CS540 Introduction to Artificial Intelligence

Lecture 17

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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Bridge and Torch Game

Motivation

- Four people with one flashlight (torch) want to go across a river. The bridge can hold two people at a time, and they must cross with the flashlight. The time it takes for each person to cross the river:

A	B	C	D
1	2	4	5

- What is the minimum total time required for everyone to cross the river?
- A : 10, B: 11, C: 12, D: 13, E: 14

Due Dates and Grades

Admin

- Next Monday: $M8$, $M9$, $P4$
- Next, next Monday: $M10$, $M11$, $P5$
- Next, next Thursday and Friday: exams

Bridge and Torch Game States

Motivation

Search Problem Applications

Motivation

- Puzzles and games.
- Navigation: route finding.
- Motion planning.
- Scheduling.

Wolf, Sheep, Cabbage Example

Motivation

8 Puzzle Example

Motivation

Sizes of State Space

Motivation

- Tic Tac Toe: 10^3
- Checkers: 10^{20}
- Chess: 10^{50}
- Go: 10^{170}

Water Jugs Example

Definition

Performance

Definition

- A search strategy is complete if it finds at least one solution.
- A search strategy is optimal if it finds the optimal solution.
- For uninformed search, the costs are assumed to be 1 for all edges $c = 1$.

Complexity

Definition

- The time complexity of a search strategy is the worst case maximum number of vertices expanded.
- The space complexity of a search strategy is the worst case maximum number of states stored in the frontier at a single time.
- Notation: the goals are d edges away from the initial state. This means assuming a constant cost of 1, the optimal solution has cost d. The maximum depth of the graph is D.
- Notation: the branching factor is b, the maximum number of actions associated with a state.

$$b = \max_{s \in V} \left| s'(s) \right|$$

Breadth First Search

Description

- Use Queue (FIFO) for the frontier.
- Remove from the front, add to the back.

BFS Example 1

Quiz

- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a BFS expansion sequence?

BFS Example 1 Diagram

Quiz

BFS Example 2

Quiz

- Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?
- A : 10
- B : 11
- C : 12
- D : 1023
- E : 1024

BFS Example 3

Quiz

- Suppose the states are integers between 1 and $2^{10} - 1 = 1023$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a BFS search?
- A : 10
- B : 11
- C : 12
- D : 1023
- E : 1024

Breadth First Search Performance

Discussion

- BFS is complete.
- BFS is optimal with $c = 1$.

Breadth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d.

$$T = b + b^2 + ... + b^d$$

- Space complexity: the worst case is storing all vertices at depth d is in the frontier.

$$S = b^d$$

Depth First Search

Description

- Use Stack (LIFO) for the frontier.
- Remove from the front, add to the front.

DFS Example 1

Quiz

- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a DFS expansion sequence?

DFS Example 1 Diagram

Quiz

DFS Example 2

Quiz

- Suppose the states are integers between 1 and $2^{10} = 1024$. The initial state is 1, and the goal state is 1024. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?
- A : 10
- B : 11
- C : 12
- D : 1023
- E : 1024

DFS Example 3

Quiz

- Suppose the states are integers between 1 and $2^{10} - 1 = 1023$. The initial state is 1, and the goal state is 1023. The successors of a state i are $2i$ and $2i + 1$, if exist. How many states are expanded during a DFS search?
- A : 10
- B : 11
- C : 12
- D : 1023
- E : 1024

Depth First Search Performance

Discussion

- DFS is incomplete if $D = \infty$.
- DFS is not optimal.

Depth First Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the root of the last subtree expanded in the whole graph.

$$T = b^{D-d+1} \ldots + b^{D-1} + b^{D}$$

- Space complexity: the worst case is storing all vertices sharing the parents with vertices in the current path.

$$S = (b-1)\, D + 1$$

Iterative Deepening Search

Description

- DFS but stop if path length > 1
- repeat DFS but stop if path length > 2
- ...
- repeat DFS but stop if path length $> d$

IDS Example 1

Quiz

- Suppose the states are positive integers between 1 and 10, initial state is 1, goal state is 9, successors of i is $2i$ and $2i + 1$ (if exist). What a IDS expansion sequence?

IDS Example 1 Diagram

Quiz

Iterative Deepening Search Performance

Discussion

- IDS is complete.
- IDS is optimal with $c = 1$.

Iterative Deepening Search Complexity

Discussion

- Time complexity: the worst case occurs when the goal is the last vertex at depth d.

$$T = db + (d-1)\, b^2 + ... + 3b^{d-2} + 2b^{d-1} + 1b^d$$

- Space complexity: it has the same space complexity as DFS.

$$S = (b-1)\, d$$

Configuration Space

Discussion

Summary

Discussion

- Search:

1. Uninformed: Breadth first search → Add states at the end → Remove states from the front → Complete + Optimal.
2. Uninformed: Depth first search → Add states to the front → Remove states to the front → Incomplete + Not optimal.
3. Uninformed: Itervative deepening search → DFS with depth limits $1, 2, \ldots$ → Complete + Optimal.
4. Informed: Uniform cost search
5. Informed: Best first greedy search
6. Informed: A search
7. Informed: A star search