CS540 Introduction to Artificial Intelligence

Lecture ~~12~~ 10

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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Special Bayesian Network for Sequences

Motivation

- A sequence of features $X_1, X_2, \ldots$ can be modeled by a Markov Chain but they are not observable.
- A sequence of labels $Y_1, Y_2, \ldots$ depends only on the current hidden features and they are observable.
- This type of Bayesian Network is called a Hidden Markov Model.

HMM

HMM Applications Part 1

Motivation

- Weather prediction.
- Hidden states: $X_1, X_2, \ldots$ are weather that is not observable by a person staying at home (sunny, cloudy, rainy).
- Observable states: $Y_1, Y_2, \ldots$ are Badger Herald newspaper reports of the condition (dry, dryish, damp, soggy).
- Speech recognition.
- Hidden states: $X_1, X_2, \ldots$ are words.
- Observable states: $Y_1, Y_2, \ldots$ are acoustic features.

HMM Applications Part 2

Motivation

- Stock or bond prediction.
- Hidden states: $X_1, X_2, \ldots$ are information about the company (profitability, risk measures).
- Observable states: $Y_1, Y_2, \ldots$ are stock or bond prices.
- Speech synthesis: Chatbox.
- Hidden states: $X_1, X_2, \ldots$ are context or part of speech.
- Observable states: $Y_1, Y_2, \ldots$ are words.

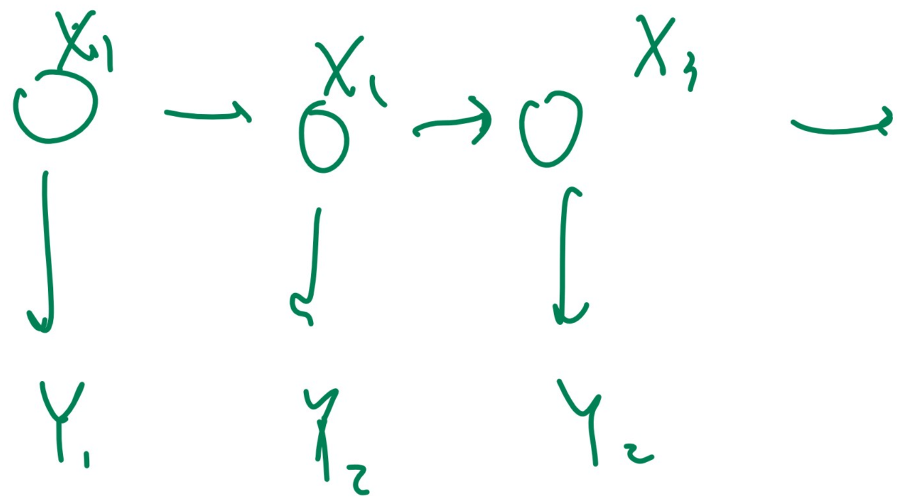
Other HMM Applications

Motivation

- Machine translation.
- Handwriting recognition.
- Gene prediction.
- Traffic control.

Hidden Markov Model Diagram

Motivation



Transition and Likelihood Matrices

Motivation

- An initial distribution vector and two-state transition matrices are used to represent a hidden Markov model.

1. Initial state vector: π.

$$\pi_i = \mathbb{P}\{X_1 = i\}, i \in 1, 2, ..., |X|$$

Markov

2. State transition matrix: A.

$$A_{ij} = \mathbb{P}\{X_t = j | X_{t-1} = i\}, i, j \in 1, 2, ..., |X|$$

3. Observation Likelihood matrix (or output probability distribution): B.

$$B_{ij} = \mathbb{P}\{Y_t = i | X_t = j\}, i \in 1, 2, ..., |Y|, j \in 1, 2, ..., |X|$$

Evaluation and Training

Motivation

- There are three main tasks associated with an HMM.

1. Evaluation problem: finding the probability of an observed sequence given an HMM: $y_1, y_2, \ldots$
2. Decoding problem: finding the most probable hidden sequence given the observed sequence: $x_1, x_2, \ldots$
3. Learning problem: finding the most probable HMM given an observed sequence: $\pi, A, B, \ldots$

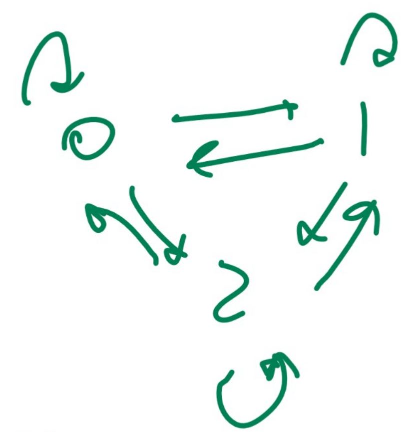
Expectation-Maximization Algorithm

Description

- Start with a random guess of π, A, B.
- Compute the forward probabilities: the joint probability of an observed sequence and its hidden state.
- Compute the backward probabilities: the probability of an observed sequence given its hidden state.
- Update the model π, A, B using Bayes rule.
- Repeat until convergence.
- Sometimes, it is called the Baum-Welch Algorithm.

Hidden Markov Model Example 1

Definition



- Compute $\mathbb{P}\{\underline{X_4 = 1}, \underline{X_5 = 2} | \underline{X_3 = 0}\}$.

$$\Pr\{X_4 = 1 | X_3 = 0\} \cdot \Pr\{X_5 = 2 | X_4 = 1, \cancel{X_3 = 0}\}$$

$$= \Pr\{0 \to 1\} \cdot \Pr\{1 \to 2\}$$

✓ ✓

Hidden Markov Model Example 1 Computations

Definition

Hidden Markov Model Example 2

Definition

$$\begin{array}{ccc} X_1 & \rightarrow & X_2 \\ \downarrow & & \downarrow \\ Y_1 & & Y_2 \end{array}$$

- Compute $\mathbb{P}\{Y_1 = 0, Y_2 = 1\}$.

$Pr\{Y_1 = 0, Y_2 = 1, X_1 = 0, X_2 = \boxed{0}, 1, 2\}$

$Pr\{Y_1 = 0 \mid X_1 = 0\}$ ✓ $Pr\{X_1 = 0\} \rightarrow 1$

$+ \; Pr\{Y_2 = 1 \mid X_2 = 0\}$ ✓ $Pr\{X_2 = 0 \mid X_1 = 0\}$

$0 \rightarrow 0$

Hidden Markov Model Example 2 Computations

Definition

Hidden Markov Model Example 3

Definition

- Compute $\mathbb{P}\{X_1 = 0, X_2 = 2 | Y_1 = 0, Y_2 = 1\}$.

$$\frac{\Pr\{X_1 = 0, X_2 = 2, Y_1 = 0, Y_2 = 1\}}{\Pr\{Y_1 = 0, Y_2 = 1\}}$$

Hidden Markov Model Example 3 Computations

Definition

Dynamic System

Motivation

- The hidden units are used as the hidden states.
- They are related by the same function over time.

$$h_{t+1} = f\left(h_t, w\right)$$
$$h_{t+2} = f\left(h_{t+1}, w\right)$$
$$h_{t+3} = f\left(h_{t+2}, w\right)$$
$$\ldots$$

Dynamic System with Input

Motivation

- The input units can also drive the dynamics of the system.
- They are still related by the same function over time.

$$
\begin{aligned}
h_{t+1} &= f\left(h_t, x_{t+1}, w\right) \\
h_{t+2} &= f\left(h_{t+1}, x_{t+2}, w\right) \\
h_{t+3} &= f\left(h_{t+2}, x_{t+3}, w\right) \\
&\cdots
\end{aligned}
$$

Dynamic System with Output

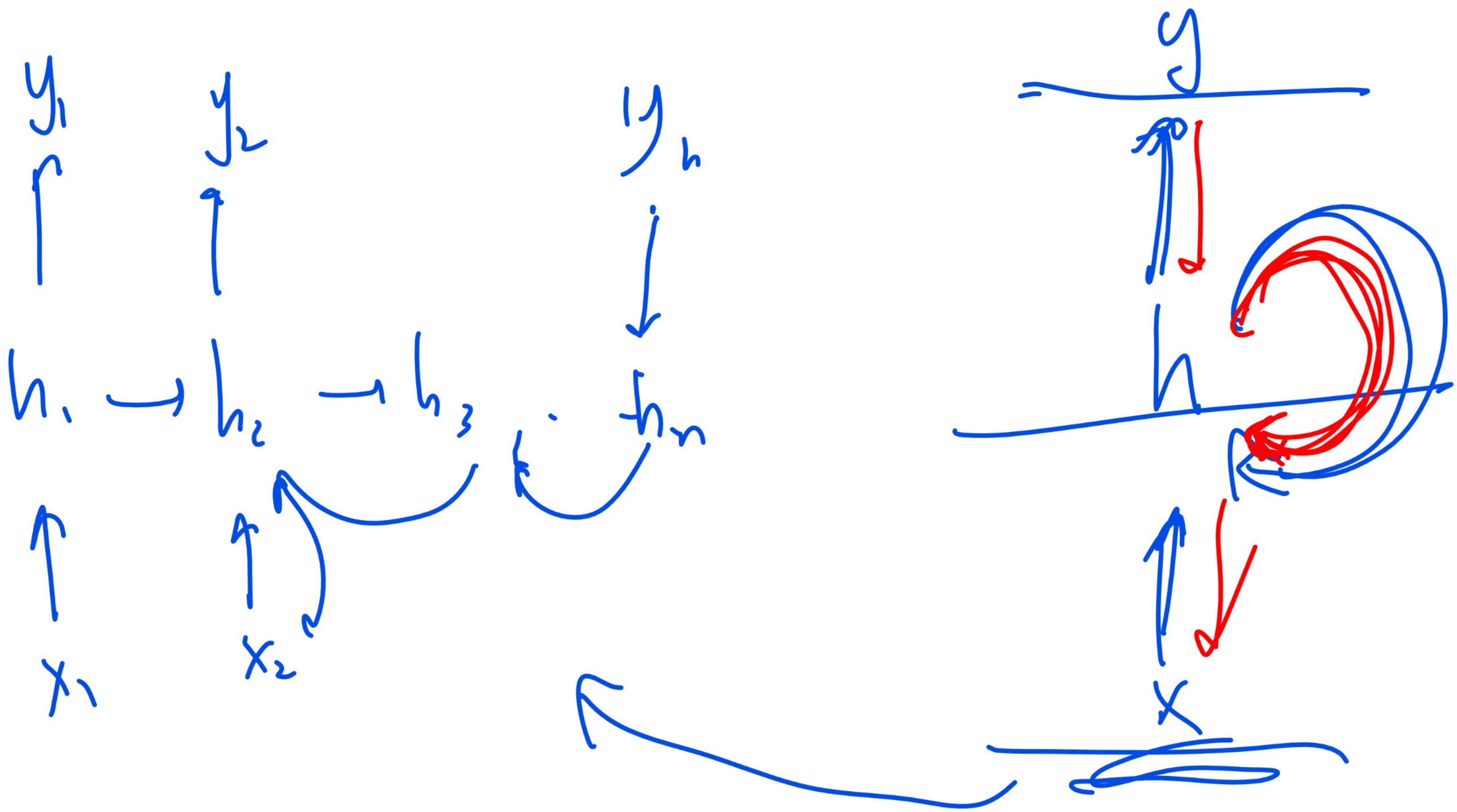
Motivation

- The output units only depend on the hidden states.

$$
\begin{aligned}
y_{t+1} &= f\left(h_{t+1}\right) \\
y_{t+2} &= f\left(h_{t+2}\right) \\
y_{t+3} &= f\left(h_{t+3}\right) \\
&\dots
\end{aligned}
$$

Dynamic System Diagram

Motivation



Recurrent Neural Network Structure Diagram

Motivation

Activation Functions

Definition

- The hidden layer activation function can be the tanh activation, and the output layer activation function can be the softmax function.

$$
\begin{aligned}
z_t^{(x)} &= W^{(x)} x_t + W^{(h)} a_{t-1}^{(x)} + b^{(x)} \\
a_t^{(x)} &= g\left(z_t^{(x)}\right), g(\boxed{\cdot}) = \tanh(\boxed{\cdot}) \\
z_t^{(y)} &= W^{(y)} a_t^{(x)} + b^{(y)} \\
a_t^{(y)} &= g\left(z_t^{(y)}\right), g(\boxed{\cdot}) = \mathrm{softmax}(\boxed{\cdot})
\end{aligned}
$$

Cost Functions

Definition

- Cross entropy loss is used with softmax activation as usual.

$$C_t = H\left(y_t, a_t^{(y)}\right)$$

$$C = \sum_t C_t$$

BackPropogation Through Time

Definition

- The gradient descent algorithm for recurrent neural networks is called BackPropogation Through Time (BPTT). The update procedure is the same as standard neural networks using the chain rule.

$$w = w - \alpha \frac{\partial C}{\partial w}$$
$$b = b - \alpha \frac{\partial C}{\partial b}$$

Unfolded Network Diagram

Definition

Vanishing and Exploding Gradient

Discussion

- If the weights are small, the gradient through many layers will shrink exponentially. This is called the vanishing gradient problem.
- If the weights are large, the gradient through many layers will grow exponentially. This is called the exploding gradient problem.
- Fully connected and convolutional neural networks only have a few hidden layers, so vanishing and exploding gradient is not a problem in training those networks.
- In a recurrent neural network, if the sequences are long, the gradients can easily vanish or explode.

RNN Variants

Discussion

- Long Short Term Memory (LSTM): gated units to keep track of long term dependencies.
- Gated Recurrent Unit (GRU): different gated units.
- Transformers (BERT, GPT): no recurrent units, positional encoding, attention mechansim.