_3$ $3_5$ $4_3$

$2+2$ $2+2\sqrt{2}$ $2\sqrt{2}+2$

total cost from 2 to current state.

choice D.

No need to submit F0A/B!

19A Q36, 39, 40

See TA's note.

36. Continue from the previous question, how many positions (actions) are optimal for player $O$ given both players are using the optimal strategy in all subgames?

| | | |
|---|---|---|
| $X$ | $O$ | $X$ |
| $O$ | $X$ | |
| | | |

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

value of game = 1 for X
$\boxed{-1 \text{ for } O}$
indep of O's choice.

~~19~~ 20. Let the states be the following five cities: Fitchburg, Madison, Middleton, Verona, Waunakee. The locations on the map are given in the following table. In the search digraph, the edge costs are the Euclidean distance between the cities corresponding to the state. The initial state is Madison, and the goal state is Verona. What is a vertex expansion sequence if Uniform Cost Search (UCS) is used? Reminder: when there are ties, the state with a smaller index has priority.

| Index | City | X-coordinate | Y-coordinate | Successors |
|---|---|---|---|---|
| 1 | Fitchburg | -1 | -1 | Middleton, Verona |
| 2 | Madison | 0 | 0 | Fitchburg, Middleton, Waunakee |
| 3 | Middleton | -2 | 0 | Verona |
| 4 | Verona | -2 | -2 | – |
| 5 | Waunakee | 0 | 2 | Middleton |

combining $3_2$, $3_1$ is more efficient when implementing

Queue: $\cancel{2}$ $\cancel{1_2}$, $\cancel{3_2}$, $5_2$, $\cancel{3_1}$, $\cancel{4_1}$, $4_3$, $4_1$

h: 0 0 0 0 0 0 0 0

2, 1, $3_1$, $3_2$, $4_1$

on this year's final: keep repeated nodes.

fitness → max

cost → min

score → ? } need to specify

$\boxed{n^* = 0} \Rightarrow n^* = 1$ N $\leftarrow$ V $\leftarrow$

0 $\geq$ $-c$

## Question 3 [0 points]

- Imagine a population of $N$ = 160 individuals. Each of them simultaneously chooses between taking the vaccine and not. All individuals have the same payoffs. Suppose there are $n$ people who choose not to take the vaccine, then the payoff from not taking the vaccine is $-\alpha \cdot \frac{n}{N}$, and the payoff from taking the vaccine is $-c - \beta \cdot \frac{n}{N}$, $\alpha$ = 19 is the herd immunity coefficient, $\beta$ = 5 measures the ineffectiveness of the vaccine, and $c$ = 5 is the cost of getting the vaccine. In a Nash equilibrium, what is the largest number of individuals who choose to take the vaccine.
- Answer: [ ]. Calculate

NE $n^*$ players pick No, $N - n^*$ players pick V

$\geq 1$

for each of $n^*$ players: No

$$-\alpha \cdot \frac{n^*}{N} \geq -c - \beta \cdot \frac{n^* - 1}{N}$$

for each of $N - n^*$ players, $-c - \beta \frac{n^*}{N} \geq -\alpha \frac{n^*+1}{N}$

$\underline{\phantom{xxxx}} \leq \underline{n^*} \leq \underline{\phantom{xxxx}}$ floor of this.

$br_i$ ( $n_{-i}$ # players not vaccinated ) $= \begin{cases} V & \text{if } -c - \beta \frac{n_{-i}}{N} \geq -\alpha \frac{n_{-i}+1}{N} \\ N & \text{o.w.} \end{cases}$

$n_{-i}$ = # players other than $i$ that picked choice NoVac.

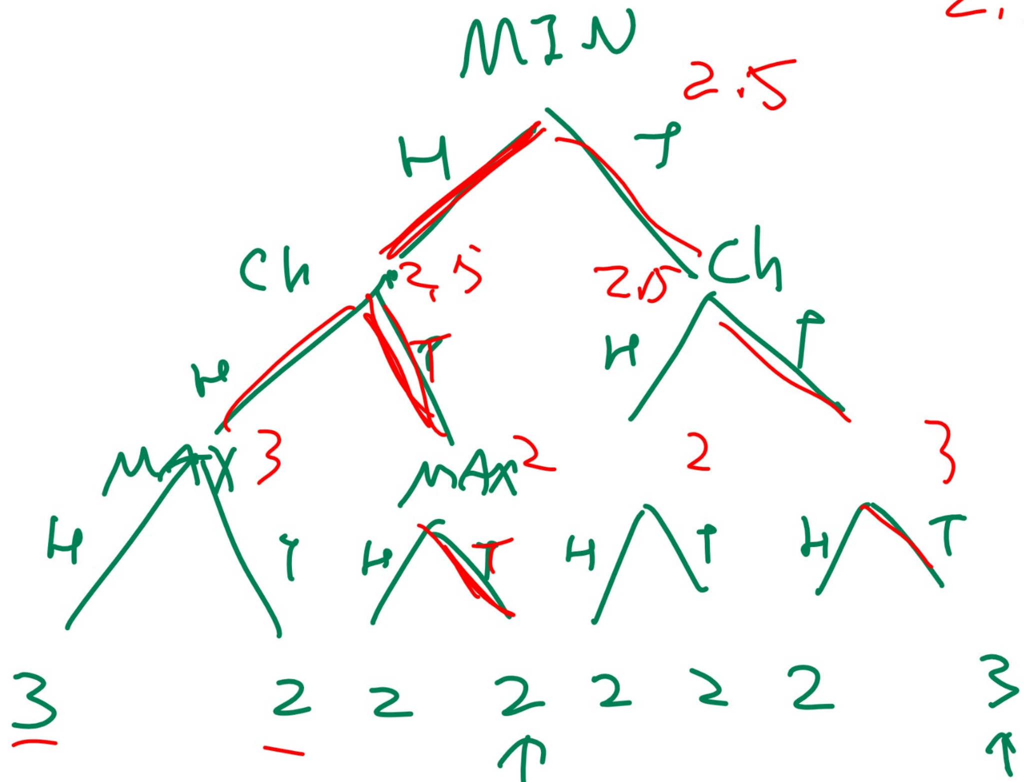
Notes post on E24

19A

24

28. Consider a sequential game with Chance. Player 1, ~~MAX~~ min, chooses an action $H$ or $T$ first. Then a fair coin is flipped, the outcome is either $H$ (heads) or $T$ (tails). Player 2, MIN, observes the outcome of the coin and chooses an action $H$ or $T$. Suppose the number of outcomes or actions that are $H$ is $x$ and the number of outcomes or actions that are $T$ is $3-x$, then the value of the terminal state is $\max\{x, 3-x\}$. In other words, the value for any path with $x$ number of $H's$ and $(3-x)$ number of $T's$ is $\max\{x, 3-x\}$, $x = 0, 1, 2, 3$. What is the value of the whole game?

2.5 if MIN player moves first.




max(1, 2)

max(0, 3)

$x=1$ $3-x=2$

$x=0$, $3-x=3$

Q1 → Q3 on F1A AND F1B

with diff randomization

$\underbrace{\text{Q4}}_{\text{from F0A/B}}$ on F2A, F2B as Q9.

## Question 6 [2 points]

21

- (Fall 2017 Final Q24) Consider n + 1 = 21 + 1 states. The initial state is 1, the goal state is n. State 0 is a dead-end state with no successors. For each non-0 state i, it has two successors: i + 1 and 0. There is no cycle check nor CLOSED list (this means we may expand (or goal-check) the same nodes many times, because we do not keep track of which nodes are checked previously). How many goal-checks will be performed by Breadth First Search? Break ties by expanding the node with the smaller index first.
- Answer: [ ] . Calculate

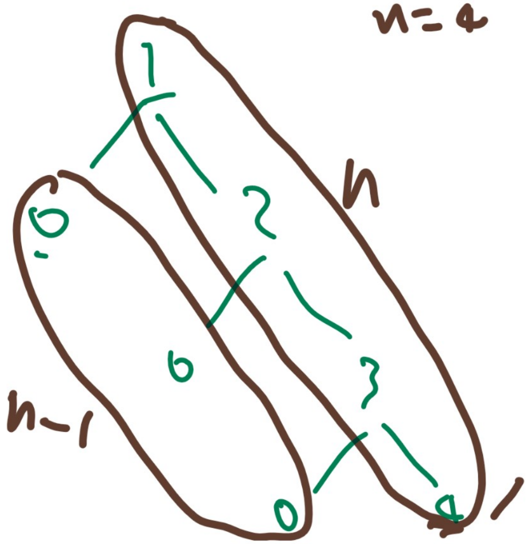
keep repeated nodes

$3 \to 3+2$

$n \to n + (n-1)$

$2n-1$

n=4



goal check — expand — dequeue (pop)

Stop when dequeue / expand the goal state

↳ not enqueue successors.

Space complex. not want total

\# stuff in queue.

37. Given the following simultaneous move game payoff table, what is the complete set of pure strategy Nash equilibria? Both players are MAX players. In each entry of the table, the first number is the reward to the ROW player and the second number is the reward to the COLUMN player.

| | | $q$ | $1-q$ |
|---|---|---|---|
| | – | L | R |
| $p$ | U | (1,1) | (0,1) |
| $1-p$ | D | (0,0) | (1,1) |

$(U, L)$, $(D, R)$

pure NE.

expected payoff from U — D

$$1 \cdot q + 0(1-q) \geq 0q + 1 \cdot (1-q)$$

$$q \geq 1-q$$

$$q \geq \frac{1}{2}$$

$$q = \frac{1}{2}$$

$$q \leq \frac{1}{2}$$

$$br_{ROW}(q) = \begin{cases} U & p=1 \\ p \in [0,1] & \\ D & p=0 \end{cases}$$

always weak inequality

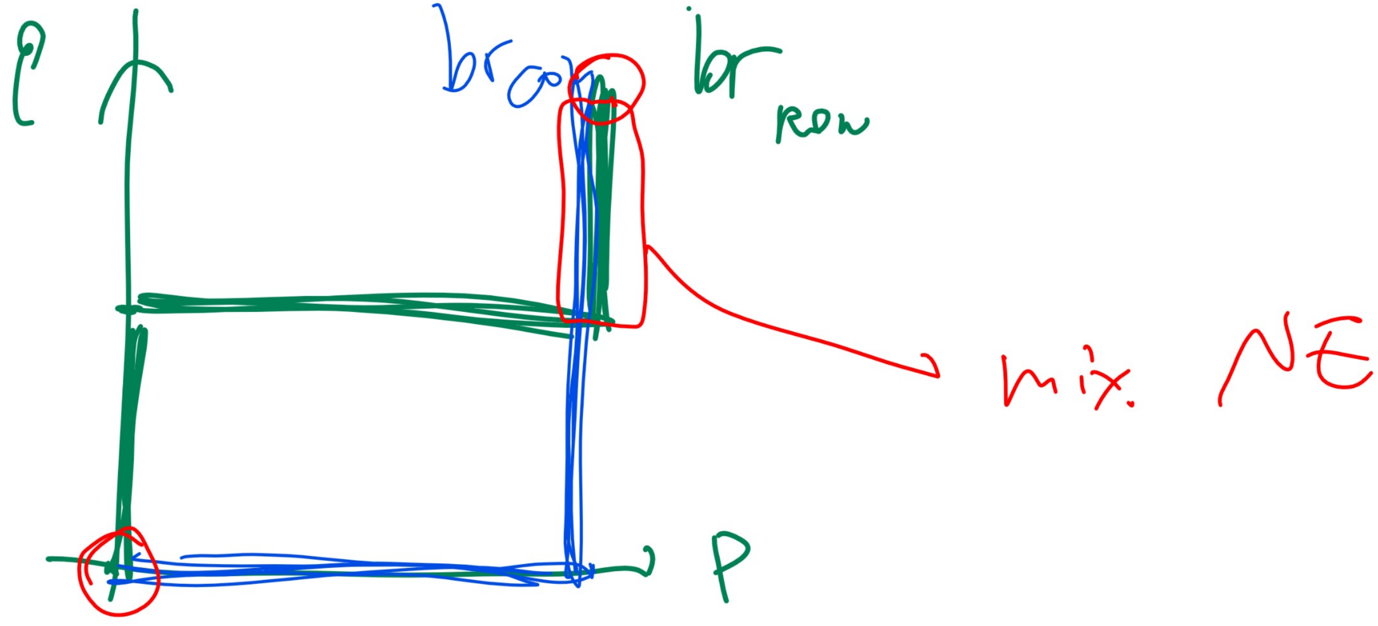
$$1 \cdot p + 0(1-p) \geq 1$$

$$p \geq 1$$

$$p = 1$$

$$p \leq 1$$

$$br_{COL}(p) = \begin{cases} L & q=1 \\ q \in [0,1] & \\ R & q=0 \end{cases}$$




$q \geq \frac{1}{2}, \; p = 1$

$$NE = \left( U, \; L^{q \geq \frac{1}{2}} R^{1-q} \right)$$

OH 12:30 – 1:45 am tonight → no guest link on Canvas BBCU

12:30 – 1:45 pm tomorrow.

## Question 3 [2 points]

• (Fall 2018 Midterm Q3) Consider a 3-puzzle where, like in the usual 8-puzzle game, a tile can only move to an adjacent empty space. Tiles cannot move diagonally. Which of the following initial states can reach the goal state $\begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$ (0 means "no tile")?

312

• Choices:

- [ ] $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ → 312
- [ ] $\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$ → 312
- [ ] $\begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$ → 312
- [ ] $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ → 312
- [ ] $\begin{bmatrix} 0 & 3 \\ 2 & 1 \end{bmatrix}$ → 312
- [ ] None of the above

312

$\begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix} \rightarrow 321$

## Question 2 [0 points]

- There are 8 lights in a row. The initial state is $\begin{bmatrix}1 & 0 & 1 & 0 & 1 & 1 & 0 & 0\end{bmatrix}$, 0 is "off", 1 is "on". A valid move finds two adjacent lights where one is one the the other is off, and switches them while keeping all other lights the same. That is, locally, you may do 01 to 10 or 10 to 01. What is the smallest number of moves to reach the goal state $\begin{bmatrix}0 & 0 & 0 & 0 & 1 & 1 & 1 & 1\end{bmatrix}$.
- Answer: ______ . Calculate

4 + 3 + 2 + 2

0 + 1 + 2 + 2

[1 0 1 0 1 0 0 0]

[1 0 1 0 1 0 1 0]

[1, 1, 1, 1, 0, 0, 0, 0]

[1 0 1 0 1 1 0 0]

[0 0 0 0 1 1 1 1]

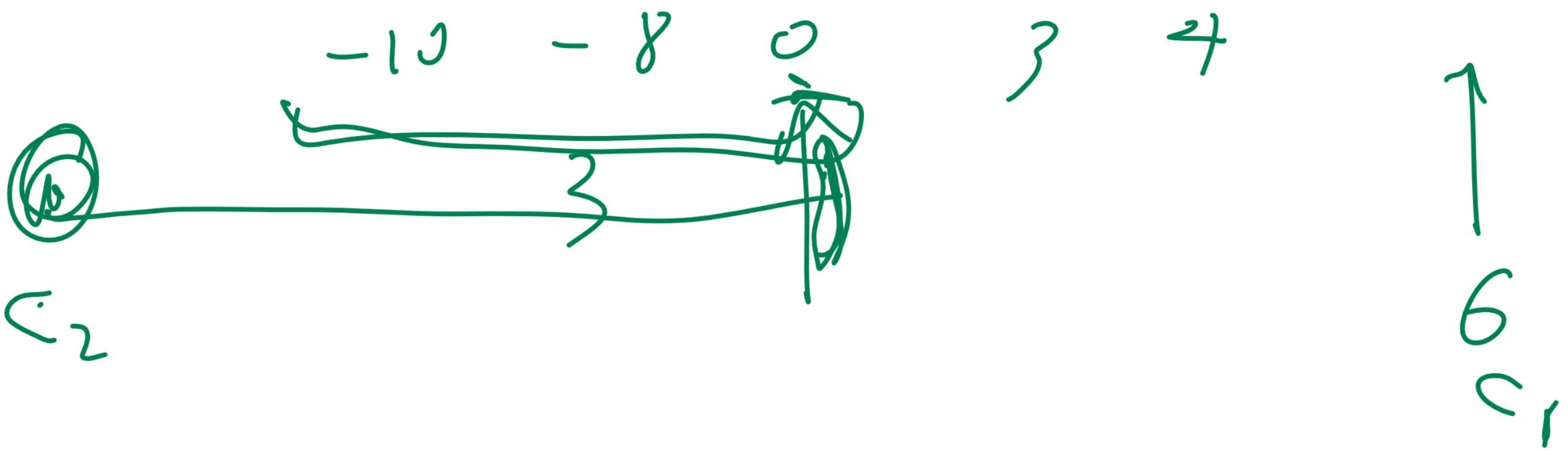
2 + 2 + 3 + 4

L' dot (initial - goal)

## Question 1 [0 points]

- Suppose K-Means with $K = 2$ is used to cluster the data set $[-10 \quad -8 \quad 0 \quad 3 \quad 4]$ and initial cluster centers are $c_1 = 6$ and $c_2 = x$. What is the smallest value of $x$ if cluster 1 has ~~3~~ 2 points. Break ties by assigning the point to cluster 2.
- Answer: [ ] Calculate

initially,




$$\frac{x+6}{2} = 0 \Rightarrow \boxed{x = -6}$$

## Question 1 [0 points]

- Suppose K-Means with $K = 2$ is used to cluster the data set $[-10 \quad -8 \quad 0 \quad 3 \quad 4]$ and initial cluster centers are $c_1$ = 6 and $c_2$ = $x$. What is the smallest value of $x$ if cluster 1 has 3 points. Break ties by assigning the point to cluster 2.
- Answer: ________. Calculate

initially.

$x \qquad -10 \qquad -8 \qquad 0 \qquad 3 \quad 4 \qquad\qquad 6$

$c_2 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad c_1$

Smallest $= -8 = \dfrac{x+6}{2} \Rightarrow x = -22$

$x = -21 \rightarrow$ same

$x = -23 \rightarrow$ 4 points in $c_1$

largest $= 0 = \dfrac{x+6}{2}$

$x < -6$

$x = -6.0001 \rightarrow$ 3 points in $c_1$

$x = -6 \rightarrow$ 2 points in $c_1$

this will be post on E24 page.

## Question 3 [0 points]

• Imagine a population of $N$ = 200 individuals. Each of them simultaneously chooses between taking the vaccine and not. All individuals have the same payoffs. Suppose there are $n$ people who choose not to take the vaccine, then the payoff from not taking the vaccine is $-\alpha \cdot \frac{n}{N}$, and the payoff from taking the vaccine is $-c - \beta \cdot \frac{n}{N}$, $\alpha$ = 17 is the herd immunity coefficient, $\beta$ = 8 measures the ineffectiveness of the vaccine, and $c$ = 2 is the cost of getting the vaccine. In a pure Nash equilibrium, what is the largest number of individuals who choose to take the vaccine.

• Answer: [ ]. Calculate

Suppose $n^*$ players choose NoVaccine.

These $n^*$ players should prefer NoV to V.
NoVac

$$\underbrace{-\alpha \cdot \frac{n^*}{N}} \geq \underbrace{-c - \beta \cdot \frac{n^* - 1}{N}}$$

For the remaining $N - n^*$ players,

prefer $V$ to NoV

~~$n^* = f.2$~~

$$-C - \beta \cdot \frac{n^*}{N} \geq -\alpha \frac{n^*+1}{N}$$

$$[redacted] \leq n^* \leq \text{floor}(\;[redacted]\;)$$

~~$n^* = 1$~~

stay not vaccinated $\longrightarrow$ if I get vaccine

$$-\alpha \frac{1}{N} \qquad -C - \beta \frac{\cancel{1}\; 0}{N}$$

$n' + 0 \qquad n' + 1$

$$br_i(\underline{\underline{n'}} \text{ players } \underline{\underline{\text{other than}}}\ \underline{i} \text{ not taking vaccine}) = \begin{cases} V & -c-\beta\frac{n'}{N} \geq -\alpha\frac{n'+1}{N} \\ \underline{N} & \text{o.tw.} \end{cases}$$

I am player $i$

| action | total # of unvaccinated | payoff to $i$ |
|---|---|---|
| $\underline{V}$ | $n'+0$ | $-c-\beta\frac{n'}{N}$ |
| $\underline{N}$ | $n'+1$ | $-\alpha\frac{n'+1}{N}$ |

$\Xi$ me

$$br_i(n') = \begin{cases} V & \text{if } -c-\beta\frac{n'}{N} \geq -\alpha\frac{n'+1}{N} \\ N & \text{if } -\alpha\frac{n'+1}{N} \geq -c-\beta\frac{n'}{N} \end{cases}$$

In NE $n^*$ player not vaccinated

For $n^*$ players $n' = n^* - 1$

$$-\alpha\frac{n^*-1+1}{N} \geq -c-\beta\frac{n^*-1}{N}$$ from br.

for other players $n' = n^*$

$$-c-\beta\frac{n^*}{N} \geq -\alpha\frac{n^*+1}{N}$$

37. Given the following simultaneous move game payoff table, what is the complete set of pure strategy Nash equilibria? Both players are MAX players. In each entry of the table, the first number is the reward to the ROW player and the second number is the reward to the COLUMN player.

19A

Q37-40

| | | $q$ | $1-q$ |
|---|---|---|---|
| | – | L | R |
| $p$ | U | (1,1) | (0,1) |
| $1-p$ | D | (0,0) | (1,1) |

(expected)

payoff U $\geq$ payoff D

$1 \cdot q + 0(1-q) \geq 0 \cdot q + 1(1-q)$

$q \geq \frac{1}{2}$

$q = \frac{1}{2}$

$q \leq \frac{1}{2}$

$$br_{Row}(q) = \begin{cases} U \Rightarrow p=1 \\ p \in [0,1] \\ D \Rightarrow p=0 \end{cases}$$

maps to a set

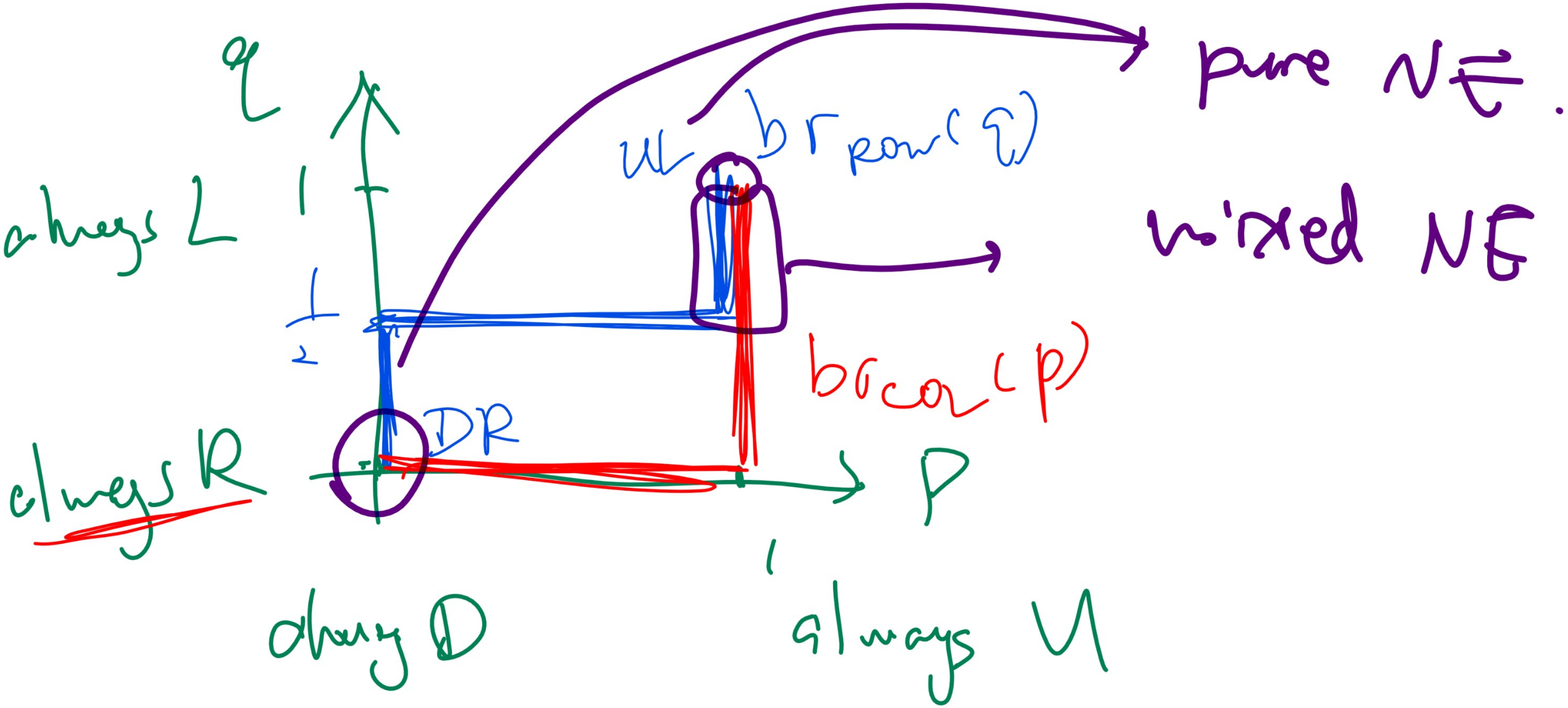
payoff L $\geq$ payoff R

$1 \cdot p + 0 \cdot (1-p) \geq 1$

$p \geq 1$

$p \leq 1$

$$br_{COL}(p) = \begin{cases} L \\ q \in [0,1] \\ R \end{cases}$$




mixed NE $= \{ (U, L^{q} R^{1-q}) : q \geq \frac{1}{2} \}$

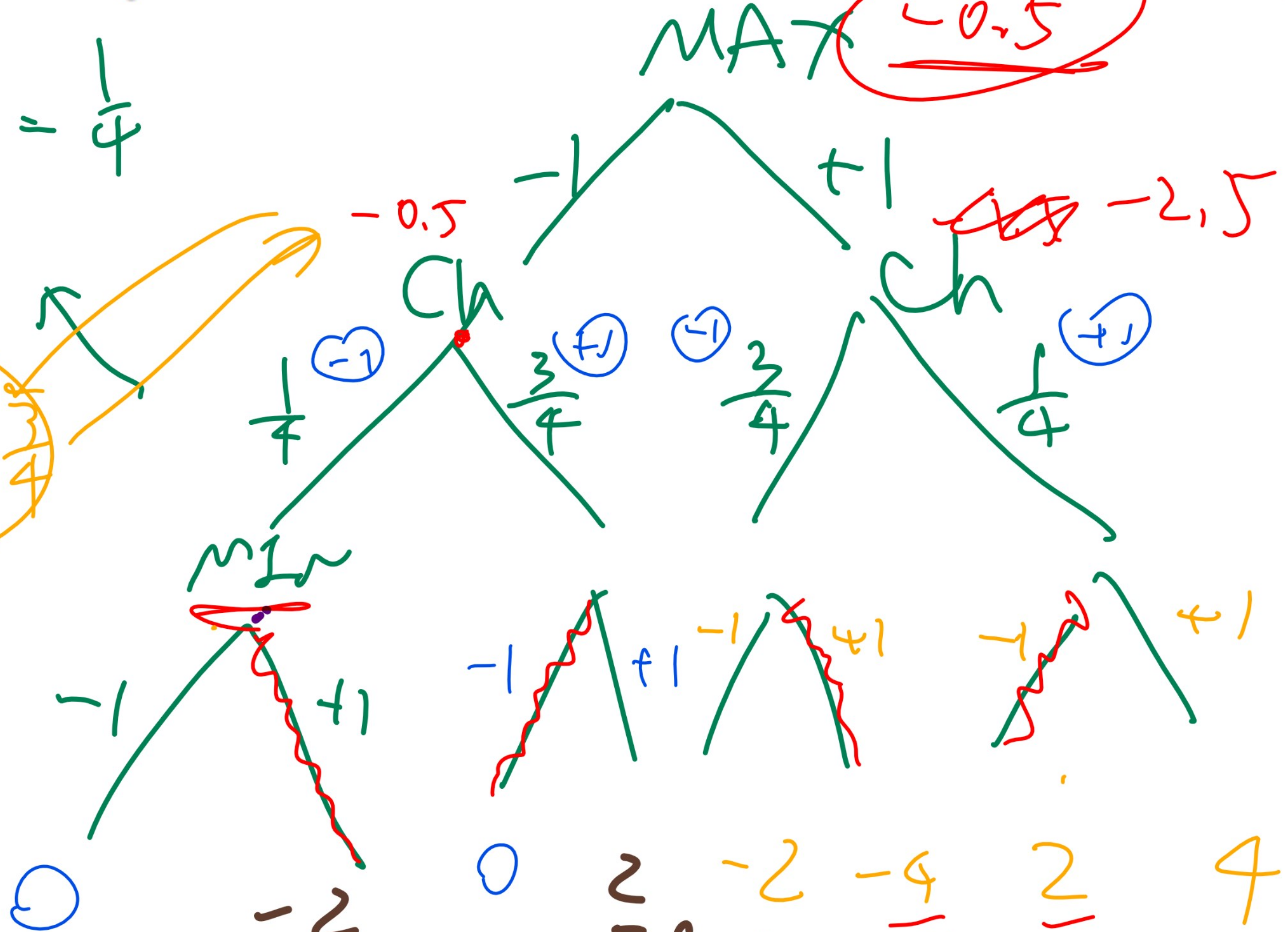
UL, DR.

a mix NE is $( U^{1} D^{0}, L^{\frac{1}{2}} R^{\frac{1}{2}} )$

34. Consider a zero-sum sequential move game with Chance. Player MAX first chooses between actions $x_1 \in \{-1,+1\}$, then Chance chooses $x_2 \in \{-1,+1\}$, $\begin{cases} -1 & \text{with probability } \frac{1}{2}+\frac{x_1}{4} \\ +1 & \text{with probability } \frac{1}{2}-\frac{x_1}{4} \end{cases}$, and at the end, player MIN chooses between actions $x_3 \in \{-1,+1\}$. The value of the terminal states corresponding to the actions $(x_1, x_2, x_3)$ is $(x_1 + 2 + x_3) \cdot x_2$. Not a typo: $\left(x_1 + \boxed{2}\,(\text{ not } x_2) + x_3\right) \cdot x_2$. What is the value of the whole game?

$\frac{1}{2} + \frac{-1}{4} = \frac{1}{4}$

$-2 \cdot \frac{1}{4} + 0 \cdot \frac{3}{4}$




$-4 \cdot \frac{3}{4} + 2 \cdot \frac{1}{4}$

$-3 + 0.5$

$(-1+2+1) \cdot 1 = 2$

$(-1+2+1) \cdot (-1) = -2$

## Question 8 [4 points]

- (Spring 2017 Midterm Q4) You are given the distance table. Consider the next iteration of hierarchical agglomerative clustering (another name for the hierarchical clustering method we covered in the lectures) using complete linkage. What will the new values be in the resulting distance table corresponding to the four new clusters? If you merge two columns (rows), put the new distances in the column (row) with the smaller index. For example, if you merge columns 2 and 4, the new column 2 should contain the new distances and column 4 should be removed, i.e. the columns and rows should be in the order (1), (2 and 4), (3), (5).

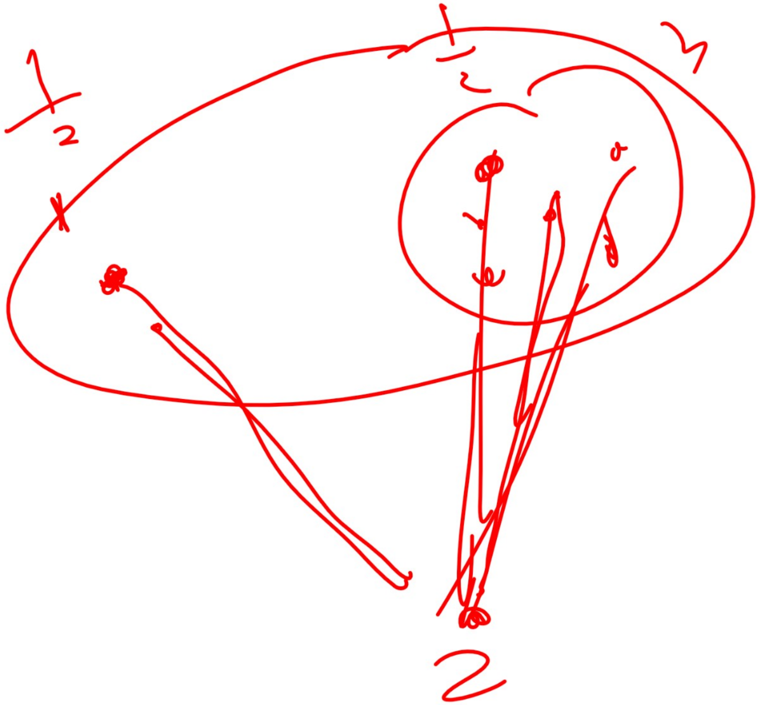
$$d = \begin{array}{cc} \begin{bmatrix} 0 & 19 & 2 & 64 & 14 \\ 19 & 0 & 91 & 100 & 82 \\ 2 & 91 & 0 & 68 & 84 \\ 64 & 100 & 68 & 0 & 67 \\ 14 & 82 & 84 & 67 & 0 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \\ \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 \end{matrix} & \end{array}$$

combine 1 and 3

- Hint: the resulting matrix should have 4 columns and 4 rows.

$$d(\underbrace{1,3}_{\text{new cluster}}, 2) = \underbrace{\max}_{\text{complete}}(d(1,2), d(3,2))$$

$$= \underbrace{\min}_{\text{single}}(d(1,2), d(3,2))$$




$$= \cancel{\frac{1}{2}\left(d(1,2) . d(3,2)\right)}$$

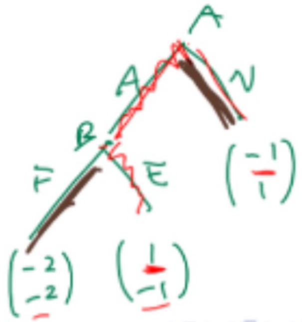
average linkage

## Non-credible Threat Example 1

Quiz

- Country A can choose to Attack or Not attack country B. If country A chooses to Attack, country B can choose to Fight back or Escape. The costs are the largest for both countries if they fight, but otherwise, A prefers attacking (and B escaping) and B prefers A not attacking. What are the Nash equilibria?
- A: (A, F)
- B: (A, E)
- C: (N, F)
- D: (N, E)
- E: (N)

Solution SPE




extensive form



## Non-credible Threat Example 1 Derivation

Quiz

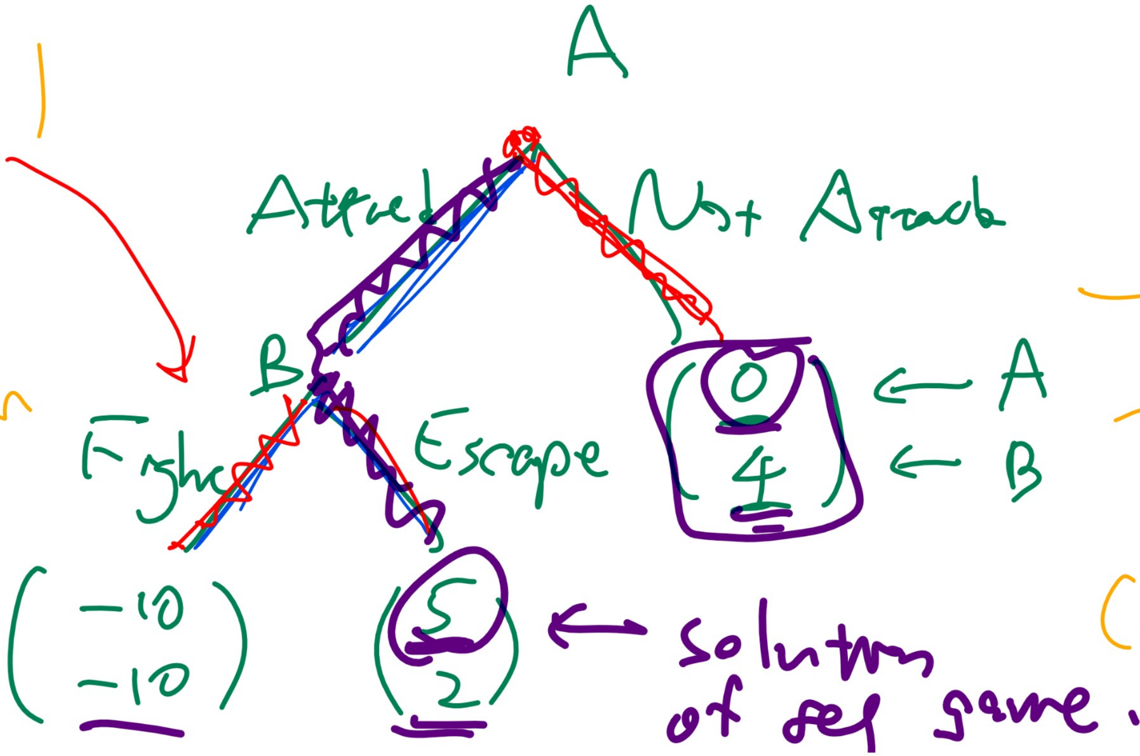
normal form

| | F | E |
|---|---|---|
| A | -2, -2 | 1, -1 |
| N | -1, 1 | -1, 1 |

(N, F) , (A, E)

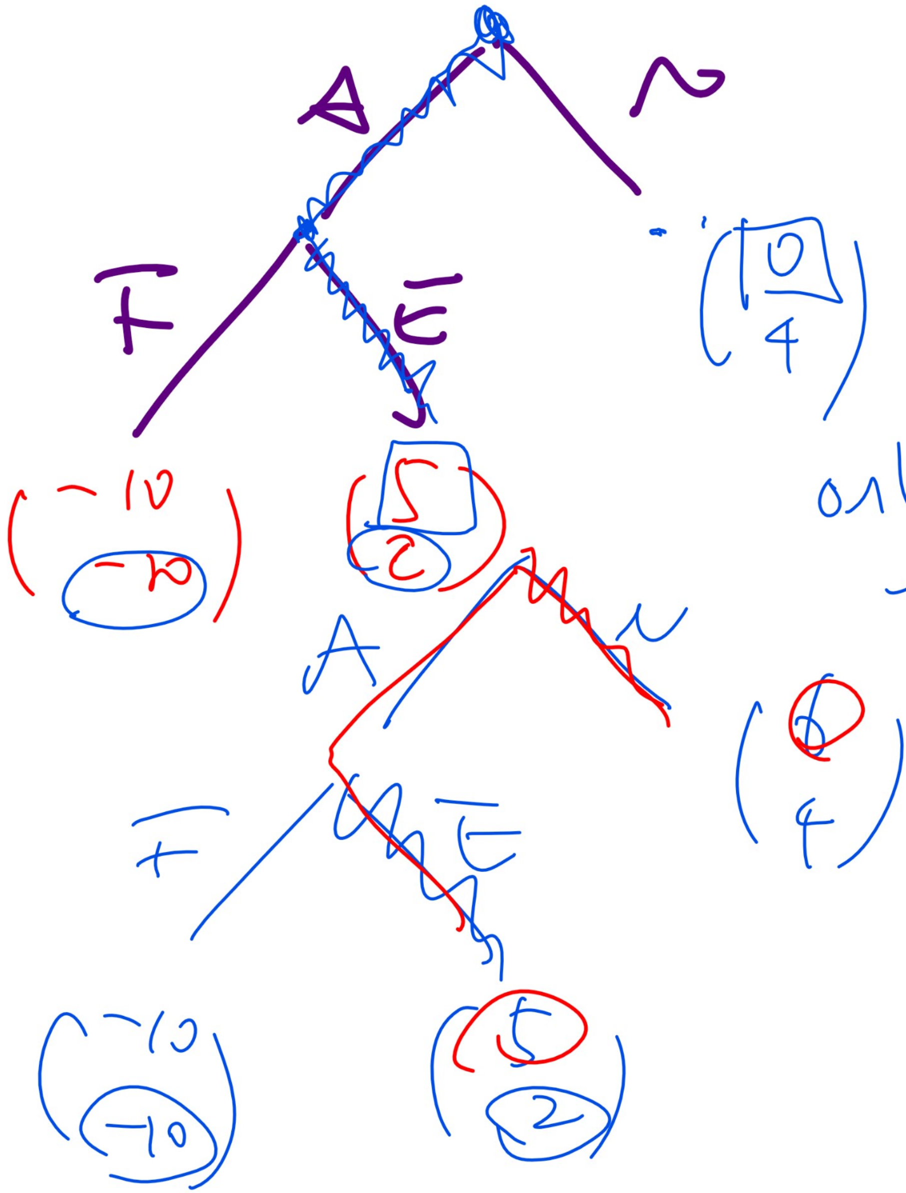
solution

(N, F) is NE not a solution




| A\B | F | E |
|---|---|---|
| A | -10, -10 | 5, 2 |
| N | 0, 4 | 0, 4 |

(A, E) → solution of seq. game




$br_B(N) = \{F, E\}$

SPE

only solution to sequential game is $(A, E)$

$(N, E)$

solution of game

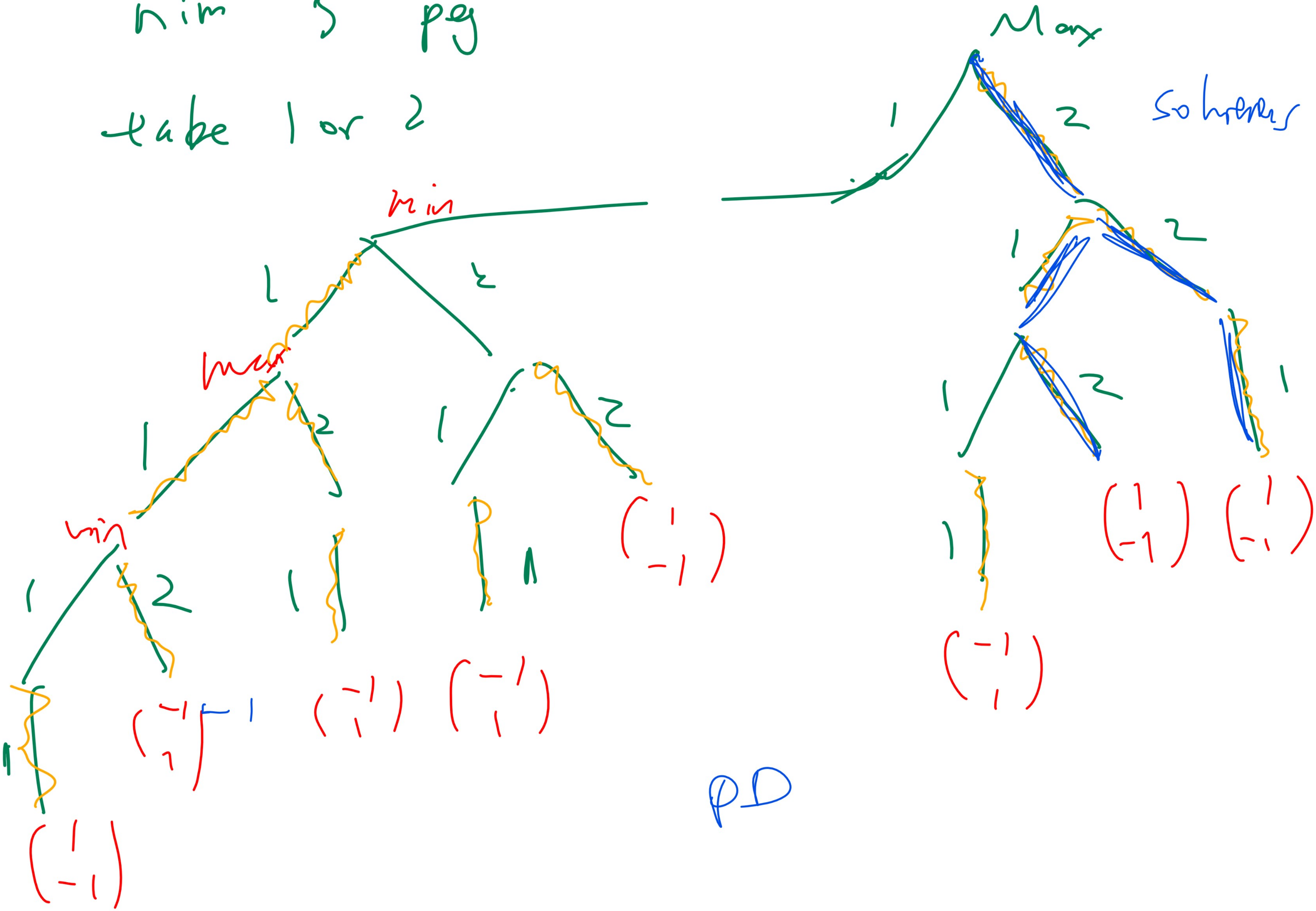
| A\B | F | E |
|---|---|---|
| A | $-10, -10$ | $5, 2$ |
| N | $0, 4$ | $0, 4$ |

pure NE

$$\left.\begin{aligned} br_B(N) &= F \\ br_A(F) &= N \end{aligned}\right\} \text{mutual br.}$$

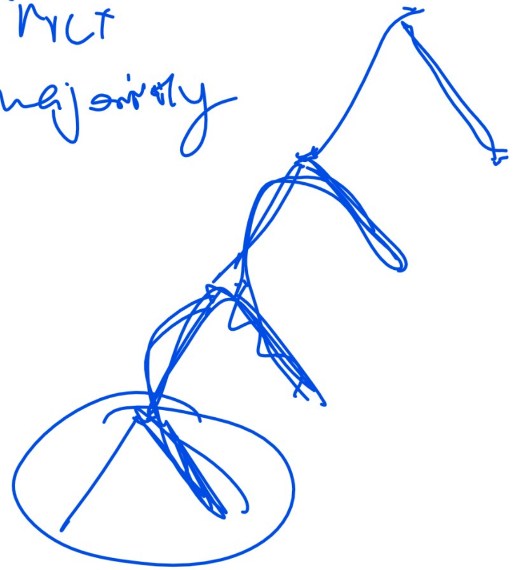
nim 5 peg

take 1 or 2



100 $ pirates

strict majority



1    0, 0, 0, 0, 100

2    0, 0, 0, 0, 100

3    0, 0, 99, 1, 0

need 3 vote

4    0, 97, 0, 2, 1

5    97, 0, 1, 0, 2

yes no yes no yes

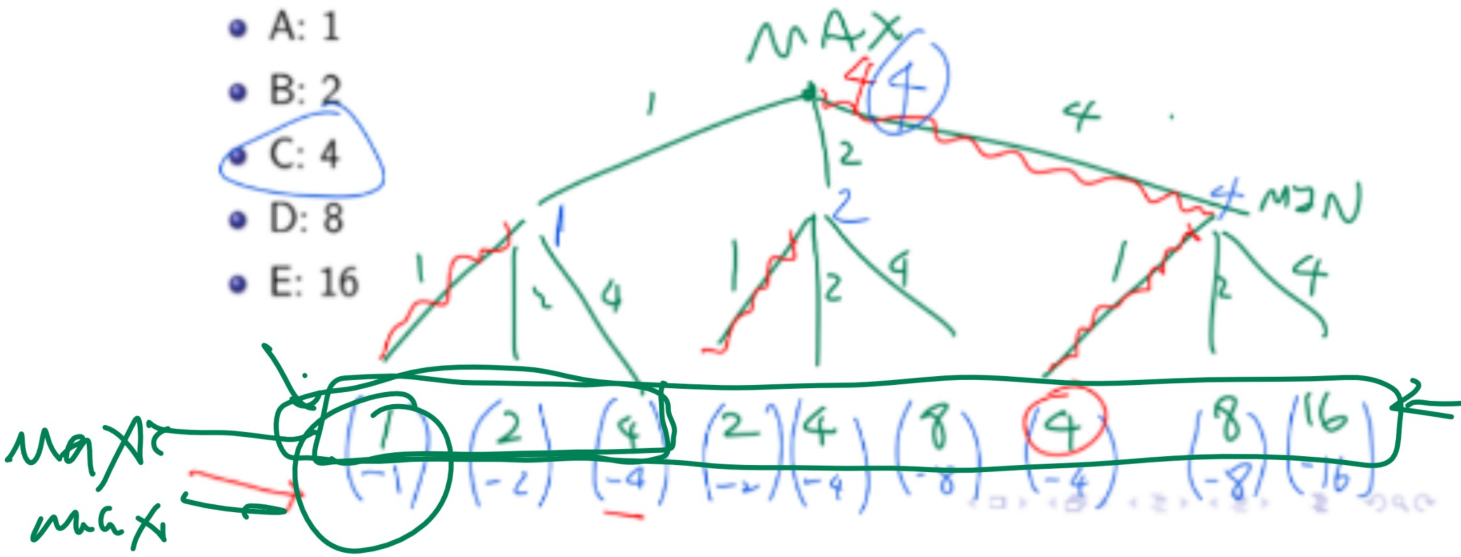
need 3 votes

# Minimax Example 2

## Quiz

Q2

- For a zero-sum game, the value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. What is the value of the game?
- A: 1
- B: 2
- C: 4
- D: 8
- E: 16




max max

min → min

Min player → min. max's payoff

min player → max min's payoff

max player $\rightarrow$ max max's payoff

for a game, we only specify $\boxed{\text{max's payoff}}$

min's payoff = − max payoff in zero sum games.

Juliet

| Romeo | | $q$ Bach | $1-q$ Stravinsky |
|---|---|---|---|
| $p$ | B | 3, 5 | 0, 0 |
| $1-p$ | S | 0, 0 | 5, 3 |

B, B

S, S

are pure NE

payoff B S

$$3q + 0(1-q) \geq 0q + 5(1-q)$$

$$q \geq \frac{5}{8}$$

$$br_{Romeo}(q) = \begin{cases} B & q > \frac{5}{8} \\ p \in [0,1] & q = \frac{5}{8} \\ S & q < \frac{5}{8} \end{cases}$$

payoff B S

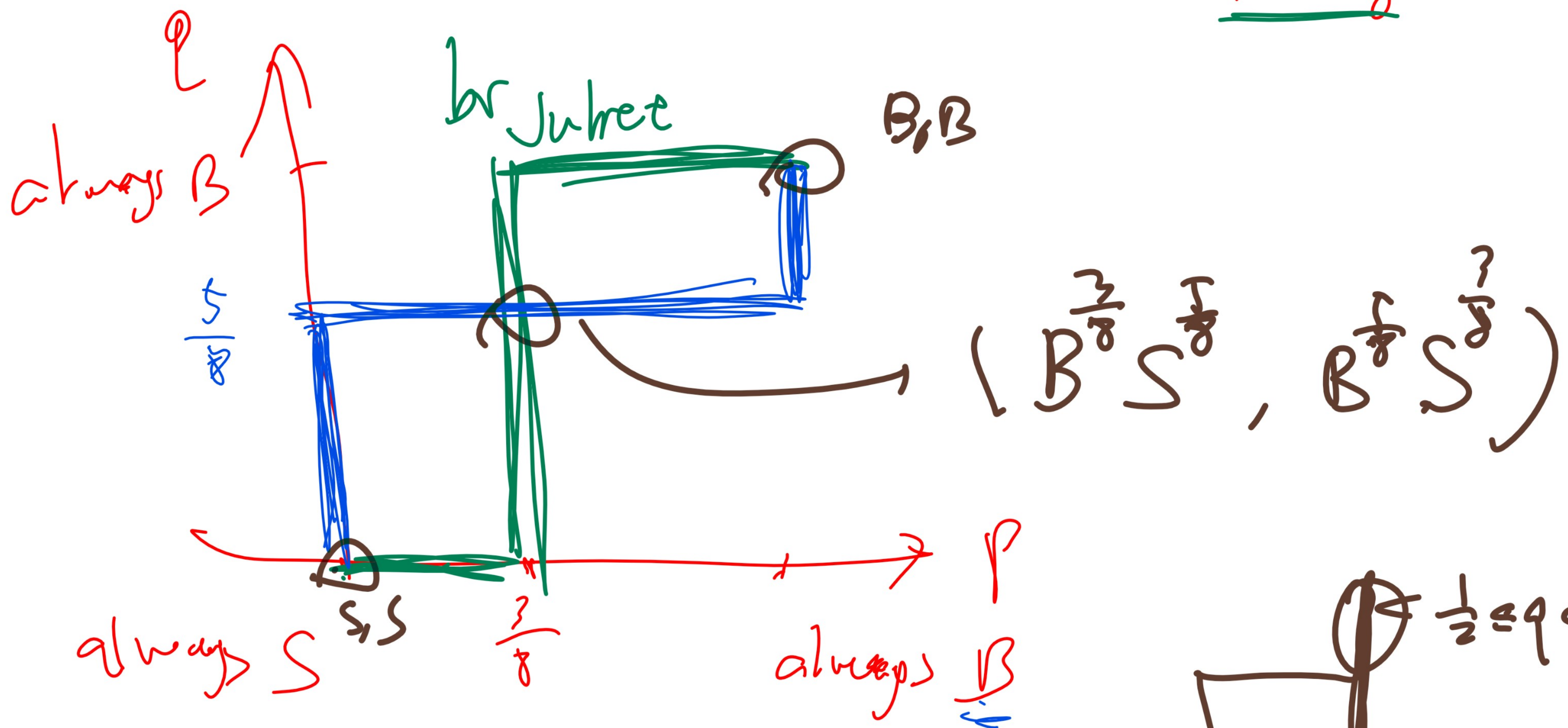
$$br_{Juliet}(p) = \begin{cases} B \\ q \in (0,1] \\ \underline{S} \end{cases}$$

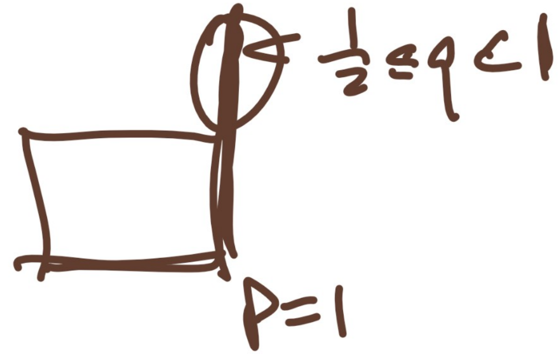
$5p \geq 3(1-p)$

$p \geq \frac{3}{8}$

$p = \frac{3}{8}$

$\underline{p \leq \frac{3}{8}}$



7. What is the projected sample variance of $\left\{\begin{bmatrix}2\\4\\6\end{bmatrix}, \begin{bmatrix}-2\\1\\0\end{bmatrix}\right\}$ onto $\begin{bmatrix}1\\2\\3\end{bmatrix}$? Note: projected sample variance is the sample variance of the magnitude of the projection of the data points onto a principal component, which $\begin{bmatrix}1\\2\\3\end{bmatrix}$ in this question. Use the maximum likelihood estimator of $\sigma^2$:

length

$$(\hat{\sigma})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2, \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\begin{pmatrix}\frac{1}{\sqrt{14}}\\ \frac{2}{\sqrt{14}}\\ \frac{3}{\sqrt{14}}\end{pmatrix}$$

length $= x_1^T u$ — only if $u$ is unit

$$\Rightarrow x_1^T u = \frac{2+8+18}{\sqrt{14}} = \frac{28}{\sqrt{14}} = 2\sqrt{14}$$

$$x_2^T u = \frac{-2+2}{\sqrt{14}} = 0$$

$$\mu = \sqrt{14}$$

$$2\sqrt{14} - \underset{\mu}{\sqrt{14}} \qquad 0 - \underset{\mu}{\sqrt{14}}$$

$$\sigma^2 = \frac{1}{2}\left[(\sqrt{14})^2 + (\sqrt{14})^2\right] = \frac{1}{2}\cdot 2\cdot 14$$

$$= 14$$

# Question 4 [4 points]

- (Fall 2017 Final Q22, Fall 2014 Final Q20, Fall 2013 Final Q14) Consider the four points: $x_1 = \begin{bmatrix} -10 \\ \cancel{-10} \end{bmatrix}$, $x_2 = \begin{bmatrix} -8 \\ \cancel{10} \end{bmatrix}$, $x_3 = \begin{bmatrix} -6 \\ \cancel{10} \end{bmatrix}$, $x_4 = \begin{bmatrix} 4 \\ \cancel{3} \end{bmatrix}$. Let there be two initial cluster centers $c_1 = \begin{bmatrix} -4 \\ \cancel{3} \end{bmatrix}$, $c_2 = \begin{bmatrix} -3 \\ \cancel{-3} \end{bmatrix}$. Use Euclidean distance. Break ties in distances by putting the point in the cluster with the smaller index (i.e. favor cluster 1). Write down the cluster centers after one iteration of k-means, the first cluster center (comma separated vector) on the first line and the second cluster center (comma separated vector) on the second line.

- Answer (matrix with multiple lines, each line is a comma separated vector): -8 4 . Calculate

$\underbrace{-10, \ -8, \ -6}_{-4}$ $\qquad$ $\underbrace{4}_{-3}$

newline

-8 , 0

4 , 0

OR

-8 , 0 ;; 4 , 0

$c_1 \begin{pmatrix} -8 \\ 0 \end{pmatrix}$ $\qquad$ $c_2 \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\sum \left( \sqrt{(x_1 - c_{x_1})^2 + (x_2 - c_{x_2})^2} \right)^2 = \text{distortion}$

Manhattan → no square $\sum |x - c_x|$

Euclidean → yes square sum = distortion.

19. Let the states be the following five cities: Fitchburg, Madison, Middleton, Verona, Waunakee. The locations on the map are given in the following table. In the search digraph, the edge costs are the Euclidean distance between the cities corresponding to the state. The initial state is Madison, and the goal state is Verona. What is a state expansion sequence if $A^\star$ Search is used with heuristics equal to the Euclidean distance between the state and the goal state, for all states? Reminder: when there are ties, the state with a smaller index has priority.

$$h\left(s=(x,y)\right)=\sqrt{(x+2)^2+(y+2)^2}\leqslant h^\star(s)$$

| Index | City | X-coordinate | Y-coordinate | Successors |
|---|---|---|---|---|
| 1 | Fitchburg | -1 | -1 | Middleton, Verona |
| 2 | Madison | 0 | 0 | Fitchburg, Middleton, Waunakee |
| 3 | Middleton | -2 | 0 | Verona |
| 4 | Verona | -2 | -2 | – |
| 5 | Waunakee | 0 | 2 | Middleton |

- A: Madison, Fitchburg, Verona
- B: Madison, Fitchburg, Middleton, Verona
- C: Madison, Fitchburg, Middleton, Waunakee, Verona
- D: Madison, Fitchburg, Middleton, Waunakee, Middleton, Verona
- E: Madison, Fitchburg, Middleton, Waunakee, Middleton, Fitchburg, Verona

|  | ~~2~~ | ~~1~~, | $3_2$, | $5_2$ |
|---|---|---|---|---|
| g | 0 | $\sqrt{2}$ | 2 | 2 |
| h | $2\sqrt{2}$ | $\sqrt{2}$ | 2 | large |
| g+h |  |  |  |  |

| $3_1$ | ~~$4_1$~~ |
|---|---|
| $\sqrt{2}+1$ large | $\sqrt{2}+\sqrt{2}$ |
| 2 | 0 |

2, 1, 4.

Q14 A ✓

Q27 B ✓

Q9, 10, 19B

Q21, 22 of 19B ✓

Queue.

FoA Q2, ✓

E18 last question ✓

M8 Q3, ✓ MW Q8 ✓ M12, Q2,3 ✓

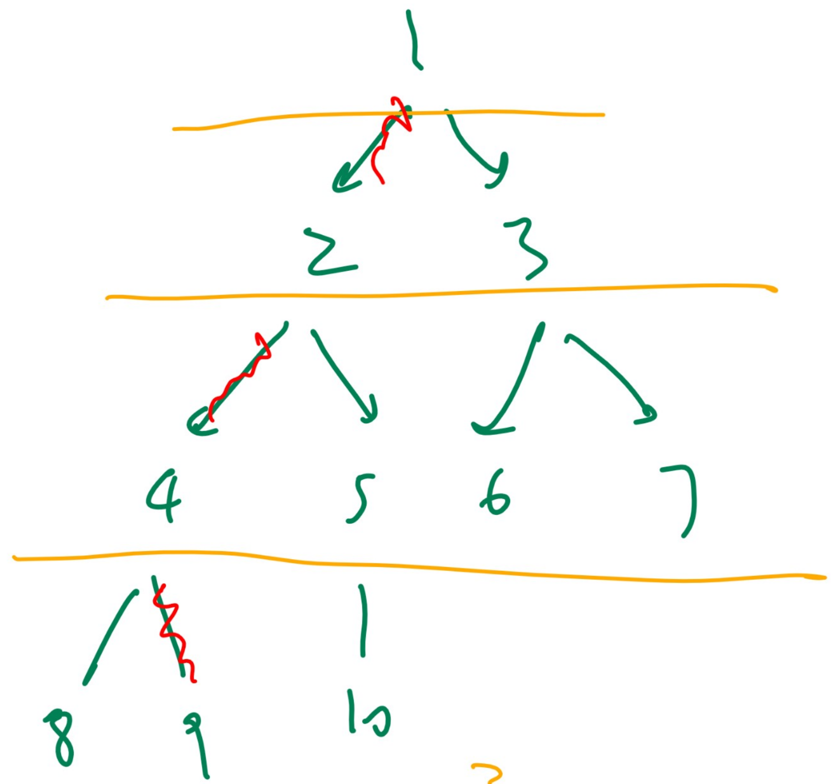
# IDS Example 1

## Quiz

- Fall 2018 Midterm Q2, Fall 2017 Midterm Q13, Fall 2010 Final Q2
- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of $i$ is $2i$ and $2i+1$ (if exist). What is IDS expansion sequence?

path

same.




$\overbrace{1}^{0},\ \overbrace{1,2,3}^{1}.\ \overbrace{1,2,4,5,3,6,7}^{2},\ \underbrace{\overbrace{1,2,4,8,9}^{3}}_{\text{DFS}}$

solution.

$\boxed{1,2,4,9}$

Dan's OH 5-6.

5, 10 in if 10 is goal.

DFS expansion path = 1,2,4,8 9

19B Q21

$\cancel{h_1(s)} = 1 - \mathbb{1}_{\{h^*(s)<0\}}$

✓ $h_2(s) = 1 - \mathbb{1}_{\{h^*(s)<1\}}$ $h^* = 0$

✓ $h_3(s) = \mathbb{1}_{\{h^*(s)<0\}}$

✓ $h_4(s) = \mathbb{1}_{\{h^*(s)>1\}}$

$\cancel{h_5(s)} = \mathbb{1}_{\{h^*(s)\geq 0\}}$

admissible → for all states.

$h_1(s) = 1 \rightarrow h_1(\text{goal}) = 1 > 0$

$$0 \leq h_2(s) = \begin{cases} 1 \ (\leq h^*) & h^*(s) \geq 1 \text{ o/w} \\ 0 \ (\leq h^*) & \text{if } h^*(s) = 0 \end{cases} \leq h^*$$

$h^*(s) < 1$

0.4

dominated ✓

$\boxed{h_3(s) = 0}$

$h_5(\text{goal})$

$= h^* = 0) = 1 > 0$

$$h_4(s) = \begin{cases} 1 \ (\leq h^*) & h^*(s) > 1 \\ 0 \ (< h^*) & h^*(s) \leq 1 \end{cases} \leq h^*$$

$h^* = 1$

not possible $h = 1$, while $h^* = 0.5$

$h = 0$

for $h^*_{(s)} = 1 \rightarrow h_4(s) = \infty \le h_2(s) = 1$

$h_4$ is dominated by $h_2$

if none of $h^*_{(s)}$ is $1 \longrightarrow h_4 = h_2$.

if $\underbrace{h_i \le h_j \ \forall s}_{\text{admissible}} \Rightarrow h_j$ dominates $h_i$

$\rightarrow h_j \ne h_i \ \forall s$

$$\boxed{h_6 = \begin{cases} 1 - \\ h^* < 0.5 \end{cases} \quad - \text{ when } \frac{h^* = 0.6}{h_6 = \infty}}$$

## Question 2 [0 points]

- There are 12 lights in a row. The initial state is $[1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0]$, 0 is "off", 1 is "on". A valid move finds two adjacent lights where one is one the the other is off, and switches them while keeping all other lights the same. That is, locally, you may do 01 to 10 or 10 to 01. What is the smallest number of moves to reach the goal state $[0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]$.
- Answer: ______ . Calculate

1 can move at most 1 step.

1, 1, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0

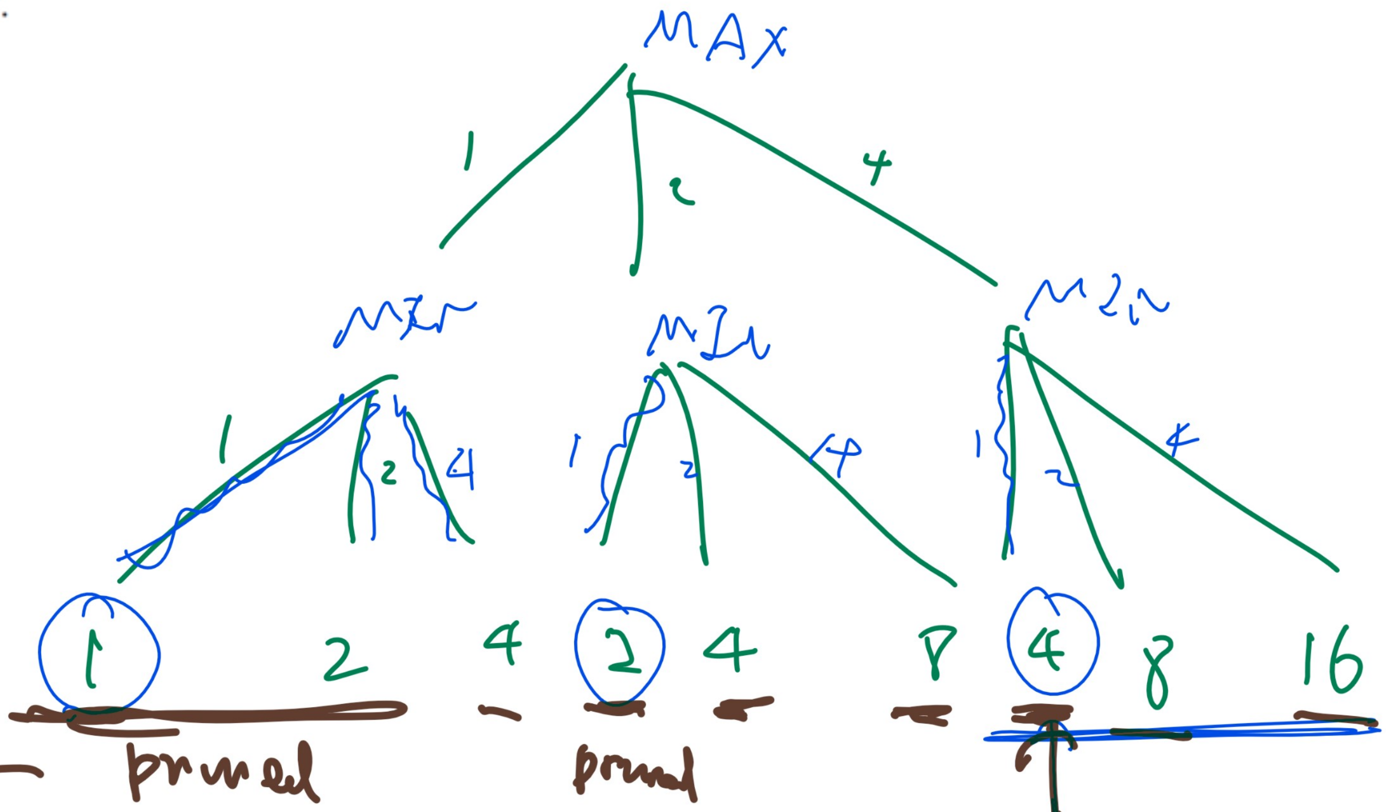
0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1

1 + 1 + 5 + 6 + 6 + 6

27. Consider a zero-sum sequential move game in which player MAX moves first, and MIN moves second. Each player has three actions labeled 1, 2, 4. The value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. Alpha-Beta pruning is used. What is the number of branches (states) that can be pruned? During the search process, the actions with smaller labels are searched first. Note that a branch is pruned if $\alpha = \beta$.

- A: 0
- B: 1
- C: 2
- D: 3
- E: 4

Q28




Q28 ←

prune at most 4

13. Suppose the states are integers between 0 and 9. The initial state is 1, and the goal state is 5. The successors of a state $i$ are the first digit and the last digit of $i \cdot 7$. For example, the first digit of 14 is 1, the last digit is 4, and the first digit of 7 is 0, the last digit is 7. What is a state expansion sequence if Breadth-First Search (BFS) is used? Use the convention that a smaller integer is always enQueued before a larger integer, and a list of visited states are stored so that the same state is never enQueued twice. For example, enQueueing $\{3, 5, 7\}$ into the Queue with $\{1, 7, 9\}$ (from front to back) results in $\{1, 7, 9, 3, 5\}$ (from front to back). Use this convention for all search questions.

19A

Q 14

DFS

$$0 \cdot 7 = 00, 1 \cdot 7 = 07, 2 \cdot 7 = 14, 3 \cdot 7 = 21, 4 \cdot 7 = 28$$

$$5 \cdot 7 = 35, 6 \cdot 7 = 42, 7 \cdot 7 = 49, 8 \cdot 7 = 56, 9 \cdot 7 = 63$$

Stack

CLOSED.

no repeat

~~5~~, 6 ~~7~~, ~~8~~ ~~4~~. 9 ~~0~~, ~~7~~, ~~1~~

1, 0, 7, 4, 2, 8, 5

## Question 1 [0 points]

- Suppose K-Means with $K = 2$ is used to cluster the data set $[-10 \quad 0 \quad 2 \quad 3 \quad 6]$ and initial cluster centers are $c_1 = 10$ and $c_2 = x$. What is the smallest value of $x$ if cluster 1 has 3 points. Break ties by assigning the point to cluster 2.
- Answer: . Calculate

largest → −6

strictly less than −6,

(initial)

$$\frac{x+10}{2} = 2$$

$$\Rightarrow x = -6$$

$\leftarrow$

$x \quad -10 \quad 0 \quad [\,2 \quad 3 \quad d \quad 6\,] \quad \frac{10}{c_1}$

$\vec{c_2}$

find x such that 0 belongs to $c_2$

$$\frac{x+10}{2} = 0 \Longrightarrow \boxed{x = -10}$$

$x = -9.9$ ✓

$d(0, -10.1) = 10.1$

$d(0, 10) = 10 < 10.1$

~~x~~ $= -10.1 \rightarrow 0$ belongs $c_1$

$c_1$ will have 4 points

$X = -6.00001 \longrightarrow 2$ belongs to $C_1$ ✓

~~$X = -6$~~ $\rightarrow 2$ belongs $C_2$ tie breaking

$C_1$ will have 2 points,

$x \qquad\qquad 0 \qquad\qquad 10$

0 is mid point $\quad \dfrac{x+10}{2} = 0$

$$n^\alpha(\alpha - \beta) \le CN - \beta$$

$$\Rightarrow \quad n^\alpha \le \frac{CN - \beta}{\alpha - \beta}$$

$\Rightarrow \alpha \ge \beta$

$$n^\alpha(\beta - \alpha) \le \alpha - CN$$

$$\Rightarrow \quad n^\alpha \cancel{\le} \ge \frac{\alpha - CN}{\beta - \alpha}$$

$\Rightarrow \beta \ge \alpha$

$$\underline{\qquad} \le n^\alpha \le \underline{\qquad}$$

# UCS Example 2

Quiz

Q5 (last)

- Given that cost from state $i$ to $j$ is $2^{j-i-1}$ for $j > i$. The initial state is 1 and goal state is 5. What is a vertex expansion sequence if Uniform Cost Search (UCS) is used?
- A: 1, 5
- B: 1, 2, 3, 4, 5
- C: 1, 2, 3, 4, 4, 5
- D: 1, 2, 3, 3, 4, 4, 5
- E: 1, 2, 3, 3, 4, 4, 4, 5

4

1, 2


Greedy

min Priority Queue

$\cancel{1}$ 2, 3, 4, $\cancel{5}$

h 4 3 2 1 0

Expansion sequence $\boxed{1, 5}$

min cost

A, A* $h \le h^*$

$h \approx h^*$

UCS Greedy

A* is A if h is admissible

# UCS

$\cancel{1}$ $\cancel{2_1}$ $\cancel{3_1}$ $\cancel{4_1}$ $5_1$, $\cancel{3_2}$, $\cancel{4_2}$, $5_2$

g 0 1 2 4 8, 2, 3, 5

$\cancel{4_3}$, $\cancel{5_3}$, $\cancel{4_3}$, $5_3$, $5_4$, $5_4$, $5_4$, $5_4$

3 4 3, 4 4 4 4 5

↳ 1, 2, 3, 3, 4, 4, 4, 4, 5

|  | 1 | 2 | 3 | 4 | 5 |
| --- | --- | --- | --- | --- | --- |
| PS |  | 1 | 1 | 3 | 4 |

9

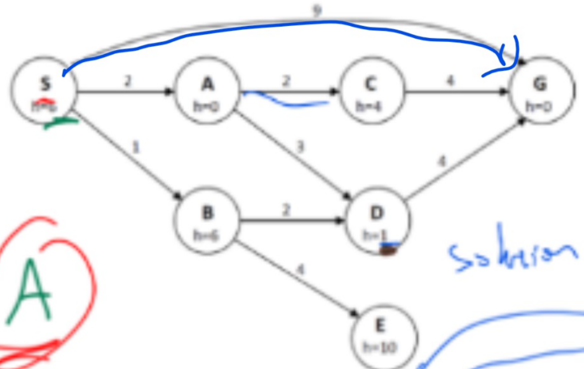
back tracker

M8 – M12
P1 – P6 ] due July 28 midnight

(July 29 morning is ok)

# A Search Example 1 Diagram

Quiz




A

expansion path : S, A, D, B, D, G

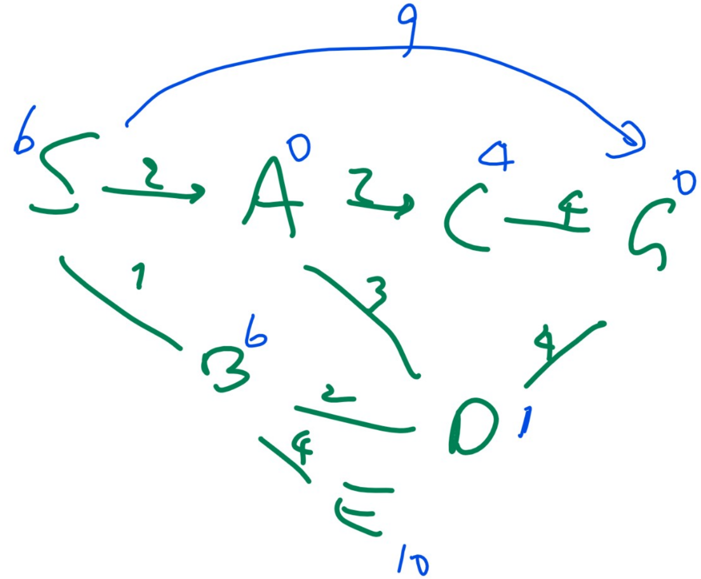
solution

SBDG

| Q | $A_S$ | $D_B$ | ~~$D_A$~~ | ~~S~~ | ~~$B_S$~~ | ~~$G_D$~~ | $C_A$ | $G_S$ | $G_D$ | $E_B$ |
|---|---|---|---|---|---|---|---|---|---|---|
| g | 2 | 1+2 | 2+3 | 0 | 1 | 3+4 | 2+2 | 9 | 5+4 | 1+9 |
| h | 0 | 1 | 1 | 6 | 6 | 0 | 4 | 0 | 0 | 10 |
| g+h | 2 | 4 | 6 | 6 | 7 | 7 | 8 | 9 | 9 | 15 |

back tracker

| S | A | B | C | D | G |
|---|---|---|---|---|---|
| Initial | S | S | A | ~~A~~ B | ~~S~~ D |




| | ~~S~~ | ~~$A_S$~~ | ~~$B_S$~~ | $G_S$ | $C_A$ | ~~$D_A$~~ | $G_D$ | ~~$D_B$~~ | $E_B$ | ~~$G_D$~~ |
|---|---|---|---|---|---|---|---|---|---|---|
| g | 0 | 2 | 1 | 9 | 4 | 5 | 9 | 3 | | 7 |
| h | 6 | 0 | 6 | 0 | 4 | 1 | 0 | 1 | deadend | 0 |
| g+h | 6 | 2 | 7 | 9 | 8 | 6 | 9 | 4 | | 7 |