Math Homework 6

CS540

June 29, 2019

1 Instruction

Please submit your answers on Canvas → Assignments → M6. Late submission will not be accepted.
Please add a file named "comments.txt", and in the first line of the file, grade yourself: $1, 1.5, 2$ (for the entire homework, not for individual questions). In your submission, please do not write your name if you do not want other students to see it (in the case it is posted as a sample solution).

Grade	Meaning
1	You attempted something but mostly incorrect.
1.5	You attempted something but there are mistakes.
2	You have the correct answers + permission to post as a sample solution.

You can put 2.5 if you already got 2 in the Quizzes for the week.

2 Questions

2.1 Question 1

Spring 2017 Midterm Q4
Given the distance between the clusters so far. Compute the resulting distance table for the next two iterations. This is hierachical clustering with single-linkage Euclidean distance between clusters.

–	A	B	C	D	E
A	0	1075	2013	2054	996
B	1075	0	3272	2687	2037
C	2013	3272	0	808	1307
D	2054	2687	808	0	1059

2.2 Question 2

Spring 2018 Midterm Q5
Given data $\{5, 7, 10, 12\}$ and initial cluster centers $c_1 = 3, c_2 = 13$, find the final clusters, their centers, and the total distortion. This is $K = 2, 2$-means clustering with Euclidean distance.

2.3 Question 3

Compute the projection of $\begin{bmatrix}1\\0\\0\end{bmatrix}$ onto $\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\1\\1\end{bmatrix}$ onto $\begin{bmatrix}1\\0\\0\end{bmatrix}$.

The answer should be $\begin{bmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\end{bmatrix}$ and $\begin{bmatrix}1\\0\\0\end{bmatrix}$.

2.4 Question 4

Given the following spectral decomposition of $\hat{\Sigma}$, write down the first and second principal components.

$$\hat{\Sigma} = \begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\-1 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\-1 & 0 & 1\end{bmatrix}^{-1}$$

The answer should be $\begin{bmatrix}\frac{1}{\sqrt{2}}\\0\\\frac{1}{\sqrt{2}}\end{bmatrix}$ and $\begin{bmatrix}0\\1\\0\end{bmatrix}$

2.5 Question 5

2017 Fall Final Q10

If the principal components are $u_1 = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{bmatrix}^T$ and $u_2 = \begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{bmatrix}^T$. If one original instance is $x = \begin{bmatrix}1 & 2 & 3\end{bmatrix}^T$. What is the new 2 dimensional PCA feature representation of x ?