CS540 Introduction to Artificial Intelligence
Lecture 22

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Traveler's Dilemma

Quiz

A1

each

- Two identical antiques are lost. The airline only knows that its value is at most 100 dollars, so the airline asks their owners (travelers) to report its value (non-negative integers, $\geqslant 2$). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive 2 dollars in reward. If you are one of the travelers, what will you report?

an integer between 2 and 100

tie ⇒ receive what they report.

Traveler's Dilemma, Rationalizability

Quiz

$\boxed{\cancel{100}}$ is never-best-response. (dominated action)

~~$br(100) = 99$~~

$br(99) = 98$

$\boxed{\cancel{99}}$

$\cancel{98}$

$\vdots$

$\cancel{3}$

$\boxed{2}$ → rationalizable,

$\vdots$

$\cancel{3}$

$\boxed{2}$

$(2,2)$ → Nash.

only rationalizable action

Alpha-Beta Pruning Example

Admin

- Another pruning example at the end of lecture, if we don't have time, I will do it on Friday.
- M1 to M11 and P1 to P5 are all ready.
- M12 Course Evaluation: waiting for the email from the department.

Remind Me to Start Recording

Admin

- The messages you send in chat will be recorded: you can change your Zoom name now before I start recording.

Guess Average Game

Motivation

1-rationalizable

0 66

- Write down an integer between 0 and 100 that is the closest to two thirds (2/3) of the average of everyone's (including yours) integers.

never best response. ⇒ dominated strategy
action

~~100~~

avg = ~~100~~

99

assume I am rational

nbr

~~66~~

44

0

Other player [0, 66)
if they are rational.

Guess Average Game Derivation

Motivation

$[0, 1] \longrightarrow$ rationalizable set

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Rationalizability Example

Quiz

- Fall 2005 Final Q6
- Both players are MAX players. Which actions are rationalizable for the ROW player? Choose E if none of the actions are rationalizable.

Q2

IESDS

COL

Row

–	A	B	C
~~A~~	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
~~C~~	(4, 1)	(2, 8)	(5, 3)
D	(3, 6)	(4, 0)	(1, 9)

no matter what Row is picking

C > A

Row COL value

B.C 1- rationalizable

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$\begin{cases} br_{MAX}\left(s_{MIN}\right) = \arg\max_{s \in S_{MAX}} c\left(s, s_{MIN}\right) \\ br_{MIN}\left(s_{MAX}\right) = \arg\min_{s \in S_{MIN}} c\left(s_{MAX}, s\right) \end{cases}$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$\begin{cases} s_i \succ_{MAX} s_{i'} \text{ if } c(s_i, s) > c(s_{i'}, s) \ \forall s \in S_{MIN} \\ s_i \succ_{MIN} s_{i'} \text{ if } c(s, s_i) < c(s, s_{i'}) \ \forall s \in S_{MAX} \end{cases}$$

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example 1

Quiz

- Find the value of the Nash equilibrium of the following ~~zero-sum~~ game.

MIN

br

value to MAX

MAX

–	I	II	III
I	-4	-7	-3
II	9	1	7
III	-6	-1	5

- A: -7 , B: 9 , C: -3 , D: 1, E: -4

max of each COL

min of each ROW

minimax

$\boxed{\text{NE}}$

Nash Equilibrium Example 1

Quiz

- Find the value (of MAX player) of the Nash equilibrium of the following zero-sum game.

MIN

MAX

–	I	II	III
I	(-4, 4)	(-7, 7)	(-3, 3)
II	(9, –9)	(1, –1)	(7, –7)
III	(-6, 6)	(-1, 1)	(5, –5)

br_{MAX} br_{MIN} NE.

Nash Equilibrium Example 2

Quiz

Q3

of games

- Find the value of the Nash equilibrium of the following zero-sum game.

MIN

MAX

–	I	II	III
I	1	2	3
II	4	5	6
III	7	8	9

NE

$br_{MAX}(I)$

- A: 1 , B: 3 , C: 5 , D: 7, E: 9

max of each col

min of each row

Mixed Strategy Nash Equilibrium

Definition

RPS → $\underbrace{\frac{1}{3}R, \frac{1}{3}P, \frac{1}{3}S.}$

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A $\boxed{\text{pure strategy}}$ is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Chance → value → expected value.

Battle of the Sexes Example

Discussion

- Battle of the Sexes (BoS, also called Bach or Stravinsky) is a game that models coordination in which two players have different preferences in which alternative to coordinate on.

–	Bach	Stravinsky
Bach	(x, y)	(0, 0)
Stravinsky	(0, 0)	$\boxed{(y, x)}$

Romeo

Julie

$y > x$

Battle of the Sexes Example 1

Quiz

- Find all Nash equilibria of the following game.

	q	$1-q$
–	I	II
p I	(3, 5)	(0, 0)
$1-p$ II	(0, 0)	(5, 3)

Row

$3q > 5 - 5q$

$8q > 5$

$q > \frac{5}{8}$

$$br_R(q) = \begin{cases} \cancel{\mathrm{I}}\ p = 1 & \text{if } 3q > 5(1-q) \quad q > \frac{5}{8} \\ \cancel{\mathrm{I}, \mathrm{II}}\ p \in [0,1] & \text{if } q = \frac{5}{8} \\ \cancel{\mathrm{II}}\ p = 0 & \text{if } 5(1-q) > 3q \quad q < \frac{5}{8} \end{cases}$$

→ in terms p

R

I → $3q$

II → $5(1-q)$

Battle of the Sexes Example 1 Derivation 1

Quiz

$$br_J(p) = \begin{cases} \cancel{I} \quad q=1 & \text{if } p > \frac{3}{8} \\ \cancel{I, II} \quad q \in [0,1] & \text{if } p = \frac{3}{8} \\ \cancel{II} \quad q=0 & \text{if } p < \frac{3}{8} \end{cases}$$

in terms of q

–	I	II
I (p)	(3,5)	(0,0)
II ($1-p$)	(0,0)	(5,3)

I $\to 5p = \frac{15}{8}$

II $\to 3(1-p)$ $\frac{15}{8}$

$5p > 3(1-p)$

$p > \frac{3}{8}$

$0 = br_R(0)$ $\quad$ $0 = br_J(0)$

$(0, 0) \to NE$ $\quad$ (II, II)

$(1, 1)$ $\quad$ (I, I)

Battle of the Sexes Example 1 Derivation 2

one of br

Quiz

$br_J\left(\frac{3}{8}\right) = \frac{5}{8}$ $\qquad br_R\left(\frac{5}{8}\right) = \frac{3}{8}$

$\left(\frac{3}{8}, \frac{5}{8}\right) \rightarrow NE \rightarrow \left(I^{\left(\frac{3}{8}\right)} II^{\left(\frac{5}{8}\right)}, I^{\left(\frac{5}{8}\right)} II^{\left(\frac{3}{8}\right)}\right)$

–	I	II
I	$(3,5)$	$(0,0)$
II	$(0,0)$	$(5,3)$

Mixed Strategy Example 1

Quiz

- Which ones of the following are Nash equilibria?

col

–	L	R
U	(3,1)	(0,0)
D	(0,1)	(1,1)

row

$br_R(\frac{1}{2},\frac{1}{2}) = U$

$U \rightarrow \frac{1}{2}\cdot 3 + \frac{1}{2}\cdot 0 = 1.5$

$D = \frac{1}{2}\cdot 0 + \frac{1}{2}\cdot 0 = 0.5$

$br_{col}(U) = L$

Q4

- A: $\left(\text{always } U, \left(L\ \frac{1}{2} \text{ of the time}, R\ \frac{1}{2} \text{ of the time}\right)\right)$ br
- B: $\left(\text{always } D, \left(L\ \frac{1}{2} \text{ of the time}, R\ \frac{1}{2} \text{ of the time}\right)\right)$
- C: $\left(\left(U\ \frac{1}{2} \text{ of the time}, D\ \frac{1}{2} \text{ of the time}\right), \text{always } L\right)$ $br(L) = U$
- D: $\left(\left(U\ \frac{1}{2} \text{ of the time}, D\ \frac{1}{2} \text{ of the time}\right), \text{always } R\right)$ $= U$
- E: $\left(\left(U\ \frac{1}{2} \text{ time}, D\ \frac{1}{2} \text{ time}\right), \left(L\ \frac{1}{2} \text{ time}, R\ \frac{1}{2} \text{ time}\right)\right)$ br

Mixed Strategy Example 1 Derivation

Quiz

$br(L(\frac{1}{8})\ R(\frac{7}{8})) = D$

NE = {always D, $L(\frac{1}{4})\ R(\frac{3}{4})$}

–	L	R
U	(3, 1)	(0, 0)
D	(0, 1)	(1, 1)

$U \to \frac{3}{8}$

$D \to \frac{7}{8}$

$U \to \frac{1}{4} \cdot 3 + \frac{3}{4} \cdot 0 = \frac{3}{4}$

$D \to \frac{1}{4} \cdot 0 + \frac{3}{4} \cdot 1 = \frac{3}{4}$

$D \geq U$

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Alpha Beta Example 4

Quiz

Alpha Beta Example 4 Continued

Quiz

Static Evaluation Function

Definition

- A static board evaluation function is a heuristics to estimate the value of non-terminal states.
- It should reflect the player's chances of winning from that vertex.
- It should be easy to compute from the board configuration.

Linear Evaluation Function Example

Definition

- For Chess, an example of an evaluation function can be a linear combination of the following variables.

1. Material.
2. Mobility.
3. King safety.
4. Center control.

- These are called the features of the board.

Iterative Deepening Search

Discussion

- IDS could be used with SBE.
- In iteration d, the depth is limited to d, and the SBE of the non-terminal vertices are used as their cost or reward.