# Math Homework 4

CS540

June 12, 2019

## 1 Instruction

Please submit your answers on Canvas → Assignments → M4. Late submission will not be accepted.
Please add a file named "comments.txt", and in the first line of the file, grade yourself: $1, 1.5, 2$ (for the entire homework, not for individual questions). In your submission, please do not write your name if you do not want other students to see it (in the case it is posted as a sample solution).

| Grade | Meaning |
|---|---|
| 1 | You attempted something but mostly incorrect. |
| 1.5 | You attempted something but there are mistakes. |
| 2 | You have the correct answers + permission to post as a sample solution. |

You can put 2.5 if you already got 2 in the Quizzes for the week.

## 2 Questions

### 2.1 Question 1

Spring 2018 Final Q8 to Q11
Given $10 \times 10$ images, a convolutional neural network is trained with $5 \times 5$ filters in the convolutional layer, $2 \times 2$ non-overlapping filters (called stride 2 in the old exams) in the pooling layer, and one fully connected layer with 4 activation units in the output (last) layer. Suppose the pooling layer weights are fixed, how many distinct weights (including biases) must be learned in the whole network?
Hint: 129.
Note: the original question does not use zero padding, this question (and the lecture slides) uses zero padding. Zero padding means adding 0's around the images so that convolutions can be performed for pixels near the boundaries of the images.

### 2.2 Question 2

Compute one of following convolutions. Show computations for each entry of the matrix.

1. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

## 2.3 Question 3

Compute and compare the results for one of the convolutions in the previous question (choose a different one) using one two dimensional convolution vs using two one dimensional convolution.

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

## 2.4 Question 4

Show that a Gaussian filter with size $k = 3$ and standard deviation $\sigma = 0.8$ is apprximately:

$$W_\sigma \approx \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Hint: use the Gaussian filter formula:

$$(W_\sigma)_{t,t'} = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{t^2 + t'^2}{2\sigma^2}\right)$$