CS540 Introduction to Artificial Intelligence

Lecture 21

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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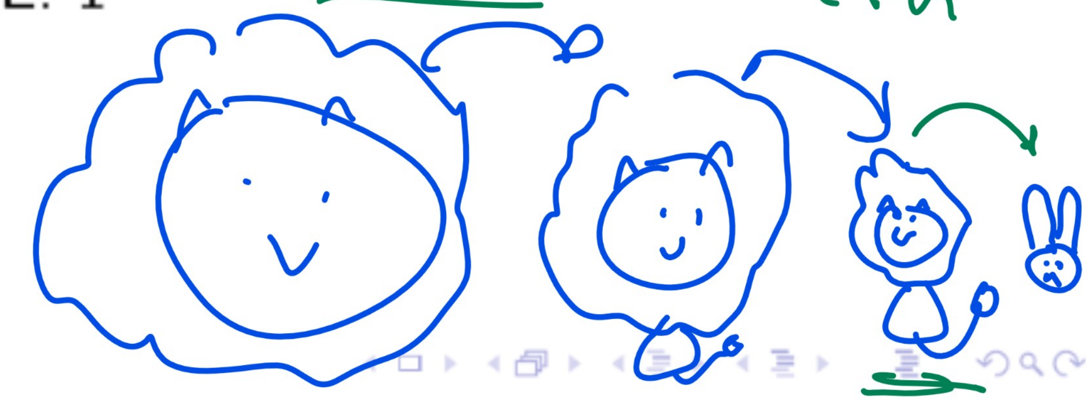
Lion Game Example

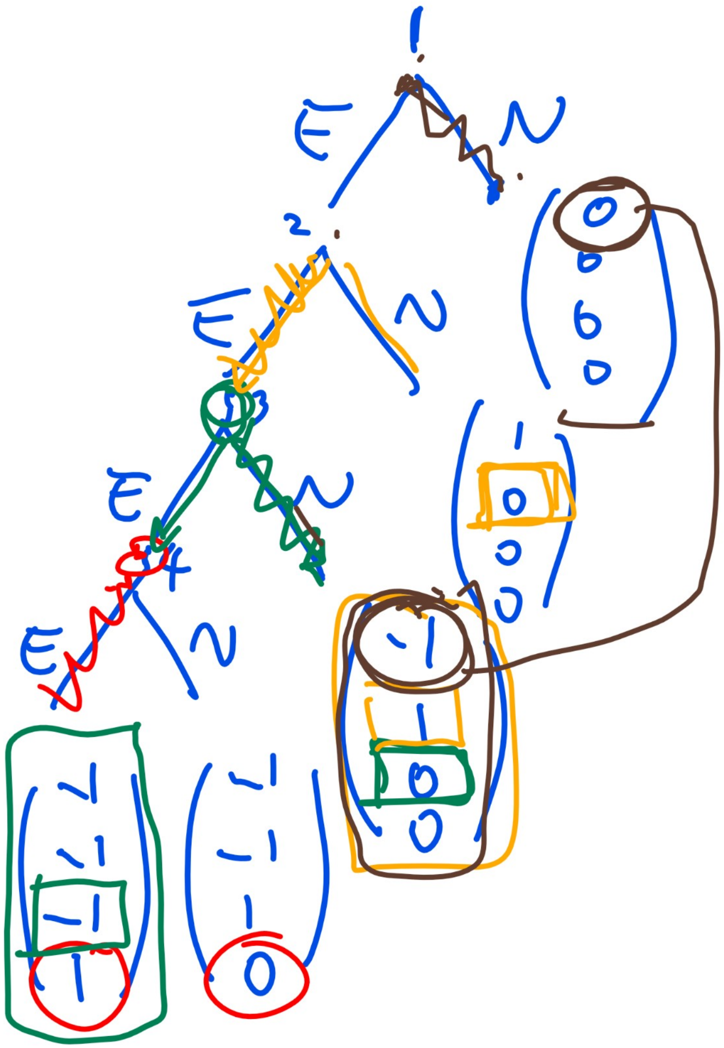
Quiz

- There are N lions, ordered by size, $i = 1, 2, 3, ..., N$, and a bunny. N takes an integer between 1 and 10 with equal probability (known to all the lions). Each lion i can choose to jump out and eat the slightly smaller lion $i - 1$, or stay hidden, and only lion 1 can eat the bunny. Each lion prefers eating to staying hungry to being eaten.
- What is the probability that the bunny is eaten?
- A: 0, B: $\frac{1}{3}$, C: $\frac{1}{2}$, D: $\frac{2}{3}$, E: 1



Lion Game Example Diagram

Quiz



Eat → 1
Hungry → 0
Eaten → -1

DFS

n = 4
bunny not eaten
n odd bunny eaten
n even not

Summary

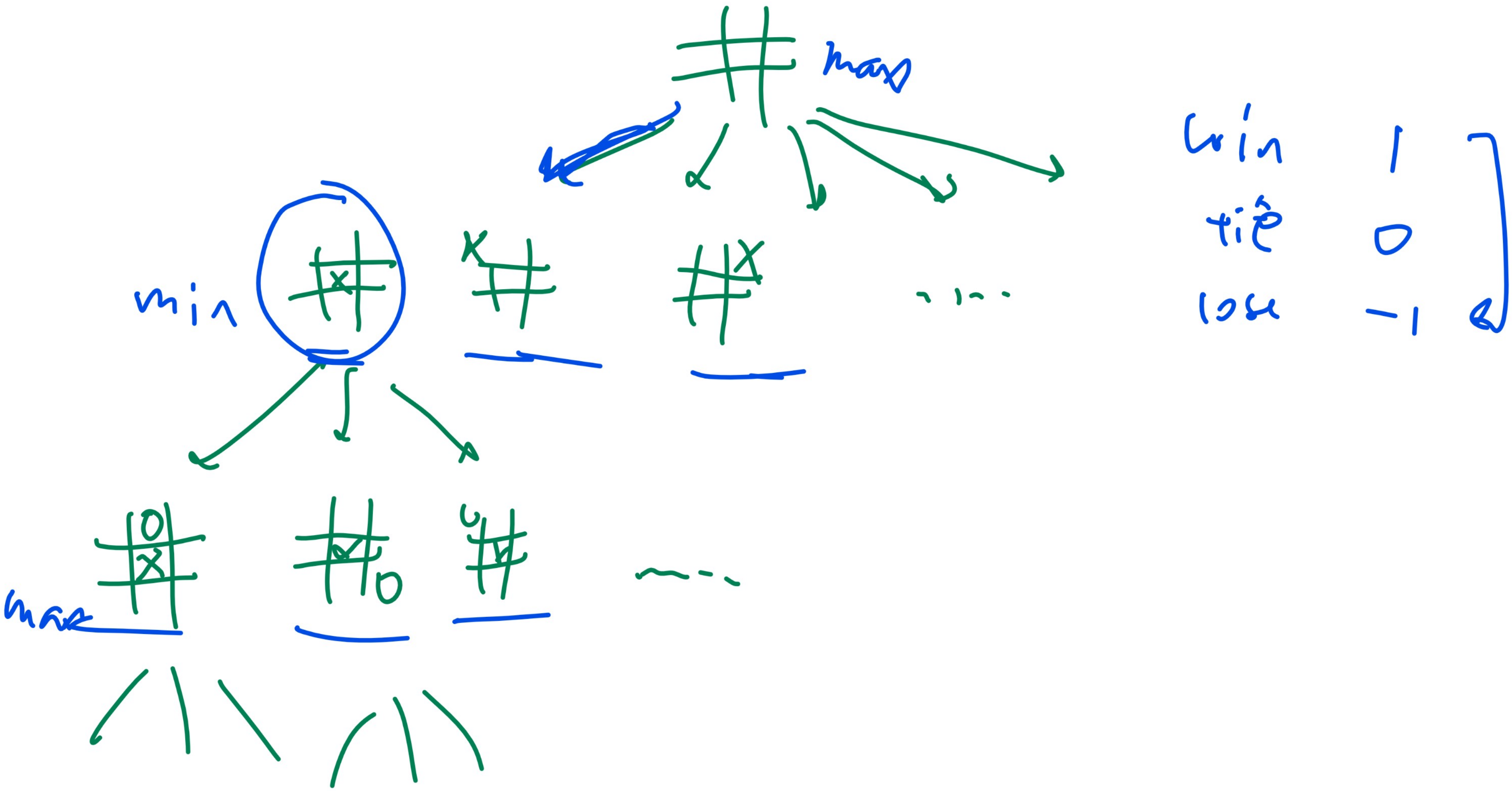
Discussion

- Search:

1. Uninformed.
2. Informed.
3. Local Search.
4. Adversarial Search: Sequential move games.
5. Adversarial Search: Simultaneous move games.

Tic Tac Toe Example

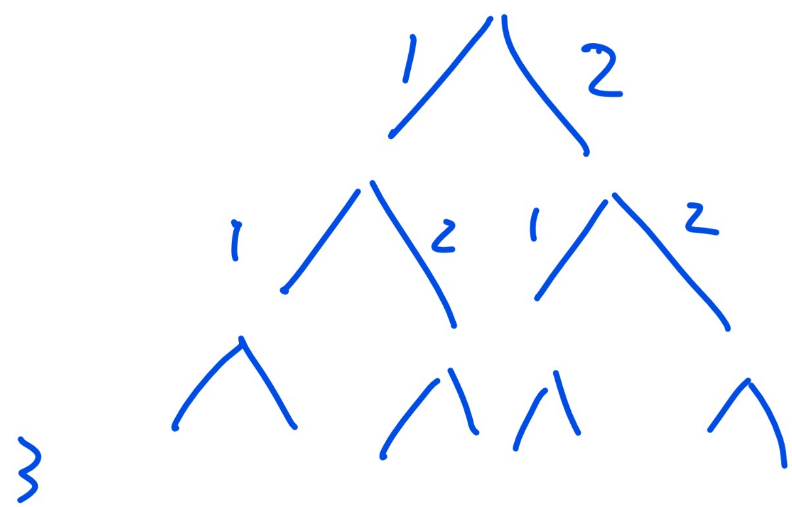
Motivation



Nim Game Example

Motivation

- Ten objects. Pick 1 or 2 each time. Pick the last one to win.
- A : Pick 1.
- B : Pick 2.
- C, D, E : Don't choose.



Minimax Algorithm

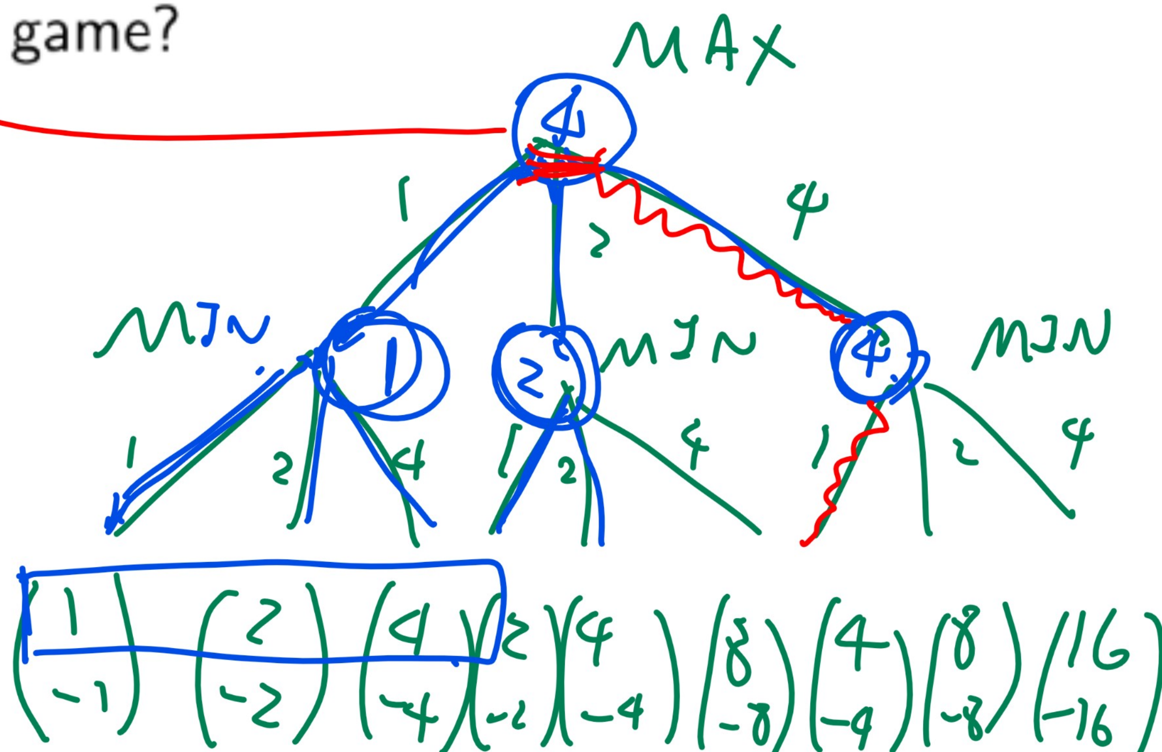
Description

- Use DFS on the game tree.

Minimax Example

Quiz

- For a zero-sum game, the value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. What is the value of the game?
- A : 1
- B : 2
- C : 4
- D : 8
- E : 16



Minimax Performance

Discussion

- The time and space complexity is the same as DFS. Note that $D = d$ is the maximum depth of the terminal states.

$$T = 1 + b + b^2 + ... + b^d$$

$$S = (b - 1) \cdot d$$

Non-deterministic Game

Discussion

- For non-deterministic games in which chance can make a move (dice roll or coin flip), use expected reward or cost instead.
- The algorithm is also called expectiminimax.

payoff score

Game Tree with Chance Example 1

Quiz

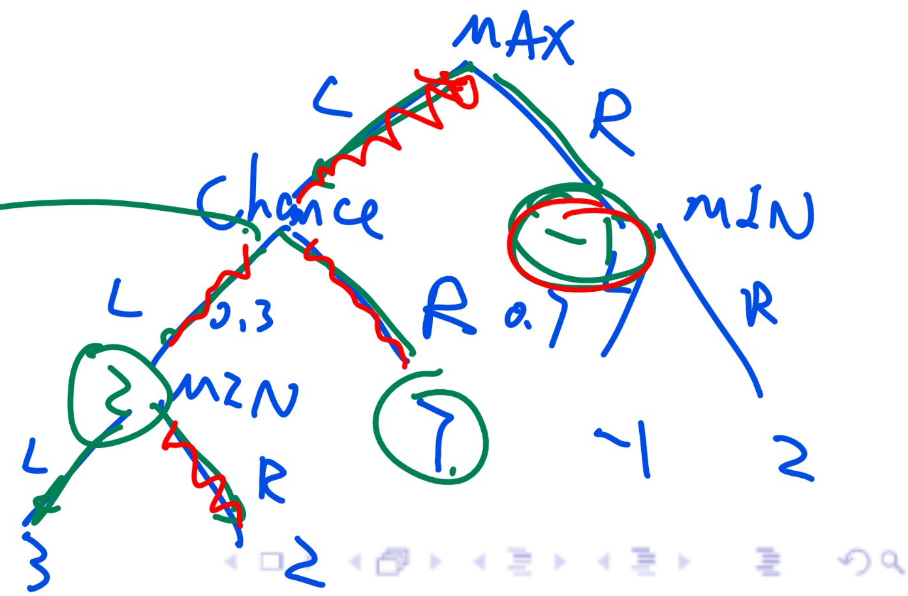
- Max can pick *L* or *R*. If Max picks *L*, Chance picks *L* with probability 0.3 and *R* with probability 0.7. If Chance picks *L*, Min picks *L* to get 3, *R* to get 2, and if Chance picks *R*, Min gets 7. If Max picks *R*, Min picks *L* to get -1 and *R* to get 2. What is the value of the game?

5.5

$7 \cdot 0.7 + 2 \cdot 0.3$

$4.9 + 0.6$

$= 5.5$



Game Tree with Chance Example 1 Diagram

Quiz

Pruning

Motivation

- Time complexity is a problem because the computer usually has a limited amount of time to "think" and make a move.
- It is possible to reduce the time complexity by removing the branches that will not lead the current player to win. It is called the Alpha-Beta pruning.

Alpha Beta Pruning

Description

- During DFS, keep track of both α and β for each vertex.
- Prune the subtree with $\alpha \geqslant \beta$.

Alpha Beta Example 1

Quiz

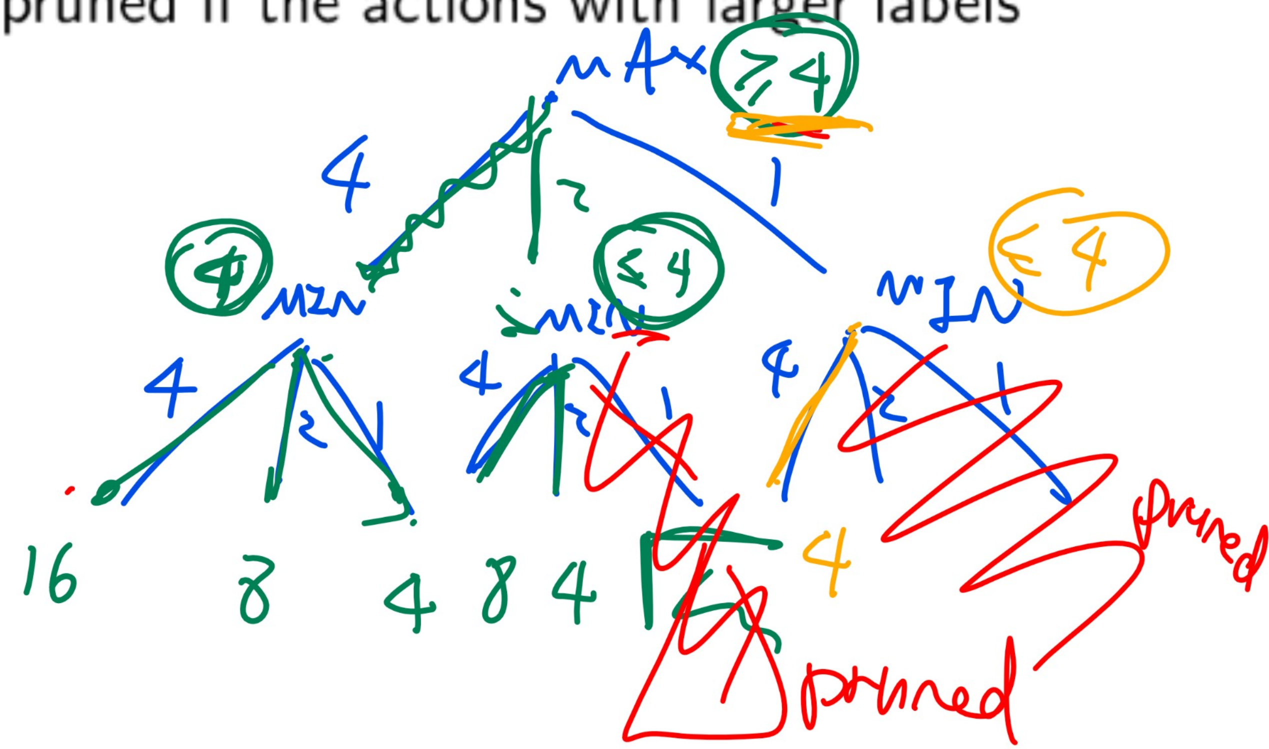
Alpha Beta Example 1 Continued

Quiz

Alpha Beta Example 2

Quiz

- For a zero-sum game, the value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. Alpha-Beta pruning is used. What is the number of branches (states) that can be pruned if the actions with larger labels are searched first?
- A : 0
- B : 1
- C : 2
- D : 3
- E : 4



Alpha Beta Example 3

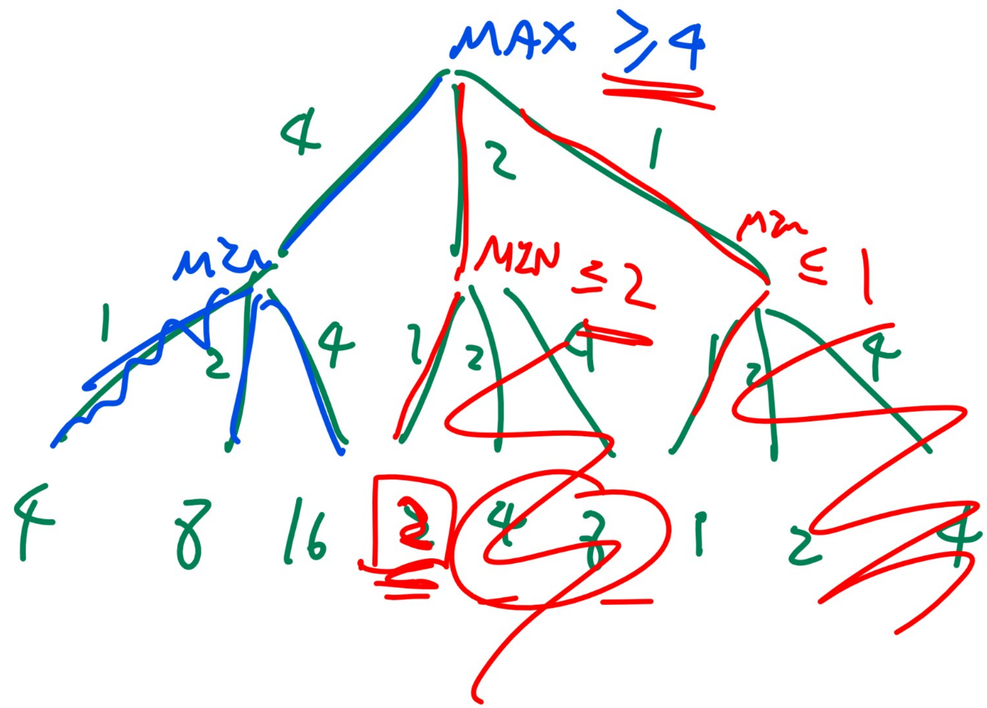
Quiz

- For a zero-sum game, the value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. Alpha-Beta pruning is used. What is the number of branches (states) that can be pruned if the actions with smaller labels are searched first?
- A : 0
- B : 1
- C : 2
- D : 3
- E : 4

Alpha Beta Example 4

Quiz

- For a zero-sum game, the value to the MAX player if MAX plays $x_1 \in \{1, 2, 4\}$ and MIN plays $x_2 \in \{1, 2, 4\}$ is $x_1 \cdot x_2$. Alpha-Beta pruning is used. What is the maximum number of branches (states) that can be pruned if the actions can be searched in any order?
- $A : 2$
- $B : 3$
- $C : 4$
- $D : 5$
- $E : 6$



Alpha Beta Example 4

Quiz

Alpha Beta Example 4 Continued

Quiz

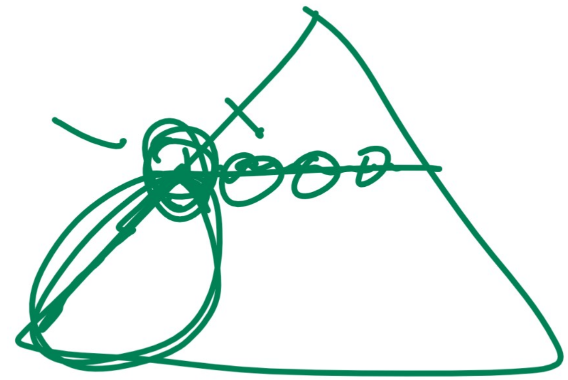
Alpha Beta Performance

Discussion

- In the best case, the best action of each player is the leftmost child.
- In the worst case, Alpha Beta is the same as minimax.

Static Evaluation Function

Definition



- A static board evaluation function is a heuristics to estimate the value of non-terminal states.
- It should reflect the player's chances of winning from that vertex.
- It should be easy to compute from the board configuration.

Linear Evaluation Function Example

Definition

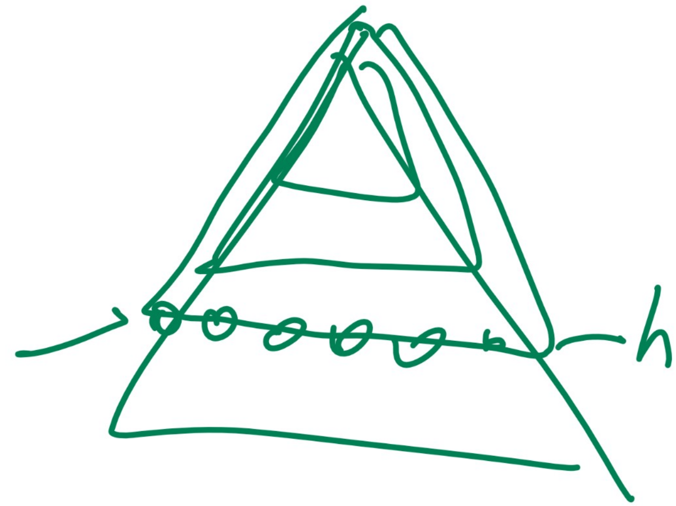
- For Chess, an example of an evaluation function can be a linear combination of the following variables.

1. Material.
2. Mobility.
3. King safety.
4. Center control.

- These are called the features of the board.

Iterative Deepening Search

Discussion



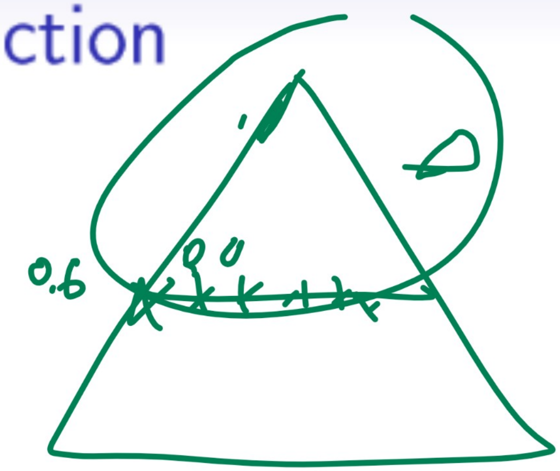
- IDS could be used with SBE.
- In iteration d, the depth is limited to d, and the SBE of the non-terminal vertices are used as their cost or reward.

IDS with SBE Diagram

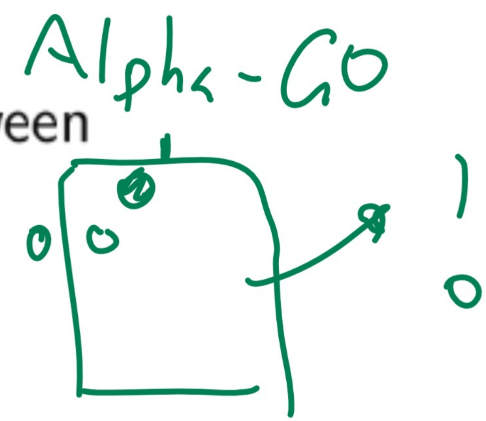
Discussion

Non Linear Evaluation Function

Discussion



- The SBE can be estimated given the features using a neural network.
- The features are constructed using domain knowledge, or a possibly a convolutional neural network.
- The training data are obtained from games between professional players.



Monte Carlo Tree Search

Discussion

- Simulate random games by selecting random moves for both players.
- Exploitation by keeping track of average win rate for each successor from previous searches and picking the successors that lead to more wins.
- Exploration by allowing random choices of unvisited successors.

Monte Carlo Tree Search Diagram

Discussion

back 7:40

Alpha GO Example

Discussion

- MCTS with $> 10^5$ play-outs.
- Convolutional neural network to compute SBE.

Summary

Discussion

- Adversarial Search:

1. Sequential Move Games: Minimax $\rightarrow$ DFS on the game tree.
2. Sequential Move Games: Alpha-Beta Pruning $\rightarrow$ DFS to keep track α and β $\rightarrow$ prune the subtree with $\alpha \Rightarrow \beta$.
3. Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability).
4. Simultaneous Move Games: Nash Equilibrium.