Math Homework 3

CS540

June 6, 2019

1 Instruction

Please submit your answers on Canvas → Assignments → M3. Late submission will not be accepted.
Please add a file named "comments.txt", and in the first line of the file, grade yourself: $1, 1.5, 2$ (for the entire homework, not for individual questions). In your submission, please do not write your name if you do not want other students to see it (in the case it is posted as a sample solution).

Grade	Meaning
1	You attempted something but mostly incorrect.
1.5	You attempted something but there are mistakes.
2	You have the correct answers + permission to post as a sample solution.

You can put 2.5 if you already got 2 in the Quizzes for the week.

2 Questions

2.1 Question 1

Fall 2005 Final Q15 and Fall 2006 Final Q15

Find the weights w_1, w_2 for the SVM classifier $\mathbb{1}_{\{w_1 x_{i1} + w_2 x_{i2} + 1 \geqslant 0\}}$ given the training data $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ with $y_1 = 1, y_2 = 0$. Compute the margin.

2.2 Question 2

Show that the subderivative of $|x|$ at $x = 0$ is $[-1, 1]$ using the following definition.

$$\partial f(x) = \left\{ v : f\left(x'\right) \geqslant f(x) + v\left(x' - x\right) \ \forall\, x' \right\}$$

2.3 Question 3

Fall 2009 Final Q2

What is the feature vector $\phi(x)$ induced by the kernel $K_{ij} = \exp(x_i + x_j) + \sqrt{x_i x_j} + 3$? You can guess $\phi(x)$

and prove $K_{ij} = \phi(x_i)^T \phi(x_j)$.

2.4 Question 4

Fall 2009 Midterm Q2

Draw the decision trees (four trees) for the following logical operators (AND $y_i = x_{i1} \wedge x_{i2}$, OR $y_i = x_{i1} \vee x_{i2}$, IMPLIES $y_i = x_{i1} \Rightarrow x_{i2}$, IF $y_i = x_{i1} \Leftarrow x_{i2}$). Split according to feature 1 first. Compute the information gain from each split for AND.

x_1	x_2	$x_1 \wedge x_2$	$x_1 \vee x_2$	$x_1 \Rightarrow x_2$	$x_1 \Leftarrow x_2$
1	1	1	1	1	1
1	0	0	1	0	1
0	1	0	1	1	0
0	0	0	0	1	1

2.5 Question 5

Spring 2018 Midterm Q7

Find the 1 Nearest Neighbor, 3 Nearest Neighbor, and 5 Nearest Neighbor labels for $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$ using the following training set with the Manhattan distance.

x_1	1	1	3	5	2
x_2	1	7	3	4	5
y	0	1	1	0	0