CS540 Introduction to Artificial Intelligence

Lecture 21

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Summary

Discussion

- Adversarial Search:

1. Sequential Move Games: Minimax $\rightarrow$ DFS on the game tree.
2. Sequential Move Games: Alpha-Beta Pruning $\rightarrow$ DFS to keep track α and β $\rightarrow$ prune the subtree with $\alpha \Rightarrow \beta$.
3. Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability).
4. Simultaneous Move Games: Nash Equilibrium.

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}(s_{MIN}) = \operatorname*{argmax}_{s \in S_{MAX}} c(s, s_{MIN})$$

$$br_{MIN}(s_{MAX}) = \operatorname*{argmin}_{s \in S_{MIN}} c(s_{MAX}, s)$$

Strictly Dominated and Dominant Strategy

Definition

- An action s_i strictly dominates another $s_{i'}$ if it leads to a better state no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c\left(s_i, s\right) > c\left(s_{i'}, s\right) \forall\ s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c\left(s, s_i\right) < c\left(s, s_{i'}\right) \forall\ s \in S_{MAX}$$

- The action $s_{i'}$ is called strictly dominated.
- An action that strictly dominates all other actions is called strictly dominant.

Weakly Dominated and Dominant Strategy

Definition

- An action s_i weakly dominates another $s_{i'}$ if it leads to a better state or a state with the same payoff no matter what the opponents' actions are.

$$s_i \succ_{MAX} s_{i'} \text{ if } c\left(s_i, s\right) \geqslant c\left(s_{i'}, s\right) \forall\ s \in S_{MIN}$$
$$s_i \succ_{MIN} s_{i'} \text{ if } c\left(s, s_i\right) \leqslant c\left(s, s_{i'}\right) \forall\ s \in S_{MAX}$$

- The action $s_{i'}$ is called weakly dominated.

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

–	C	D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.

Properties of Nash Equilibrium

Discussion

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- Rationalizable actions (the set of Nash equilibria is a subset of this) can be found be iterated elimination of strictly dominated actions.
- The above statements are not true for weakly dominated actions.

Normal Form of Sequential Games

Discussion

- Sequential games can have normal form too, but the solution concept is different.
- Nash equilibria of the normal form may not be a solution of the original sequential form game.

Fixed Point Algorithm

Description

- For small games, it is possible to find all the best responses. The states that are best responses for all players are the solutions of the game.
- For large games, start with a random action, find the best response for each player and update until the state is not changing.

Mixed Strategy Nash Equilibrium

Definition

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Rock Paper Scissors Example

Discussion

- There are no pure strategy Nash equilibria.
- Playing each action (rock, paper, scissors) with equal probability is a mixed strategy Nash.

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Fixed Point Nash Equilibrium

Algorithm

- Input: the payoff table $c(s_i, s_j)$ for $s_i \in S_{MAX}, s_j \in S_{MIN}$.
- Output: the Nash equilibria.
- Start with random state $s = (s_{MAX}, s_{MIN})$.
- Update the state by computing the best response of one of the players.

$$\text{either } s' = (br_{MAX}(s_{MIN}), br_{MIN}(br_{MAX}(s_{MIN})))$$
$$\text{or } s' = (br_{MAX}(br_{MIN}(s_{MAX})), br_{MIN}(s_{MAX}))$$

- Stop when $s' = s$.

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2. Sequential Move Games: Alpha-Beta Pruning → DFS to keep track α and β → prune the subtree with $\alpha \Rightarrow \beta$.
3. Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability) → Remove dominated actions for each player → Repeat.
4. Simultaneous Move Games: Nash Equilibrium → Compute the best response → Find strategies (pure or mixed) that are mutual best responses.