CS540 Introduction to Artificial Intelligence
Lecture 21

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Based on lecture slides by Jerry Zhu and Yingyu Liang

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Normal Form of Sequential Games

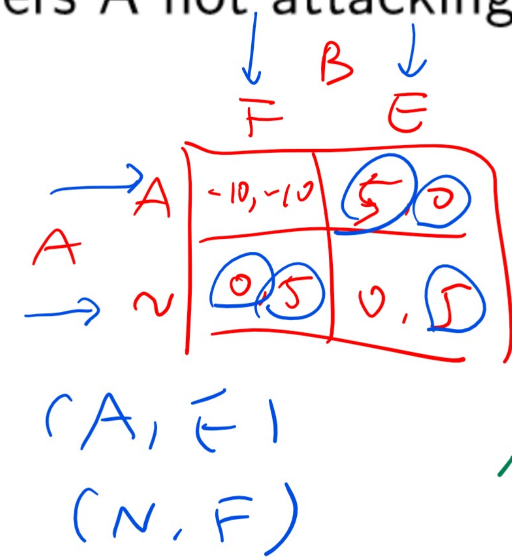
Discussion

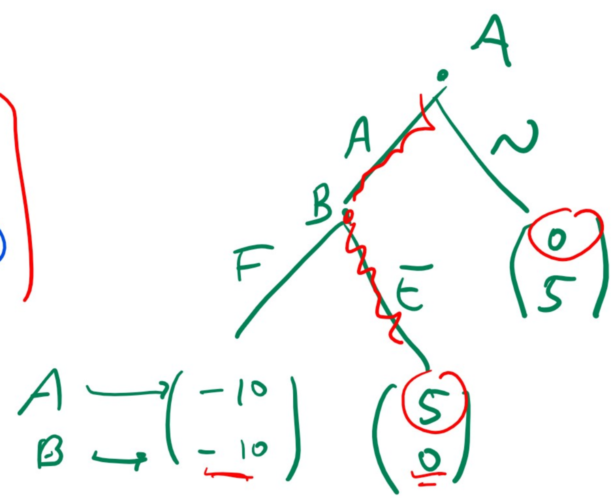
- Sequential games can have normal form too, but the solution concept is different.
- Nash equilibria of the normal form may not be a solution of the original sequential form game.

Non-credible Threat Example, Part I

Quiz (Graded)

- Country A can choose to Attack or Not attack country B. If country A chooses to Attack, country B can choose to Fight back or Escape. The costs are the largest for both countries if they fight, but otherwise, A prefers attacking (and B escaping) and B prefers A not attacking. What are the Nash equilibria?
- A: (A, F)
- B: (A, E)
- C: (N, F)
- D: (N, E)
- E: (N)

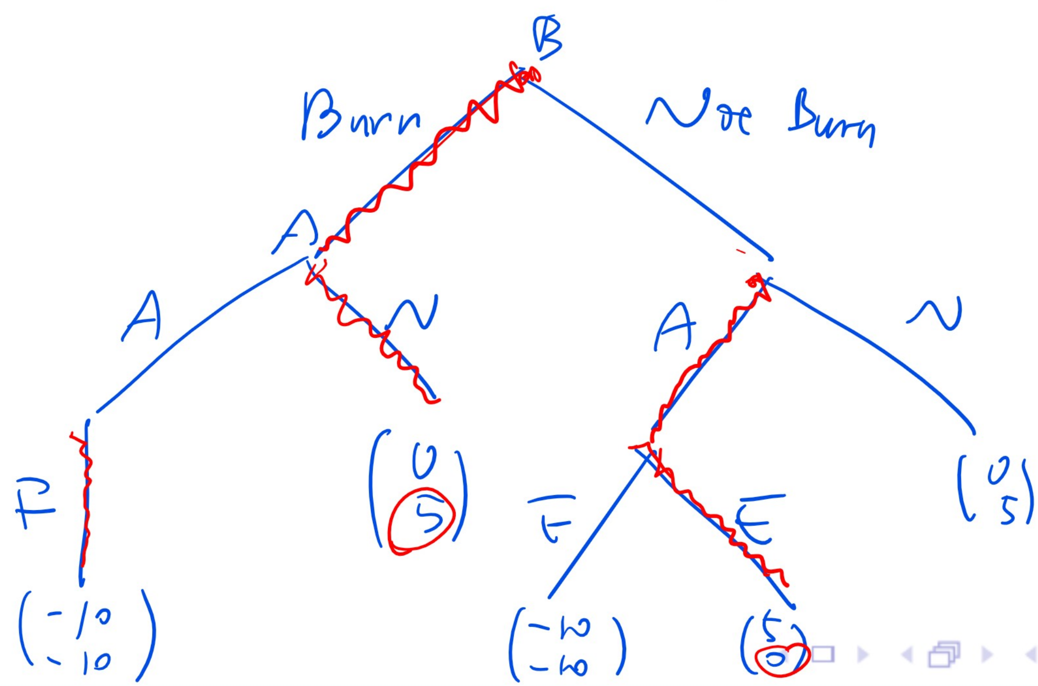


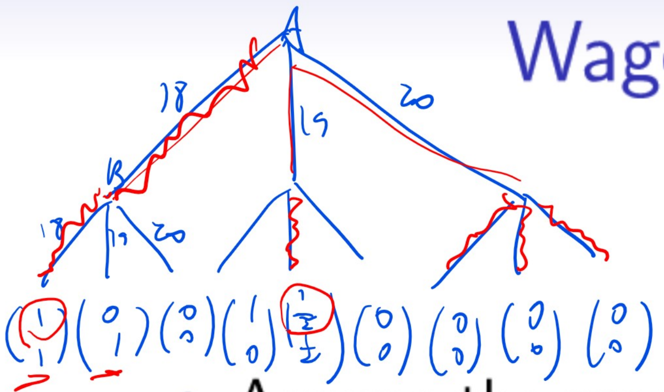


Non-credible Threat Example, Part II

Quiz (Graded)

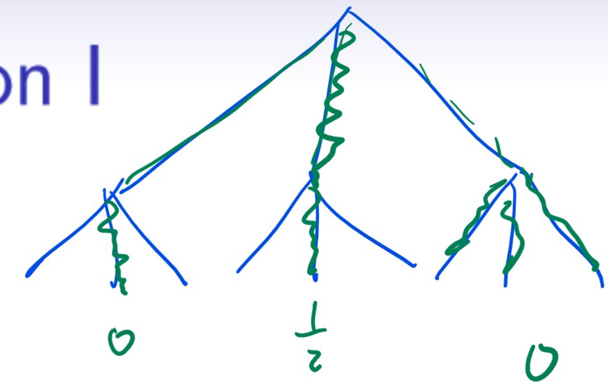
- What if country B can burn the bridge at the beginning of the game so that it cannot choose to escape?





Wage Competition, Version I

Quiz (Participation)



- Assume the productivity of the applicant is 20 dollars per hour, and in case of a tie in the offers, the applicant randomly picks each company with probability a half. What should the companies offer?
  - A: (18, 18)
  - B: (19, 19)
  - C: (20, 20)
  - D: (19, 18)
  - E: (18, 19)

Penalty Kick, Part I

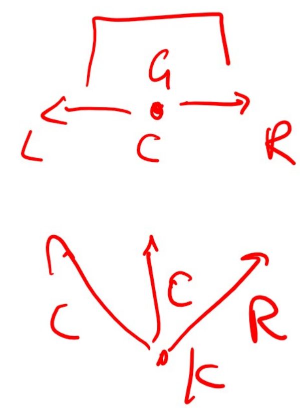
Quiz (Participation)

- The kicker (ROW) and the goalie (COL) choose L, C, R simultaneously. The following table is the estimated probability of scoring the goal given the actions. Kicker maximizes the probability and goalie minimizes the probability. Find all mixed strategy Nash.

MIN

MAX

–	L	C	R
L	0.6	0.9	0.9
C	1	0.4	1
R	0.9	0.9	0.6



no pure Nash.

Penalty Kick, Part II

Quiz (Participation)

–	L	C	R
L	0.6	0.9	0.9
C	1	0.4	1
R	0.9	0.9	0.6

- A: $\left(\left(\frac{1}{3}L, \frac{1}{3}C, \frac{1}{3}R\right), \left(\frac{1}{3}L, \frac{1}{3}C, \frac{1}{3}R\right)\right)$
- B: $\left(\left(\frac{2}{5}L, \frac{1}{5}C, \frac{2}{5}R\right), \left(\frac{1}{3}L, \frac{1}{3}C, \frac{1}{3}R\right)\right)$
- C: $\left(\left(\frac{1}{3}L, \frac{1}{3}C, \frac{1}{3}R\right), \left(\frac{2}{5}L, \frac{1}{5}C, \frac{2}{5}R\right)\right)$
- D: $\left(\left(\frac{2}{5}L, \frac{1}{5}C, \frac{2}{5}R\right), \left(\frac{2}{5}L, \frac{1}{5}C, \frac{2}{5}R\right)\right)$

if COL 1/3, 1/3, 1/3, Row's best response is ANY mixed between L, C, R

Penalty Kick, Part III

Quiz (Participation)

if Row 1/3, 1/3, 1/3

COL → L ⇒ $\frac{1}{3}\cdot 0.4 + \frac{1}{3}\cdot 0 + \frac{1}{3}\cdot 0.1 = 0.2$

$C = \frac{1}{3}\cdot 0.1 + \frac{1}{3}\cdot 0.6 + \frac{1}{3}\cdot 0.1 = \frac{0.8}{3}$

COL

		–	L	C	R
2/5	1/3	L	0.6	0.9	0.9
1/5	1/3	C	1	0.4	1
2/5	1/3	R	0.9	0.9	0.6

L: 0.4, 0, 0.1

C: 0.1, 0.6, 0.1

R: 0.1, 0, 0.4

$br_{COL}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ = always play C.

COL → $L = \frac{2}{5}\, 0.4 + \frac{1}{5}\, 0 + \frac{2}{5}\, 0.1 = 0.2$

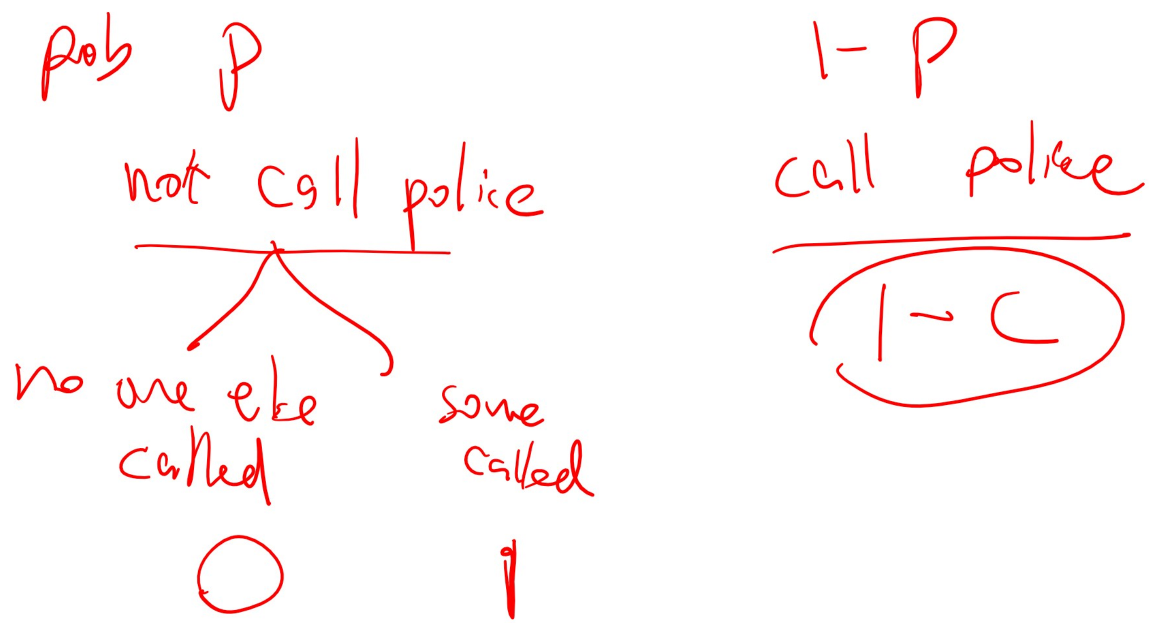
$C = \frac{2}{5}\cdot 0.1 + \frac{1}{5}\, 0.6 + \frac{2}{5}\, 0.1 = 0.2$

$R =$ ______ $= 0.2$

Volunteer's Dilemma, Part I

Quiz (Participation)

- On March 13, 1964, Kitty Genovese was stabbed outside the apartment building. There are 38 witnesses, and no one reported. Suppose the benefit of reported crime is 1 and the cost of reporting is $c < 1$. What is the probability that no one reported?
- A: c
- B: $c^{1/37}$
- C: $c^{38/37}$
- D: $c^{1/38}$
- E: $c^{37/38}$



$\Longrightarrow C = p^{37} \qquad p = C^{1/37}$

Volunteer's Dilemma, Part II

Quiz (Participation)

Public Good Game, Part I

Quiz (Participation)

- You received one free point for this question and you have two choices.
- A: Donate the point.
- B: Keep the point.
- Your final grade is the points you keep plus twice the average donation.

Public Good Game, Part II

Quiz (Participation)

Split or Steal Game

Quiz (Participation)

- Two players choose whether to split or steal a large sum of money, say x dollars. If both choose to split, each player gets $\frac{x}{2}$. If both choose to steal, each player gets 0. If one player chooses to steal, that player gets x. What is a pure strategy Nash equilibrium?
- A: (Split, Split)
- B: (Steal, Split)
- C: (Split, Steal)
- D: (Steal, Steal)

Rubinstein Bargaining Game, Part I

Quiz (Participation)

- There is a cake of size 1. Two kids bargain how to divide the cake for N rounds. The size of the cake is reduced to δ^t after t rounds of bargaining. In round t, if t is odd, kid 1 proposes the division, and kid 2 decides whether to accept or reject, and if t is even, kid 2 proposes the division, and kid 1 decides whether to accept or reject. The game ends when a proposal is accepted, and both kids get 0 if all proposals are rejected. How should the kid 1 propose in round 1? Assume kids accept when indifferent.

Rubinstein Bargaining Game, Part II

Quiz (Participation)

- How should the kid 1 propose in round 1 if $N = 2$? Assume kids accept when indifferent.
- A: $(1, 0)$
- B: $(1 - \delta, \delta)$
- C: $(1 - \delta + \delta^2, \delta - \delta^2)$
- D: $(1 - \delta + \delta^2 - \delta^3, \delta - \delta^2 + \delta^3)$
- E: $\left(\frac{1}{1-\delta}, \frac{\delta}{1-\delta}\right)$

Rubinstein Bargaining Game, Part III

Quiz (Participation)

- How should the kid 1 propose in round 1 if $N = 4$? Assume kids accept when indifferent.
- A: $(1, 0)$
- B: $(1 - \delta, \delta)$
- C: $(1 - \delta + \delta^2, \delta - \delta^2)$
- D: $(1 - \delta + \delta^2 - \delta^3, \delta - \delta^2 + \delta^3)$
- E: $\left(\frac{1}{1 - \delta}, \frac{\delta}{1 - \delta}\right)$

Rubinstein Bargaining Game, Part IV

Quiz (Participation)

- How should the kid 1 propose in round 1 if $N = \infty$? Assume kids accept when indifferent.
- A: $(1, 0)$
- B: $(1 - \delta, \delta)$
- C: $(1 - \delta + \delta^2, \delta - \delta^2)$
- D: $(1 - \delta + \delta^2 - \delta^3, \delta - \delta^2 + \delta^3)$
- E: $\left(\frac{1}{1-\delta}, \frac{\delta}{1-\delta}\right)$