CS 540: Introduction to Artificial Intelligence Summer 2020

Final Review

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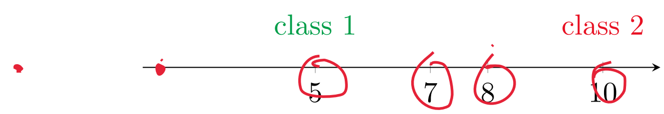
1. **(Summer 2019 Sample Final C Q4)** Suppose K-Means with $K = 2$ is used to cluster the dataset $\{5, 7, 8, 10, x\}$ and initial cluster centers $c_1 = 5, c_2 = 10$. What is x if one of the cluster centers in the next iteration is 5? Here, x can belong to either cluster 1 or 2.



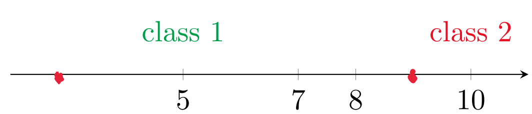
$$\frac{5+7}{2} = 6 \qquad \frac{8+10}{2} = 9$$

$$\frac{5+7+x}{3} = 5 \qquad \frac{8+10+x}{3} = 5$$

$x = 3$ ✓ $\qquad \cancel{x = -3}$

2. **(Summer 2019 Sample Final C Q5)** Continue from the previous question (but ignore the last sentence). What is x if one of the cluster centers in the next iteration is 7? Here, x can belong to either cluster 1 or 2.

Solution:



$$\frac{5+7}{2} = 6 \qquad \frac{8+10}{2} = 9$$

$$\frac{5+7+x}{3} = 7 \qquad \frac{8+10+x}{3} = 7$$

$\cancel{x = 9} \qquad \cancel{x = 3}$

3. **(M8Q8) (Spring 2017 Midterm Q4)** You are given the distance table. Consider the next iteration of hierarchical agglomerative clustering (another name for the hierarchical clustering method we covered in the lectures) using complete linkage. What will the new values be in the resulting distance table corresponding to the four new clusters? If you merge two columns (rows), put the new distances in the column (row) with the smaller index. For example, if you merge columns 2 and 4, the new column 2 should contain the new distances and column 4 should be removed, i.e. the columns and rows should be in the order (1), (2 and 4), (3), (5).

$$\begin{array}{cc} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ d = \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 86 & 63 & 78 & 1 \\ 86 & 0 & 22 & 15 & 77 \\ 63 & 22 & 0 & 17 & 43 \\ 78 & 15 & 17 & 0 & 30 \\ 1 & 77 & 43 & 30 & 0 \end{bmatrix} \end{array}$$

Hint: the resulting matrix should have 4 columns and 4 rows.

Single

	AE	B	C	D
AE	0	77	43	30
B	77	0	22	15
C	43	22	0	17
D	30	15	17	0

$d(AE, B) = \min\{d(B, A), d(B, E)\} = \min(86, 77) = 77$

$d(AE, C) = \min\{d(C, A), d(C, E)\} = \min(63, 43) = 43$

$d(AE, d) = \min\{d(d, A), d(d, E)\} = \min(78, 30) = 30$

Complete

	AE	B	C	D
AE	0	86	63	78
B	86	0	22	15
C	63	22	0	17
D	78	15	17	0

$d(AE, B) = \max\{d(B, A), d(B, E)\} = \max(86, 77) = 86$

$\max(63, 43) = 63$

$\max(78, 30) = 78$

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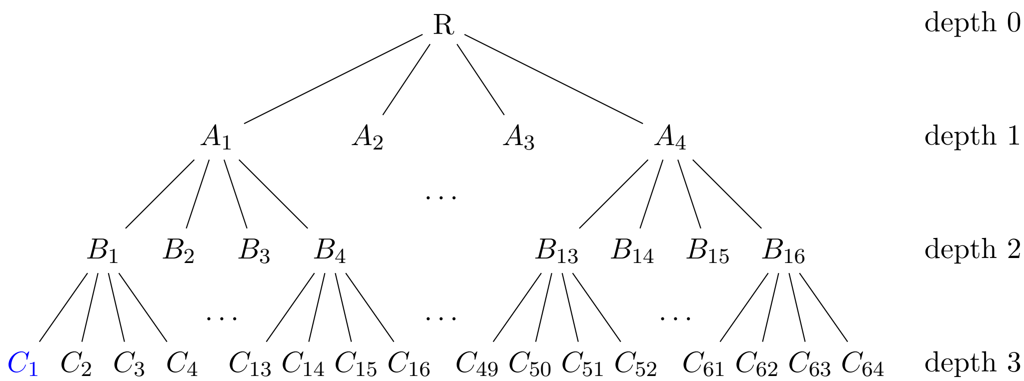
4. **(M9Q7)(Fall 2017 Final Q14)** Consider a search graph which is a tree, and each internal node has 4 children. The only goal node is at depth 3 (root is depth 0). How many total goal-checks will be performed by Iterative Deepening Search in the luckiest case (i.e. the smallest number of goal-checks)? If a node is checked multiple times you should count that multiple times



BFS

R, A1, . . . A4, B1, . . . B16, C1

$4^0 + 4^1 + 4^2 + 1$

DFS

R, A1, B1, C1

$1 + 1 + 1 + 1$

IDS

DFS @ 0

R

4^0

DFS @ 1

R, A1, . . . A4

$4^0 + 4^1$

DFS @ 2

R, A1, B1 . . . B4, A2, B5 . . . B8 . . . . A4, B13 . . . B16

$4^0 + 4^1 + 4^2$

DFS @ 3

R, A1, B1, C1

$1 + 1 + 1 + 1$

$4^0 + 4^0 + 4^1 + 4^0 + 4^1 + 4^2 + 1 + 1 + 1 + 1$

5. **(M10Q2, M10Q3)(Fall 2018 Midterm Q6, Fall 2017 Midterm Q10)** Let h_1 be an admissible heuristic from a state to the optimal goal, A* search with which ones of the following h will **(never)** be admissible?

(A) $h(n) = h_1(n) \cdot 2$ $\quad 0 \cdot 2 = 0$ $\qquad h^* = 2 \quad h_1(n) = 2 \quad h(n) = 4$

(B) $h(n) = h_1(n)^2$ $\quad 0^2 = 0$

(C) $h(n) = \sqrt{h_1(n)}$ $\quad \sqrt{0} = 0$

(D) $h(n) = h_1(n) + 1$ $\quad 0 + 1 = 1$

(E) $h(n) = h_1(n) - 1$ $\quad 0 - 1 = -1$

(F) $h(n) = \frac{h_1(n)}{2}$ $\quad \frac{0}{2} = 0$

$0 \le h(n) \le h^*(n)$

$h^*(\text{goal}) = 0$

D E NEVER admissible

F is admissible

6:30

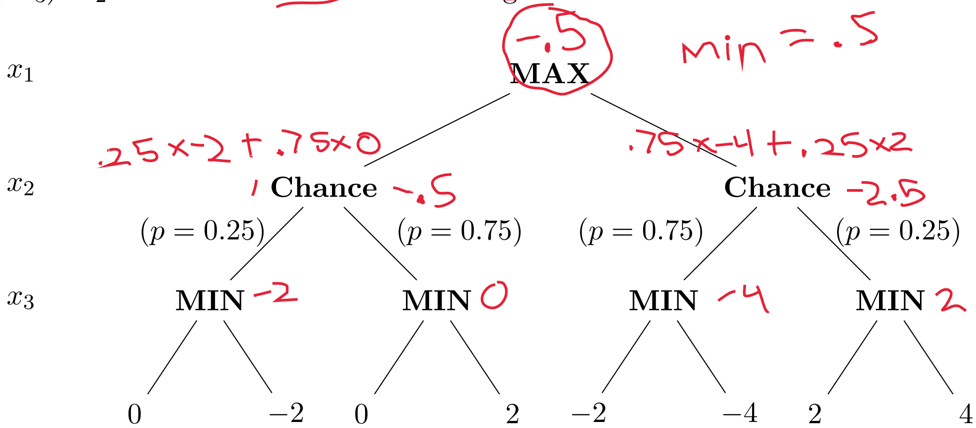
6. **(Summer 2019 Sample Final B Q26)** Suppose the states are given by five integers from $\{0,1\}$. The fitness function is the position of the last 1 in the sequence, i.e. $F(x_1,x_2,x_3,x_4,x_5) = \max\{t \in \{0,1,2,3,4,5\} : x_t = 1\}$, with $x_0 = 1$. There are in total five states. Using the genetic algorithm, the reproduction probability of first state a is $\frac{1}{5}$. What is the reproduction probability of the last state e? The reproduction probabilities are proportional to the fitness of the states.

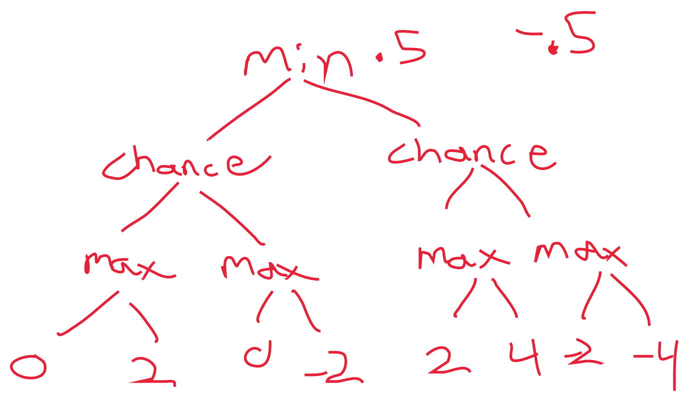
$$x_a = 1(0,0,1,0,0) \quad f(x_a) = \text{argmax}(x_0,x_3) = \max(0,3) = 3$$
$$x_b = 1(0,1,0,0,1) \quad f(x_b) = 5$$
$$x_c = 1(1,0,1,1,0) \quad f(x_c) = 4$$
$$x_d = 1(0,0,0,0,0) \quad f(x_d) = 0$$
$$x_e = ?$$

$$P_a = \frac{f(x_a)}{\sum_{i=a}^{e} f(x_i)} = \frac{3}{3+5+4+0+f(x_e)} = \frac{1}{5} \qquad f(x_e) = 3$$

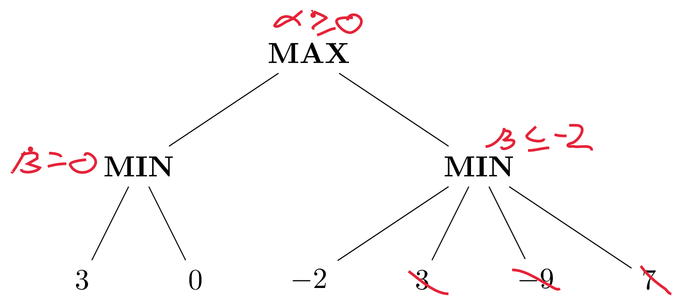
$$P_e = \frac{f(x_e)}{\sum_{i=a}^{e} f(x_i)} = \frac{3}{3+5+4+0+3} = \frac{1}{5}$$

7. **(Summer 2019 Final B Q34)** Consider a zero-sum sequential move game with Chance. Player MAX first chooses between actions $x_1 \in \{-1, +1\}$, then Chance chooses $x_2 \in \{-1, +1\}, \begin{cases} -1 \text{ with probability } \frac{1}{2} + \frac{x_1}{4} \\ +1 \text{ with probability } \frac{1}{2} - \frac{x_1}{4} \end{cases}$. At the end, player MIN chooses between actions $x_3 \in \{-1, +1\}$. The value of the terminal states corresponding to the actions (x_1, x_2, x_3) is $(x_1 + 2 + x_3) \cdot x_2$. What is the value of the whole game?



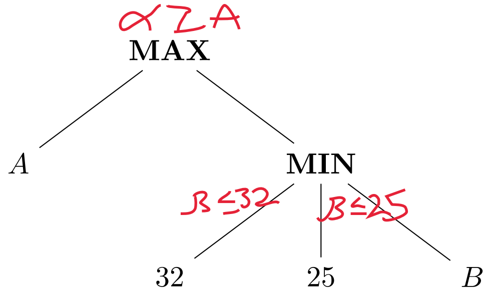


8. **(M12Q2)(Fall 2017 Midterm Q12, Fall 2016 Midterm Q8, Fall 2014 Final Q13, Fall 2012 Final Q17)** Which nodes are pruned by alpha-beta pruning? The max player moves first.



$\beta < \alpha$

9. **(M12Q1)(Fall 2018 Midterm Q7)** Select the values of A such that B will be alpha-beta pruned. In the case $\alpha = \beta$ (i.e. tie-breaking rule), prune the node.



$\alpha \geq \beta$ $\quad A \geq 25$

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10. **(Summer 2019 Final A Q37)** Given the following simultaneous move game payoff table, what is the complete set of pure strategy Nash equilibria? Both players are MAX players. In each entry of the table, the first number is the reward to the ROW player and the second number is the reward to the COLUMN player.

	L	R
U	$(1, 1)$	$(0, 1)$
D	$(0, 0)$	$(1, 1)$

{(U,L), (D,R)}

(Lecture 20 Part 3) As for a mixed strategy Nash equilibrium, suppose COLUMN player chooses L with probability p, then the probability to choose R is $1-p$.

	p L	$1-p$ R
U	(1, 1)	(0, 1)
D	(0, 0)	(1,1)

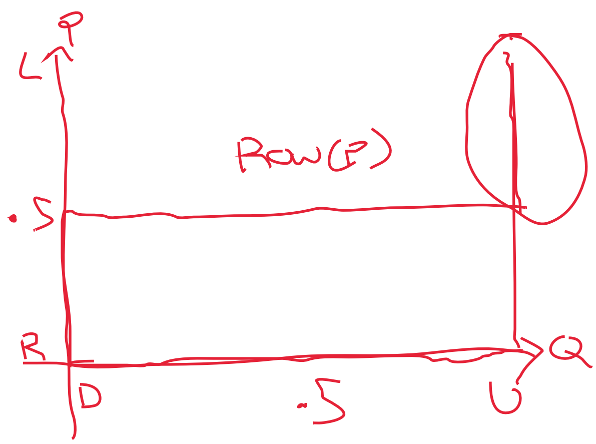
Row
expected payoff(U) $= 1 * p + 0(1-p) = p$
expected payoff(D) $= 0 * p + 1(1-p) = 1-p$

best Response $\text{Row}(p)\begin{cases} U>D,\ p>1-p \quad p>\frac{1}{2} \quad U \text{ if } p>\frac{1}{2} \\ U=D \text{ any mix } U, D \quad p=\frac{1}{2} \\ U<D \quad p<1-p \quad p<\frac{1}{2} \quad D \text{ if } p<\frac{1}{2} \end{cases}$

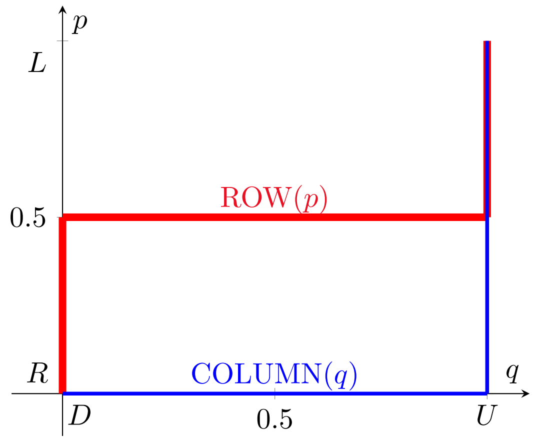
		L	R
q	U	(1, 1)	(0, 1)
$1-q$	D	(0, 0)	(1, 1)

Column
expected payoff(L) $= 1 \cdot q + 0(1-q) = q$
expected payoff(R) $= 1 \cdot q + 1 \cdot (1-q) = 1$

best Response $\text{COLUMN}(q)\begin{cases} \cancel{L>R \quad q>1} \\ L=R \text{ any mix of } L \text{ and } R \quad q=1 \\ L<R \quad q<1 \end{cases}$



(always U, ($\frac{1}{2}R$, $\frac{1}{2}L$))


p
L
ROW(p)
0.5
R
COLUMN(q)
q
D
0.5
U