CS540 Introduction to Artificial Intelligence
Lecture 22

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Coverage

- Unsupervised Learning
- Search

Unsupervised Learning

- Hierarchical Clustering
- K Means Clustering (no Gaussian mixture)

Single Complete Linkage Example

	A	B	C	D
A	0	~~1~~	2	3
B		0	4	5
C			0	~~6~~
D				0

merge A, B

	AB	C	D
AB	0	2	3
C		0	6
D			0

single

single linkage dist (A, B) and (C, D) = min (2, 3, 4, 5)

= 2

Complete — — = max { 2, 3, 4, 5 }

= 5

Principal Component Analysis

- Basic Linear (Matrix) Algebra
- Compute Projection and Variance (use formula on the formula sheet)
- Feature Reconstruction

Matrix Multiplication Example

i,j entry of $\underbrace{X^T X}_{A}$

$e_i = \begin{pmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ ← i position

$e = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ all 1's

$e_i^T X^T X e_j$

$\underbrace{\underbrace{(0\ 0\ \cdots\ 1\ \cdots\ 0\ 0)\, (X^TX)}_{\text{ith row}} \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}}_{\text{jth entry of ith row}}$

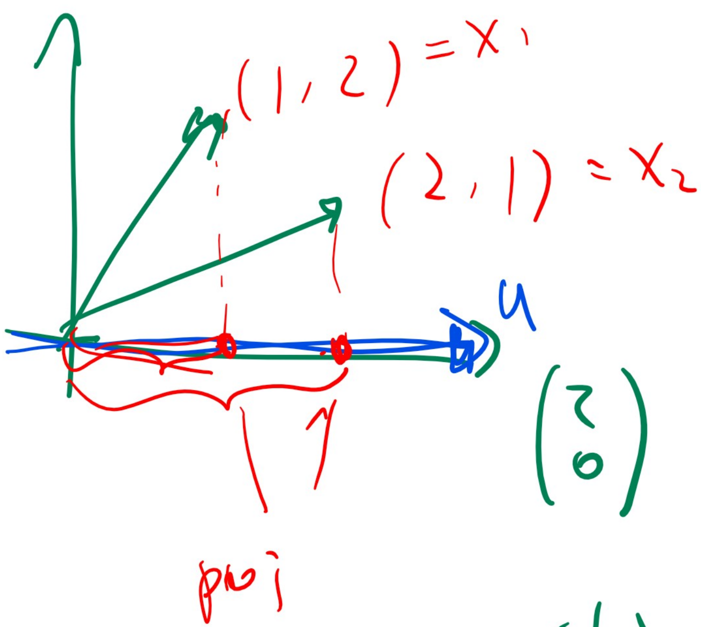
i, j entry

sum of row i - $e_i^T X^T X e$

Sum of everything $e^T X^T X e$

Projected Variance Example

variance of magnitude of projection



$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$x = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\frac{u^T x}{u^T u} u = \frac{2\cdot 1 + 0\cdot 2}{2\cdot 2 + 0\cdot 0}\cdot\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$u^T x\, u = 1\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

with $x = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad u^T x\, u = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

mag of proj 2, 1

Variance:

MLE: $\frac{1}{n}\sum (x_i - \hat{\mu})^2 \qquad \frac{1}{2}(0.5^2 + 0.5^2)$

unbiased: $\frac{1}{n-1}\sum (x_i - \hat{\mu})^2 \qquad \frac{1}{1}(0.5^2 + 0.5^2)$

Search

- Uniformed: no hueristic
- Informed: hueristic
- Local: optimization
- Adverserial: sequential move game
- Equilibrium: simultaneous move game

Uniformed Search

- BFS
- DFS
- IDS
- UCS

Informed Search

- Greedy
- A (or A Star)

Uniformed Search Counting Example

States are 1 to $\underline{1024} = 2^{10}$

initial 1

goal 1024

\# of vertices expanded

successor of i, is

$2i$, $2i+1$

BFS: 1024

DFS: 11

change to 1025

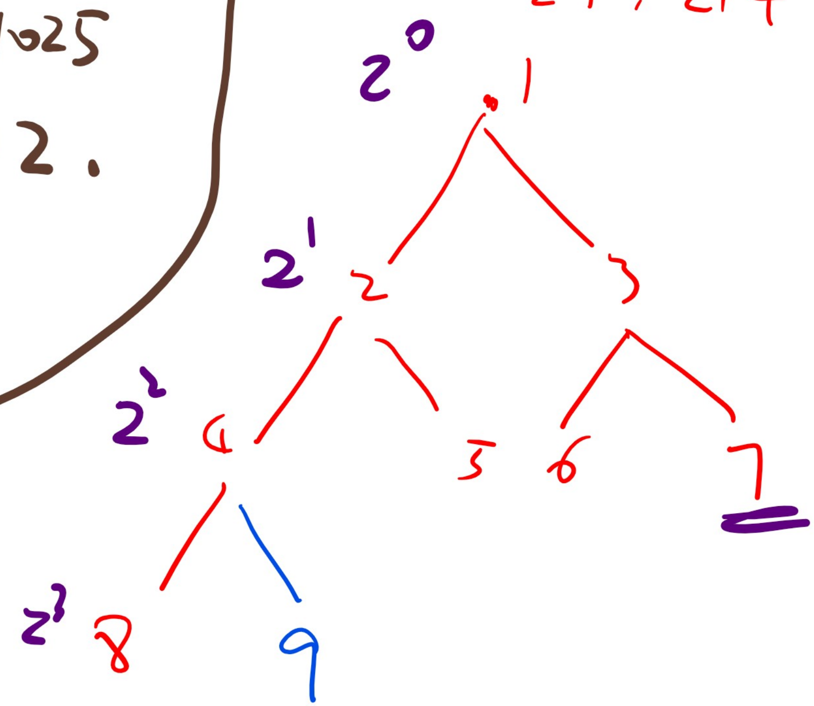
BFS: 1025

DFS: 12.

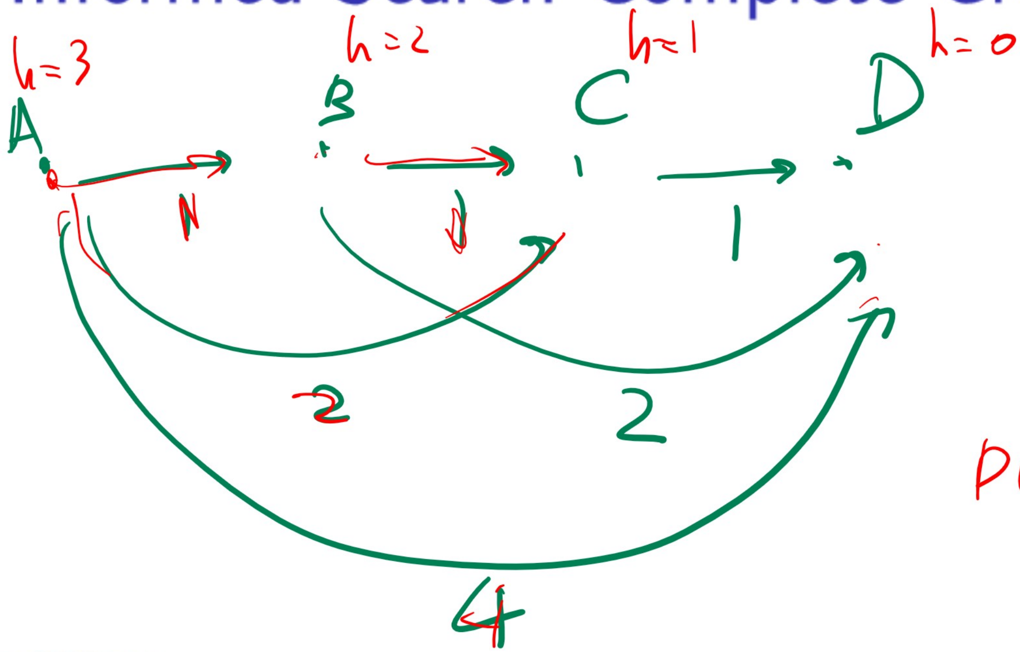
change 1024 to 1023

BFS: 1023

DFS: 1023,



Informed Search Complete Graph Example



expansion path
A, B, C, D

PQ
~~A~~, ~~B~~, ~~C~~, ~~D~~

g= 0 1 2 ~~4~~
1+1 1+2

UCS A, B, C, D

Greedy A D

h

A* A, B, C, D

g+h

cost = cost of parent
+ cost between parent and this state.

PQ: ~~A~~ ~~B~~ ~~C~~ ~~D~~ D from B

3+0 1+2 2+1 4+0 3+0

Local Search

- Hill Climbing
- Simulated Annealing
- Genetic Algorithm

SAT Local Search Example

fitness score = # of clauses satisfied

$$\underbrace{(x \vee y)}_{\text{clause } 1} \wedge \underbrace{(x \vee z)}_{2} \wedge \underbrace{(y \vee z)}_{3}$$

initial $s_0 = x, y, z = \text{False}$. $f(s_0) = 0$

→ find a random neighbor. $s_1 = (x = T,\ y, z = F)$

$f(s_1) = 2 > 0$ $\quad prob_{SA}(\text{move to } s_1) = 1$

→ find another random neighbor. $s_2 = (x, y, z = F)$

assume

$f(s_2) = 0 < 2$ $\quad prob_{SA}(\text{move to } s_2)$

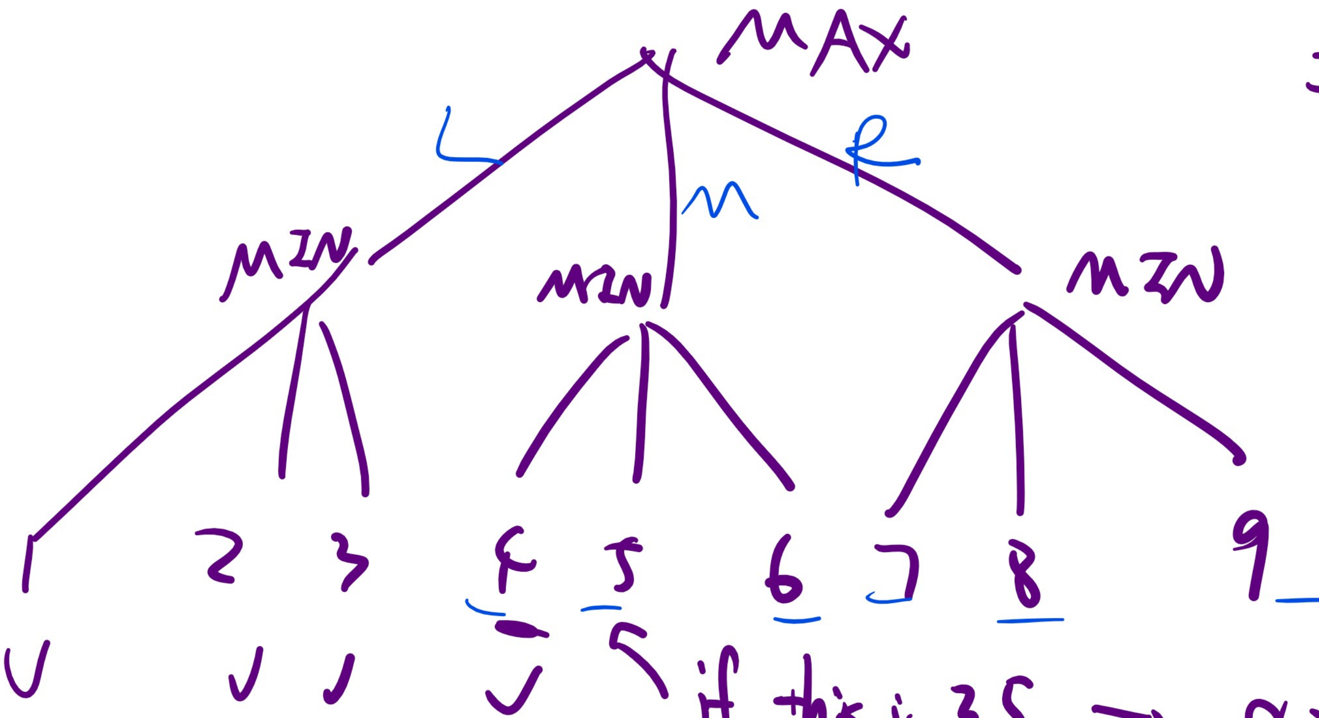
$Temp_0 = 1 \quad Temp_{t+1} = 0.9\, Temp_t$

$$= e^{\left(-\frac{|f(s_2) - f(s_1)|}{temp}\right)} = e^{-\frac{2}{0.9^2}}$$

~~Adverseial~~ Search

- Minimax
- Alpha Beta

Alpha Beta Max Pruning



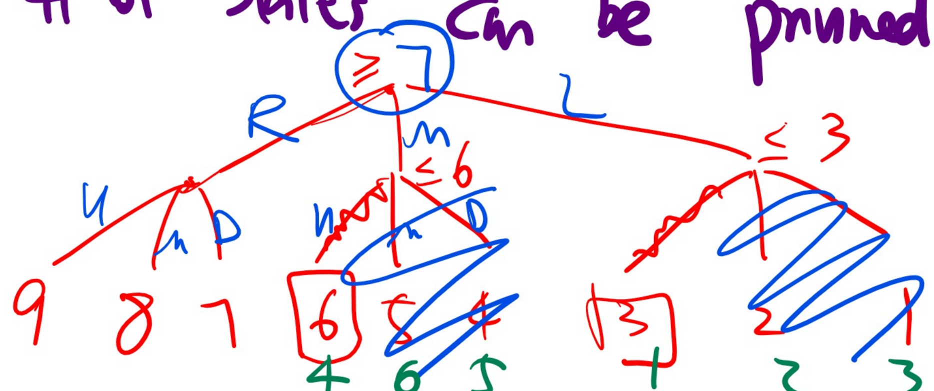
# of pruned states = 0

if this is 3.5 ⇒ α = 3.5

cannot prune.

DFS, left most first

max # of states can be pruned after re order branches,



max

= 4

Equilibrium Search

- IESDS
- Best Responses (Nash)
- Fixed Point (Nash)

Bargaining Example (see Lec 21)

2 kids try to divide cake

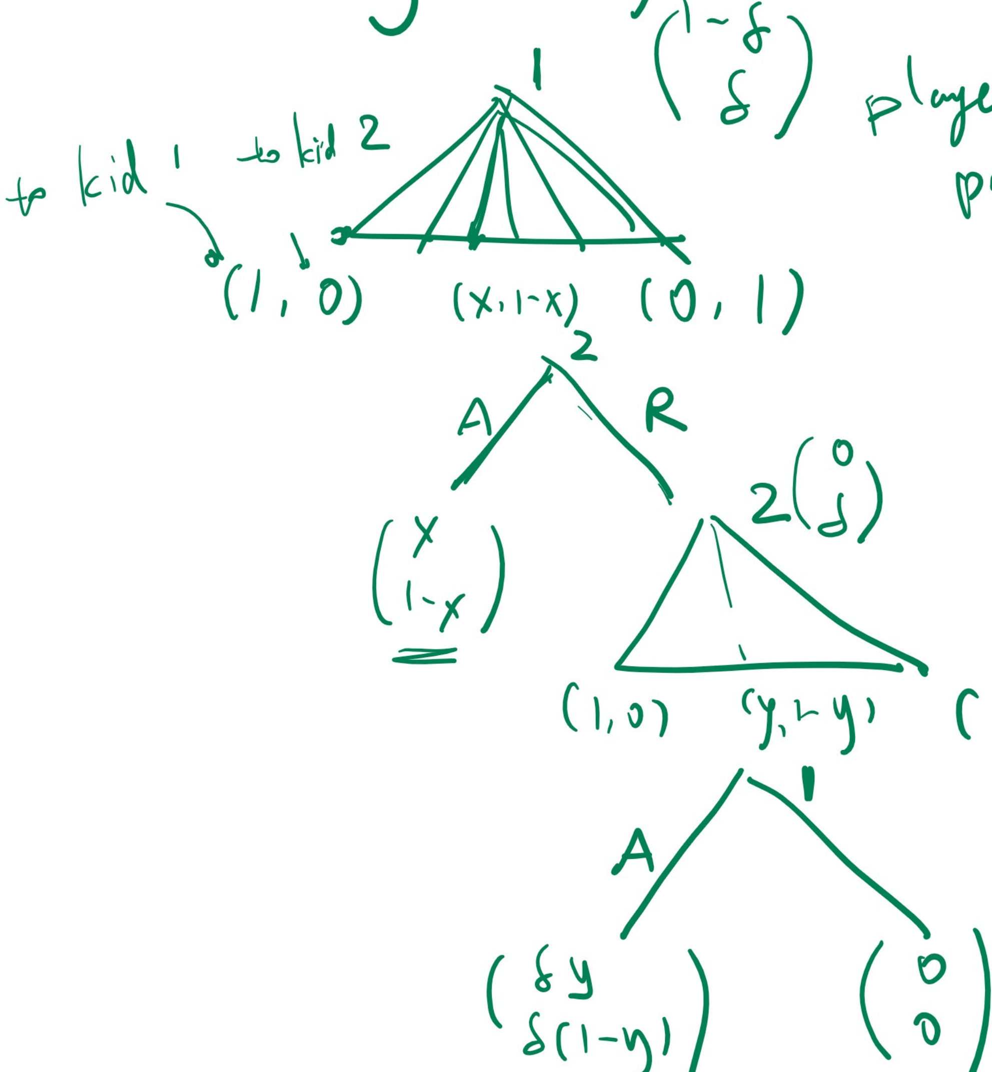
each time they disagree dad eats δ portion of cake

1

δ

δ^2

$\vdots$



player 1

propose $1-\delta$ δ

player 2

accept iff

$1-x \geq \delta$

propose $(0, 1)$

$(0, \delta)$

player 1

accept iff $\delta y \geq 0$

$y \geq 0$

$\leftarrow$ assumption

$N=2$, $\boxed{\begin{pmatrix} 1-\delta \\ \delta \end{pmatrix}}$

$N=\infty$

P1 propose $\boxed{x = 1-\delta+\delta^2 z}$

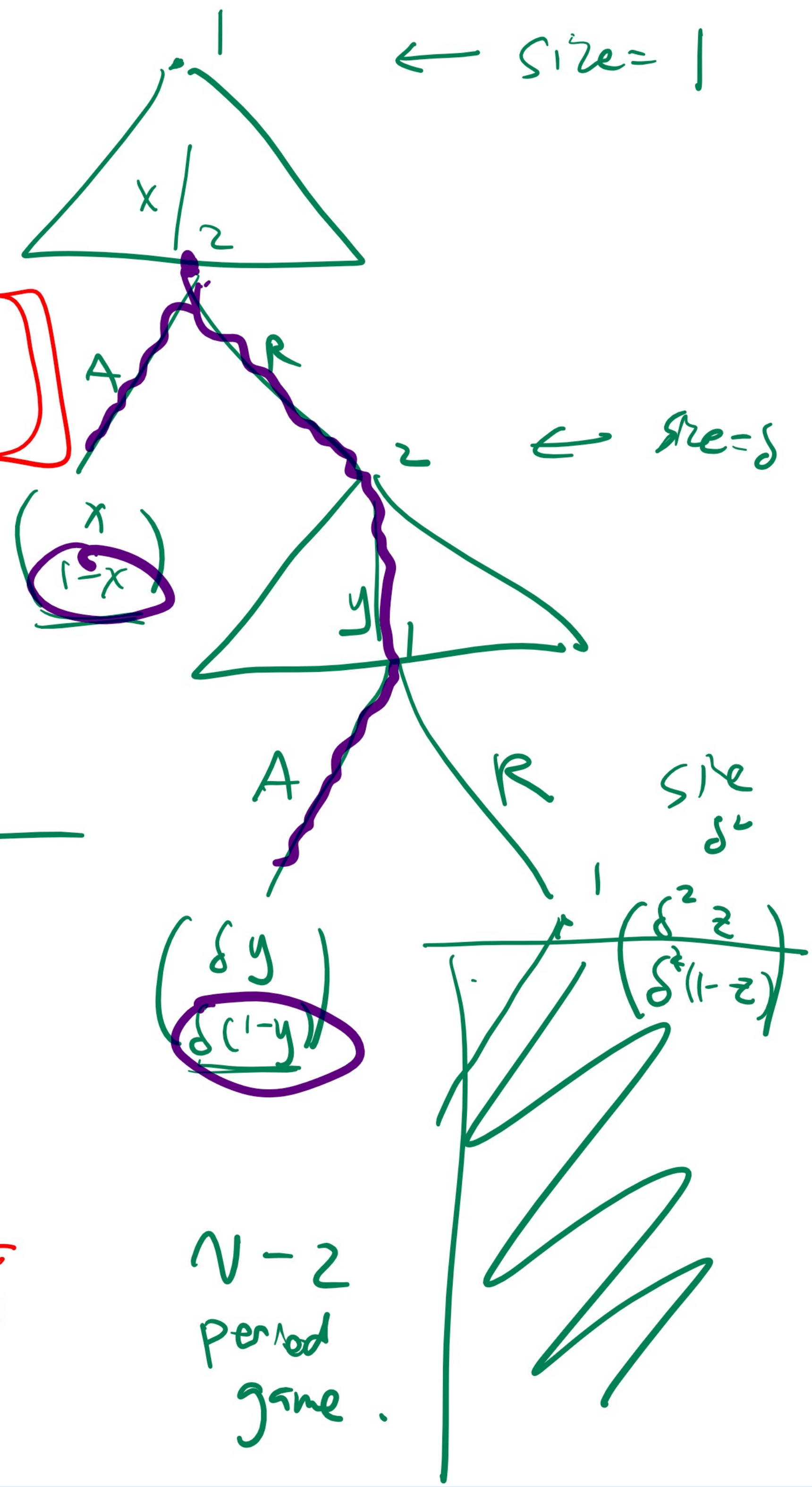
P2 accept iff $1-x \geq \delta(1-\underbrace{\delta z}_{y})$

P2 propose $y = \delta z$

P1 accept iff $\delta y \geq \delta^2 z$

$$\boxed{x = 1-\delta+\delta^2 x}$$

$\Rightarrow$ $x = \frac{1}{1-\delta}$



$N=2,\quad \begin{pmatrix} 1-\delta \\ \delta \end{pmatrix}$

2 period game

$N=4,\quad \begin{pmatrix} 1-\delta+\delta^2\left(1-\delta\right) \\ \delta-\delta^2(1-\delta) \end{pmatrix}$

$\begin{pmatrix} 1-\delta+\delta^2-\delta^3 \\ \delta-\delta^2+\delta^3 \end{pmatrix}$

$N=6,\quad \begin{pmatrix} 1-\delta+\delta^2-\delta^3+\delta^4-\delta^5 \\ \delta-\delta^2+\delta^3-\delta^4+\delta^5 \end{pmatrix}$

$\vdots$

$N=\infty\quad \begin{pmatrix} (1-\delta)(1+\delta^2+\delta^4+\delta^6+\dots) \\ \end{pmatrix}$

$$= \begin{pmatrix} (1-\delta)\cdot\dfrac{1}{1-\delta^2} \\ - \end{pmatrix} = \begin{pmatrix} \boxed{\dfrac{1}{1+\delta}} \\ \dfrac{\delta}{1+\delta} \end{pmatrix}$$

$$1 + x + x^2 + \cdots = \frac{1}{1-x}, \quad x = \delta^2$$