

CS540 Introduction to Artificial Intelligence

Lecture 22

Young Wu

Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

August 3, 2021

Traveler's Dilemma

Quiz

A1

- Two identical antiques are lost. ^{each} The airline only knows that its value is at most 100 dollars, so the airline asks their owners (travelers) to report its value (non-negative integers, ≥ 2). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive 2 dollars in reward. If you are one of the travelers, what will you report?

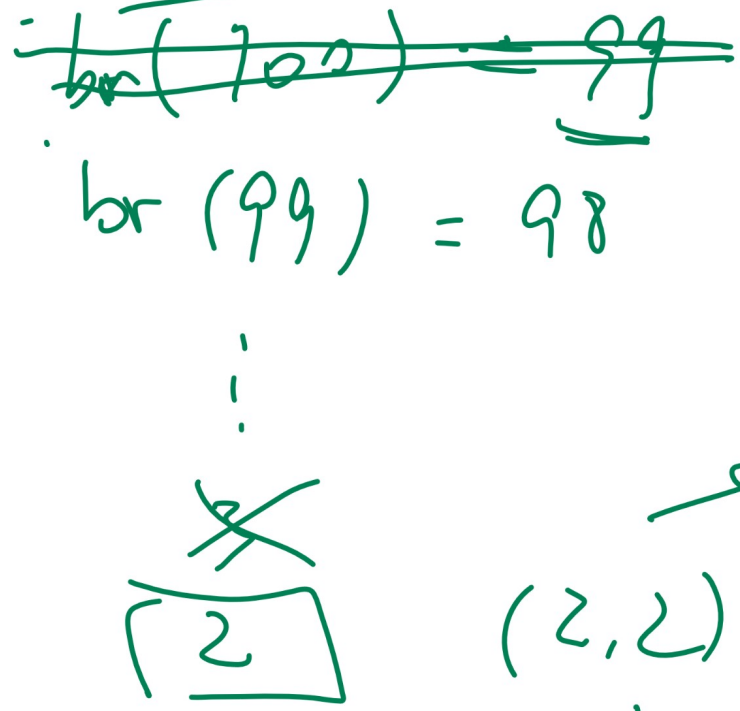
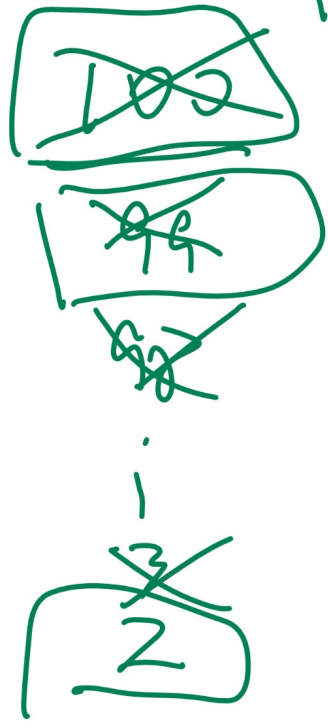
tie \Rightarrow receive what they report.

an integer $\hat{\quad}$ 2 and 100
between

Traveler's Dilemma, Rationalizability

Quiz

is never-best-response. (dominated action)



Nash.
(2,2)
only rationalizable action

→ rationalizable,

Alpha-Beta Pruning Example

Admin

- Another pruning example at the end of lecture, if we don't have time, I will do it on Friday.
- M1 to M11 and P1 to P5 are all ready.
- M12 Course Evaluation: waiting for the email from the department.

Remind Me to Start Recording

Admin

- The messages you send in chat will be recorded: you can change your Zoom name now before I start recording.

Guess Average Game

Motivation

1-rationalizable

~~0~~ 66

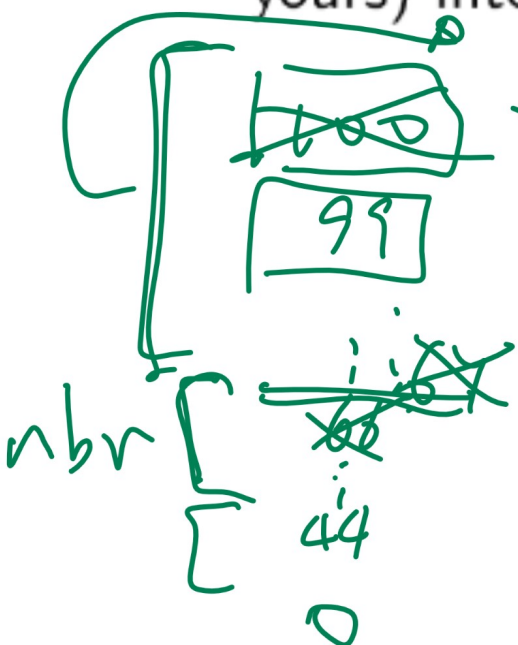
- Write down an integer between 0 and 100 that is the closest to two thirds (2/3) of the average of everyone's (including yours) integers.

never best response. \Rightarrow dominated strategy action

$\arg = \cancel{100}$

Assume I am rational

Other player (0, 66) if they are rational.



Guess Average Game Derivation

Motivation

$[0, 1]$ \rightarrow rationalizable set

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞ -rationalizable.

Rationalizability Example

Quiz

- Fall 2005 Final Q6
- Both players are MAX players. Which actions are rationalizable for the ROW player? Choose E if none of the actions are rationalizable.

Q2
IESDS

Row

	A	B	C
A	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
C	(4, 1)	(2, 8)	(5, 3)
D	(3, 6)	(4, 0)	(1, 9)

Row Col value

no matter
what Row
is picking

$C > A$

B, C 1- rationalizable

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$\left[\begin{array}{l} br_{MAX}(s_{MIN}) = \arg \max_{s \in S_{MAX}} c(s, s_{MIN}) \\ br_{MIN}(s_{MAX}) = \arg \min_{s \in S_{MIN}} c(s_{MAX}, s) \end{array} \right.$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$\left[\begin{array}{l} s_i \succ_{MAX} s_{i'} \text{ if } c(s_i, s) > c(s_{i'}, s) \quad \forall s \in S_{MIN} \\ s_i \succ_{MIN} s_{i'} \text{ if } c(s, s_i) < c(s, s_{i'}) \quad \forall s \in S_{MAX} \end{array} \right.$$

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example 1

Quiz

- Find the value of the Nash equilibrium of the following zero-sum game.

MZV

—	I	II	III
<u>I</u>	-4	-7	-3
II	9	1	7
III	-6	-1	5

br value to MAX

- A: -7 , B: 9 , C: -3 , D: 1, E: -4

max of each Col
min of each Row

minimax
NE

Nash Equilibrium Example 1

Quiz

- Find the value (of MAX player) of the Nash equilibrium of the following zero-sum game.

—	I	II	III
I	(-4, 4)	(-7, 7)	(-3, 3)
II	(9, -9)	(1, -1)	(7, -7)
III	(-6, 6)	(-1, 1)	(5, -5)

Handwritten annotations:
 - 'MAX' (green) with arrows pointing to rows I and II.
 - 'MIN' (red) with an arrow pointing to column II.
 - 'br MAX' (green) with an arrow pointing to column I.
 - 'br MIN' (red) with an arrow pointing to row II.
 - 'NE' (purple) with an arrow pointing to the (II, II) cell.
 - The (II, II) cell (1, -1) is circled in red and boxed in purple.
 - The (II, I) cell (9, -9) is circled in green.
 - The (III, I) cell (-6, 6) is circled in red.
 - The (I, II) cell (-7, 7) is circled in red.
 - The (II, III) cell (7, -7) is circled in green.

Nash Equilibrium Example 2

Quiz

Q3

of games

- Find the value of the Nash equilibrium of the following zero-sum game.

-	I	II	III
I	1	2	3
II	4	5	6
III	7	8	9

NE

- A: 1, B: 3, C: 5, D: 7, E: 9

[max of each col
min of each row

MIN

MAX

br_{MAX} (I)

Mixed Strategy Nash Equilibrium

Definition

RPS $\rightarrow \frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S$

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Chance \rightarrow value \rightarrow expected value.

Battle of the Sexes Example

Discussion

- Battle of the Sexes (BoS, also called Bach or Stravinsky) is a game that models coordination in which two players have different preferences in which alternative to coordinate on.

Romeo

—	Bach	Stravinsky
Bach	(x, y)	$(0, 0)$
Stravinsky	$(0, 0)$	(y, x)

Julia

$$y > x$$

Battle of the Sexes Example 1

Quiz

- Find all Nash equilibria of the following game.

Row

		q	$1-q$
I	P	(3, 5)	(0, 0)
II	1-P	(0, 0)	(5, 3)

$3q > 5 - 5q$
 $8q > 5$
 $q > \frac{5}{8}$

$br_R(q) =$
 $P=1$ if $3q > 5(1-q)$
 $P \in [0, 1]$ if $3q = 5(1-q)$
 $P=0$ if $5(1-q) > 3q$
 $q > \frac{5}{8}$
 $q = \frac{5}{8}$
 $q < \frac{5}{8}$

$I \rightarrow 3q$
 $II \rightarrow 5(1-q)$

$q > \frac{5}{8}$
 $q = \frac{5}{8}$
 $q < \frac{5}{8}$

Battle of the Sexes Example 1 Derivation 1

Quiz

$br_j(p) =$
in terms of q

$q=1$ if $p > \frac{3}{8}$
 ~~$q \in (0,1)$~~ if $p = \frac{3}{8}$
 ~~$q=0$~~ if $p < \frac{3}{8}$

$I \rightarrow 5p = \frac{5}{8}$
 $II \rightarrow 3(1-p) = \frac{5}{8}$
 $5p > 3(1-p)$
 $p > \frac{3}{8}$

	I	II
I	(3, 5)	(0, 0)
II	(0, 0)	(5, 3)

$0 = br_R(0) \quad 0 = br_J(0)$
 $(0, 0) \rightarrow NE \quad (I, II)$
 $(1, 1) \quad (II, I)$

Battle of the Sexes Example 1 Derivation 2

$br_j\left(\frac{3}{8}\right) = \frac{5}{8}$ one of br
 $br_k\left(\frac{5}{8}\right) = \frac{3}{8}$
 $\left(\frac{3}{8}, \frac{5}{8}\right) \rightarrow NE \rightarrow \left(\underset{\substack{(\frac{3}{8}) \\ I}}{I} \underset{\substack{(\frac{5}{8}) \\ I}}{I}, \underset{\substack{(\frac{5}{8}) \\ I}}{I} \underset{\substack{(\frac{3}{8}) \\ I}}{I} \right)$

-	I	II
I	(3, 5)	(0, 0)
II	(0, 0)	(5, 3)

Mixed Strategy Example 1

Quiz

- Which ones of the following are Nash equilibria?

	L	R
U	(3, 1)	(0, 0)
D	(0, 1)	(1, 1)

$u \rightarrow \frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 0 = 1.5$
 $\underline{D} = \frac{1}{2} \cdot 0 + \frac{1}{2} \cdot 0 = 0.5$
 $br_{col}(u) = L$

$br_R(\frac{1}{2}, \frac{1}{2}) = U$

- ~~A: (always U, (L $\frac{1}{2}$ of the time, R $\frac{1}{2}$ of the time))~~

- ~~B: (always D, (L $\frac{1}{2}$ of the time, R $\frac{1}{2}$ of the time))~~

- ~~C: ((U $\frac{1}{2}$ of the time, D $\frac{1}{2}$ of the time), always L)~~

- ~~D: ((U $\frac{1}{2}$ of the time, D $\frac{1}{2}$ of the time), always R)~~

- ~~E: ((U $\frac{1}{2}$ time, D $\frac{1}{2}$ time), (L $\frac{1}{2}$ time, R $\frac{1}{2}$ time))~~

Q4

$br(L) = U$

$= U$

Mixed Strategy Example 1 Derivation

Quiz

$\text{br}(L(\frac{1}{8}), R(\frac{7}{8})) = \underline{D}$

NE

Always D

$L(\frac{1}{4})$ $R(\frac{3}{4})$

-	L	R
U	(3, 1)	(0, 0)
D	(0, 1)	(1, 1)

U → $\frac{1}{8}$
D → $\frac{7}{8}$

$U \rightarrow \frac{1}{4} \cdot 3 + \frac{3}{4} \cdot 0 = \frac{3}{4}$

$D \rightarrow \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 1 = \frac{3}{4}$

$D \geq U$

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Alpha Beta Example 4

Quiz

Alpha Beta Example 4 Continued

Quiz

Static Evaluation Function

Definition

- A static board evaluation function is a heuristics to estimate the value of non-terminal states.
- It should reflect the player's chances of winning from that vertex.
- It should be easy to compute from the board configuration.

Linear Evaluation Function Example

Definition

- For Chess, an example of an evaluation function can be a linear combination of the following variables.
 - 1 Material.
 - 2 Mobility.
 - 3 King safety.
 - 4 Center control.
- These are called the features of the board.

Iterative Deepening Search

Discussion

- IDS could be used with SBE.
- In iteration d , the depth is limited to d , and the SBE of the non-terminal vertices are used as their cost or reward.