CS540 Introduction to Artificial Intelligence

Lecture 11

Young Wu

Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

June 27, 2022

Coordination Game

Quiz

- You are not allowed to discuss anything about this question in chat. There will be around 10 new questions on the midterm exam. I will post n of them before the exam (this weekend):
- $A : n = 0$.
- $B : n = 1$ if more than 50 percent of you choose B.
- $C : n = 2$ if more than 75 percent of you choose C.
- $D : n = 3$ if more than 95 percent of you choose D.
- $E : n = 0$.
- I will repeat this question a second time. If you fail to coordinate both times, I will not post any of the new questions.

Exam Date

Admin

- July 11 from 5 : 30 to 8 : 30.
- July 27 (online only, with the other section) from 5 : 30 to 8 : 30.
- A : I will be available on July 11.
- B : I will be available on July 27.
- C : I am not available on both dates (email me).

Exam Format

Admin

- Similar to $M2$ to $M7, X1$ to $X3$, total of 30 questions.
- No hints, auto-grading will be turned off.
- There could be minor changes to the questions in $M2$ to $M7, X1$ to $X3$, please re-read the questions carefully.
- The last question will ask you for comments.

Exam Formula Sheet

Admin

- Formulas on $W1$ to $W7$ pages: post on Piazza if you want me to add any.
- You are allowed to implement some of these formulas in Excel or a programming language and use them during the exams.
- You are NOT allowed to work with another student.

Review Session

Admin

- July 6 from $5:30$ to $8:30$ on Zoom, go through past exam questions, answer questions.
- July 7 TA Office Hours on Zoom.
- Message me if you would like more office hours.

Image Features Diagram

Motivation

One Dimensional Convolution

Definition

- The convolution of a vector $x = (x_1, x_2, ..., x_m)$ with a filter $w = (w_{-k}, w_{-k+1}, ...w_{k-1}, w_k)$ is:

$$a = (a_1, a_2, ..., a_m) = x * w$$

$$a_j = \sum_{t=-k}^{k} w_t x_{j-t}, j = 1, 2, ..., m$$

- w is also called a kernel (different from the kernel for SVMs).
- The elements that do not exist are assumed to be 0.

Two Dimensional Convolution

Definition

- The convolution of an $m \times m$ matrix X with a $(2k+1) \times (2k+1)$ filter W is:

$$A = X * W$$

$$A_{j,j'} = \sum_{s=-k}^{k} \sum_{t=-k}^{k} W_{s,t} X_{j-s,j'-t}, j, j' = 1, 2, ..., m$$

- The matrix W is indexed by (s, t) for $s = -k, -k+1, ..., k-1, k$ and $t = -k, -k+1, ..., k-1, k$.
- The elements that do not exist are assumed to be 0.

Convolution Diagram and Demo

Definition

Image Gradient

Definition

- The gradient of an image is defined as the change in pixel intensity due to the change in the location of the pixel.

$$\frac{\partial I\left(s,t\right)}{\partial s} \approx \frac{I\left(s+\frac{\varepsilon}{2},t\right)-I\left(s-\frac{\varepsilon}{2},t\right)}{\varepsilon}, \varepsilon = 1$$

$$\frac{\partial I\left(s,t\right)}{\partial t} \approx \frac{I\left(s,t+\frac{\varepsilon}{2}\right)-I\left(s,t-\frac{\varepsilon}{2}\right)}{\varepsilon}, \varepsilon = 1$$

Image Derivative Filters

Definition

- The gradient can be computed using convolution with the following filters.

$$w_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}, w_y = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Sobel Filter

Definition

- The Sobel filters also are used to approximate the gradient of an image.

$$W_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, W_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Gradient of Images

Definition

- The gradient of an image I is $(\nabla_x I, \nabla_y I)$.

$$\nabla_x I = W_x * I, \nabla_y I = W_y * I$$

- The gradient magnitude is G and gradient direction Θ are the following.

$$G = \sqrt{\nabla_x^2 + \nabla_y^2}$$

$$\Theta = \arctan\left(\frac{\nabla_y}{\nabla_x}\right)$$

Gradient of Images Demo

Definition

Convolution Example

Quiz

- Find the gradient magnitude and direction for the center cell of the following image. Use the derivative filters $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Gradient Example

Quiz

Convolution Example 1

Quiz

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

- $A : \begin{bmatrix} -1 & -3 & -3 \\ 0 & 0 & 0 \\ 1 & 3 & 3 \end{bmatrix}, B : \begin{bmatrix} -3 & -3 & 3 \\ -4 & -4 & 4 \\ -3 & -3 & 3 \end{bmatrix}$
- $C : \begin{bmatrix} -3 & -4 & -3 \\ -3 & -4 & -3 \\ 3 & 4 & 3 \end{bmatrix}, D : \begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \\ -3 & 0 & 3 \end{bmatrix}$

Convolution Example 2

Quiz

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

- $A : \begin{bmatrix} -1 & -3 & -3 \\ 0 & 0 & 0 \\ 1 & 3 & 3 \end{bmatrix}, B : \begin{bmatrix} -3 & -3 & 3 \\ -4 & -4 & 4 \\ -3 & -3 & 3 \end{bmatrix}$
- $C : \begin{bmatrix} -3 & -4 & -3 \\ -3 & -4 & -3 \\ 3 & 4 & 3 \end{bmatrix}, D : \begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \\ -3 & 0 & 3 \end{bmatrix}$

SIFT

Discussion

- Scale Invariant Feature Transform (SIFT) features are features that are invariant to changes in the location, scale, orientation, and lighting of the pixels.

Histogram Binning Diagram

Discussion

HOG

Discussion

- Histogram of Oriented Gradients features is similar to SIFT but does not use dominant orientations.

Classification

Discussion

- SIFT features are not often used in training classifiers and more often used to match the objects in multiple images.
- HOG features are usually computed for every cell in the image and used as features (in place of pixel intensities) in classification algorithms such as SVM.